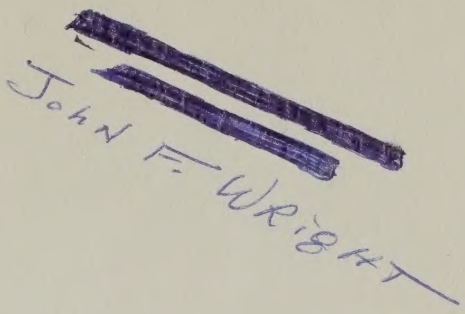
MATHEMATICS FOR TECHNICAL AND VOCATIONAL SCHOOLS

SLADE & MARGOLIS

4th edition


John F. Wright

MATHEMATICS

FOR TECHNICAL AND VOCATIONAL SCHOOLS

MATHEMATICS

for Technical and Vocational Schools

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and
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FOURTH EDITION

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PREFACE

In writing this book the authors have endeavored to produce a text on practical mathematics as applied to technical and trade work. Theoretical discussions and derivation of formulas have been eliminated or reduced to their simplest form. The large number of illustrative examples and problems will be found of great value for self-instruction as well as for class work. The tables in the back of the book have been chosen with an eye to assisting the student in solving the problems contained in this book.

The authors have found it advisable to include brief chapters on fractions and decimals, ratio and proportion, and percentage, in order to provide a rapid review of the essential parts of elementary school arithmetic which form the basis of all practical computation. In the chapter on algebra, the aim has been to enable the pupil to solve simple literal equations so that he can readily use the formulas that he needs in his work. In the chapter on geometry, no effort has been made to derive or explain the theorems upon which the constructions are based; the aim has been to show the pupil how to construct certain figures which are required in his work in the drafting room and the shop. In the chapter on trigonometry, nothing has been given but the essentials which are required for the solution of right and oblique triangles.

In the revision of the book for the fourth edition, the authors have incorporated many refinements and improvements suggested by the users of the third edition. The chapters on logarithms and graphs have been expanded and numerous problems added throughout the book to provide

additional drill and practice. The section on differential indexing has been rewritten. All problems involving costs of materials and labor have been revised to bring them into conformity with modern wage scales and costs of materials.

The authors wish to acknowledge their indebtedness to those who have taken the trouble to offer suggestions for improvement and to thank them for their constructive criticism.

SAMUEL SLADE
LOUIS MARGOLIS

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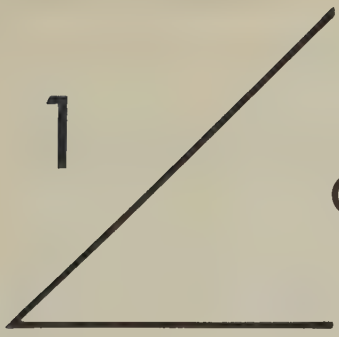


1 Common Fractions

1. Definitions

If we divide an inch into eight equal parts, each of the parts is one-eighth of an inch, written $\frac{1}{8}$ of an inch. Two such parts would be two-eighths of an inch, written $\frac{2}{8}$ of an inch; three such parts would be three-eighths of an inch, written $\frac{3}{8}$ of an inch; and so on. In each case the number 8, written below the short horizontal line, shows into how many parts the inch has been divided; while the number above the line shows how many of the parts have been taken. Thus in the expression $\frac{3}{8}$ of an inch, the number 8 below the line shows that the inch has been divided into eight equal parts; and the number 3 above the line shows that three of these equal parts are taken.

If we wish to express the fact that 15 is to be divided by 3, we write $15 \div 3 = 5$. A shorter way to indicate the division of 15 by 3 is to write it, $\frac{15}{3} = 5$. In this form the horizontal line of the division sign is retained, while the dots are omitted. It will be noticed that in the second form the dividend, or number that is to be divided, is written above the horizontal line; while the divisor, that is, the number which shows into how many parts the dividend is to be divided, is written below the horizontal line.

Such expressions as $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, and $\frac{15}{3}$ are called ***fractions.*** The number below the fraction line is called the ***denominator.*** The number above the fraction line is called the ***nu-***

merator. The numerator and denominator are called the *terms* of the fraction.

A fraction like $\frac{5}{8}$, whose value is less than unity, is called a *proper fraction.* In a proper fraction the numerator is less than the denominator.

A fraction like $\frac{8}{8}$, whose value is equal to unity, is called an *improper fraction.* Likewise a fraction like $\frac{9}{8}$, whose value is greater than unity, is called an improper fraction. In an improper fraction the numerator is equal to or greater than the denominator.

2. To Reduce Whole or Mixed Numbers to Improper Fractions

Example 1

Reduce 3 to fifths.

SOLUTION. $3 \times \frac{5}{5} = \frac{15}{5}$ EXPLANATION. Since every unit contains 5 fifths, three units will contain 3 times 5 fifths or 15 fifths, written $\frac{15}{5}$.

Definition

A mixed number is one consisting of a whole number and a fraction.

The number $5\frac{3}{8}$ is a mixed number consisting of the whole number 5 and the fraction $\frac{3}{8}$. It is read five and three-eighths and means 5 plus $\frac{3}{8}$.

Example 2

Reduce $5\frac{3}{8}$ to eighths.

SOLUTION.
$$\begin{array}{r} 5\frac{3}{8} \\ 8 \\ \hline \frac{43}{8} \end{array}$$

EXPLANATION OF PROCESS. Since in one unit there are 8 eighths, in 5 units there are 5 times 8 eighths, or 40 eighths. Hence in $5\frac{3}{8}$ there are 40 eighths plus 3 eighths or a total of 43 eighths, written $\frac{43}{8}$.

In practice we multiply the whole number by the denominator of the fraction, add the numerator, and write the result over the denominator.

PROBLEMS

1. How many halves in 3? in 4? in 5? in 12?
2. How many halves in $2\frac{1}{2}$? in $3\frac{1}{2}$? in $4\frac{1}{2}$? in $6\frac{1}{2}$? in $7\frac{1}{2}$?
3. How many eighths in 2? in 4? in 6? in 15?
4. How many eighths in $1\frac{1}{8}$? in $1\frac{7}{8}$? in $5\frac{3}{8}$? in $10\frac{5}{8}$? in $12\frac{1}{8}$?
5. How many tenths in 7? in 8? in 9? in 10?
6. How many tenths in $2\frac{3}{10}$? in $3\frac{7}{10}$? in $3\frac{9}{10}$? in $4\frac{9}{10}$?
7. How many sixteenths of an inch in 5 in.?
8. How many sixteenths of an inch in $3\frac{7}{16}$ in.? in $2\frac{1}{16}$ in.? in $2\frac{15}{16}$ in.?
9. Reduce the following mixed numbers to improper fractions: $1\frac{1}{8}$, $1\frac{1}{2}$, $1\frac{7}{8}$, $2\frac{1}{2}$, $3\frac{7}{16}$, $5\frac{7}{32}$.
10. Reduce the following mixed numbers to improper fractions: $8\frac{2}{9}$, $16\frac{2}{3}$, $4\frac{2}{7}$, $6\frac{1}{5}$, $92\frac{5}{8}$, $112\frac{17}{64}$.

3. To Reduce Improper Fractions to Whole or Mixed Numbers

Example 1

Reduce $\frac{24}{8}$ to a whole number.

SOLUTION. $\frac{24}{8} = 24 \div 8 = 3$ EXPLANATION. $\frac{24}{8}$ means 24 divided by 8; hence to reduce the fraction is to carry out the indicated division. In this case the answer is the whole number 3.

Example 2

Reduce $\frac{15}{4}$ to a mixed number.

SOLUTION. $\frac{15}{4} = 15 \div 4 = 3\frac{3}{4}$ EXPLANATION. $\frac{15}{4}$ means 15 divided by 4. Carrying out the indicated division, we get $3\frac{3}{4}$, which is a mixed number.

PROBLEMS

Reduce the following improper fractions to whole or mixed numbers.

1. $\frac{16}{8}$	2. $\frac{32}{16}$	3. $\frac{24}{4}$	4. $\frac{5}{2}$	5. $\frac{9}{8}$
6. $\frac{43}{16}$	7. $\frac{75}{32}$	8. $\frac{100}{64}$	9. $\frac{10}{3}$	10. $\frac{283}{23}$
11. $\frac{116}{12}$	12. $\frac{248}{35}$	13. $\frac{907}{180}$	14. $\frac{125664}{31416}$	15. $\frac{1187}{144}$
16. $\frac{3481}{1728}$	17. $\frac{2535}{231}$	18. $\frac{5475}{2150}$	19. $\frac{189}{12}$	20. $\frac{171}{24}$
21. $\frac{14092}{1760}$	22. $\frac{1988}{242}$	23. $\frac{1095}{360}$	24. $\frac{66076}{5280}$	25. $\frac{6247}{125}$

26. Change 73 oz. to pounds.
27. Change 47 in. to feet.
28. Change 40 sq. ft. to square yards.
29. How many cubic feet are there in 4380 cu. in.?
30. How many square feet are there in 315 sq. in.?

4. To Reduce a Fraction to Lowest Terms

Definition

A fraction is in its lowest terms when the numerator and denominator are prime to each other.

Two numbers are ***prime to each other*** when they have no common divisor or factor. For example, 5 and 7 are prime to each other because no number will divide both of them without a remainder. The numbers 10 and 12 are not prime to each other because both can be divided by 2 without a remainder.

Principle

Dividing both numerator and denominator of a fraction by the same number gives a fraction equal in value to the original fraction.

Example 1

Reduce $\frac{6}{8}$ to lowest terms.

SOLUTION. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$. EXPLANATION. Dividing both terms of the given fraction by 2 gives the fraction $\frac{3}{4}$. Since 3 and 4 contain no common divisor, the fraction $\frac{3}{4}$ is in its lowest terms.

Example 2

Reduce $\frac{70}{105}$ to lowest terms.

SOLUTION. $\frac{70 \div 5}{105 \div 5} = \frac{14}{21}$; $\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$;

or $\frac{70 \div 35}{105 \div 35} = \frac{2}{3}$

EXPLANATION. Dividing both terms of the given fraction by 5 gives the fraction $\frac{14}{21}$; dividing both terms of $\frac{14}{21}$ by 7 gives $\frac{2}{3}$. Or we can divide both terms of $\frac{70}{105}$ by 35, giving at once $\frac{2}{3}$. Since 2 and 3 contain no common divisor, the fraction $\frac{2}{3}$ is in its lowest terms.

PROBLEMS

NOTE. *All answers must be given with fractions in their lowest terms and improper fractions reduced to whole or mixed numbers.*

Reduce the following fractions to their lowest terms:

1. $\frac{2}{4}$	**2.** $\frac{6}{8}$	**3.** $\frac{8}{12}$	**4.** $\frac{9}{12}$	**5.** $\frac{18}{24}$
6. $\frac{4}{16}$	**7.** $\frac{10}{16}$	**8.** $\frac{12}{32}$	**9.** $\frac{24}{32}$	**10.** $\frac{24}{64}$
11. $\frac{56}{64}$	**12.** $\frac{36}{252}$	**13.** $\frac{2970}{5280}$	**14.** $\frac{96}{144}$	**15.** $\frac{432}{1728}$
16. $\frac{7854}{31416}$	**17.** $\frac{900}{360}$	**18.** $\frac{128}{24}$	**19.** $\frac{1485}{2673}$	**20.** $\frac{635}{1455}$

21. What part of a pound is 12 oz.?

22. Change 126 sq. in. to a fraction of a square foot.

23. Change 21 in. to a fraction of a yard.

24. What part of a cubic foot is 972 cu. in.?

25. What fraction of a cubic yard is 24 cu. ft.?

5. To Change a Fraction with a Given Denominator to One with a Higher Denominator

Example

Change $\frac{3}{4}$ to a fraction having 20 for its denominator.

Principle

Multiplying both terms of a fraction by the same number gives a fraction equal in value to the original fraction.

SOLUTION. $\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

EXPLANATION. Since the required denominator of the new fraction is 20, we must multiply both numerator and denominator by a number which will make the new denominator 20. It is evident that 5 is the required multiplier.

PROBLEMS

Convert the given fractions to equivalent fractions having the required higher denominators.

1. $\frac{1}{2}$ to 4ths	**2.** $\frac{1}{2}$ to 10ths	**3.** $\frac{3}{8}$ to 32nds
4. $\frac{5}{8}$ to 64ths	**5.** $\frac{2}{3}$ to 12ths	**6.** $\frac{3}{5}$ to 10ths
7. $\frac{3}{5}$ to 15ths	**8.** $\frac{5}{7}$ to 35ths	**9.** $\frac{11}{12}$ to 180ths
10. $\frac{5}{13}$ to 52nds	**11.** $\frac{23}{24}$ to 144ths	**12.** $\frac{23}{24}$ to 360ths
13. $\frac{11}{18}$ to 360ths	**14.** $\frac{11}{18}$ to 144ths	**15.** $\frac{9}{16}$ to 64ths
16. $\frac{9}{16}$ to 32nds	**17.** $\frac{9}{16}$ to 144ths	**18.** $\frac{5}{6}$ to 60ths
19. $\frac{3}{4}$ to 60ths	**20.** $\frac{2}{5}$ to 60ths	**21.** $\frac{5}{4}$ to 16ths
22. $\frac{7}{10}$ to 100ths	**23.** $\frac{13}{8}$ to 32nds	**24.** $\frac{7}{5}$ to 60ths

6. To Reduce Two or More Fractions to Equivalent Fractions Having a Common Denominator

Example 1

Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{16}$ to equivalent fractions having the lowest common denominator.

The lowest common denominator (L. C. D.) of the desired fractions will be the least common multiple (L. C. M.) of the given denominators. Very often, as in this case, the least common multiple can be obtained at sight. It is readily seen that the L. C. M. of 2, 4, 8, and 16 is 16; that is, 16 is the lowest number that will evenly contain 2, 4, 8, and 16.

SOLUTION.
$$\frac{1}{2} = \frac{8}{16}$$
$$\frac{3}{4} = \frac{12}{16}$$
$$\frac{5}{8} = \frac{10}{16}$$
$$\frac{5}{16} = \frac{5}{16}$$

EXPLANATION. Beside each of the given fractions write a fraction line with the L. C. D. 16 below it. Then, to obtain the new numerator, divide the L. C. D. by the given denominator and multiply the result by the given numerator.

Thus

2 into 16 gives 8; 8 times 1 gives 8, the new numerator.
4 into 16 gives 4; 4 times 3 gives 12, the new numerator.
8 into 16 gives 2; 2 times 5 gives 10, the new numerator.
16 into 16 gives 1; 1 times 5 gives 5, the new numerator.

When the least common multiple of two or more numbers cannot be obtained at sight, we proceed as follows:

Example 2

Reduce $\frac{5}{8}$, $\frac{7}{12}$, $\frac{11}{18}$ to equivalent fractions with the lowest common denominator.

SOLUTION.

(1)	2	8, 12, 18
(2)	2	4, 6, 9
(3)	2	2, 3, 9
(4)	3	1, 3, 9
(5)	3	1, 1, 3
(6)		1, 1, 1

EXPLANATION. To obtain the L. C. M. of 8, 12, and 18, write the numbers in a line as shown and divide by the smallest number that will go into one or more of the given numbers without a remainder. Thus 2 will go into 8 four times; into 12, six times; and into 18, nine times. Write the 4 under the 8, the 6 under the 12, and the 9 under the 18. We thus obtain the second line of numbers. We now divide each of these numbers by 2, and where a number does not contain the divisor an even number of times, bring down the number itself. Thus, since 2 goes into 4 twice, put the quotient 2 under the 4; since 2 goes into 6 three times, put 3 under 6; since 2 does not go evenly into 9, bring down the 9. In this way we obtain the third line. Dividing the numbers in line (3) by 2, we get line (4); dividing the numbers in line (4) by 3, we get line (5); dividing the numbers in line (5) by 3, we get line (6).

The L. C. M. of 8, 12, and 18 is the continued product of the divisors 2, 2, 2, 3, and 3; *i.e.*, 72. Therefore 72 will be the least common denominator of the given fractions.

Hence $\frac{5}{8} = \frac{45}{72}$, $\frac{7}{12} = \frac{42}{72}$, $\frac{11}{18} = \frac{44}{72}$.

PROBLEMS

In each of the following cases, change the given fractions to equivalent fractions having a least common denominator.

1. $\frac{1}{2}, \frac{3}{4}$
2. $\frac{1}{2}, \frac{1}{3}$
3. $\frac{3}{4}, \frac{5}{8}, \frac{7}{16}$
4. $\frac{2}{3}, \frac{3}{4}, \frac{3}{5}$
5. $\frac{7}{12}, \frac{3}{8}, \frac{3}{10}$
6. $\frac{9}{24}, \frac{11}{18}, \frac{23}{36}$
7. $\frac{2}{7}, \frac{20}{21}, \frac{25}{28}$
8. $\frac{5}{8}, \frac{3}{35}, \frac{7}{30}, \frac{5}{28}, \frac{17}{24}$
9. $\frac{3}{8}, \frac{13}{24}, \frac{21}{32}, \frac{17}{48}$
10. $\frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{6}{11}, \frac{8}{13}$

11. Which is greater, $\frac{3}{4}$ or $\frac{4}{5}$? $\frac{6}{7}$ or $\frac{7}{8}$? $\frac{7}{9}$ or $\frac{19}{24}$?
12. How many 32nds in each of the following? $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{16}$.

7. Addition of Fractions

It is evident that to add fractions having like denominators, that is, a common denominator, all we have to do is to add their numerators and place the sum over the common denominator. Thus $\frac{3}{8} + \frac{5}{8} + \frac{7}{8}$ means 3 eighths + 5 eighths + 7 eighths = 15 eighths = $\frac{15}{8} = 1\frac{7}{8}$. But when we are asked to add fractions with different denominators, as $\frac{3}{4} + \frac{7}{8} + \frac{5}{6}$, it is evident we cannot add fourths and eighths and sixths. In this case we must first change the given fractions to equivalent fractions having a common denominator. For the sake of convenience we find the least common denominator. This we do as explained in Article 6 as follows:

2	4,	8, 6
2	2,	4, 3
2	1,	2, 3
3	1,	1, 3
	1,	1, 1

The L. C. M. of 4, 8, and 6 is $2 \times 2 \times 2 \times 3 = 24$. Hence 24 is the L. C. D. of the three given fractions. We now reduce each of the given fractions to a fraction having 24 for its denominator, thus:

$\frac{3}{4} = \frac{18}{24}$
$\frac{7}{8} = \frac{21}{24}$
$\frac{5}{6} = \frac{20}{24}$

$\frac{59}{24} = 2\frac{11}{24}$

The sum of 18 twenty-fourths and 21 twenty-fourths and 20 twenty-fourths is evidently 18 + 21 + 20 = 59 twenty-fourths, written $\frac{59}{24}$, and reduced to the mixed number $2\frac{11}{24}$.

The addition of the fractions, after obtaining the L. C. D., may be simplified by omitting the L. C. D. and writing only the new numerator after each of the given fractions, thus:

$\frac{3}{4} = 18$
$\frac{7}{8} = 21$
$\frac{5}{6} = 20$

$\frac{59}{24} = 2\frac{11}{24}$

This method not only saves the time of repeating the new denominator, but makes the addition of the new numerators easier.

PROBLEMS

Find the sum of:

1. $\frac{1}{8} + \frac{3}{8} + \frac{5}{8} + \frac{7}{8}$
2. $\frac{3}{16} + \frac{7}{16} + \frac{5}{16} + \frac{9}{16} + \frac{11}{16}$
3. $\frac{29}{21} + \frac{13}{21} + \frac{5}{21} + \frac{38}{21}$
4. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8} + \frac{7}{16}$

5. $\frac{9}{16} + \frac{7}{8} + \frac{15}{32}$
6. $\frac{7}{12} + \frac{3}{10} + \frac{8}{15}$
7. $\frac{1}{8} + \frac{5}{12} + \frac{11}{18} + \frac{23}{24}$
8. $\frac{3}{7} + \frac{5}{14} + \frac{7}{18} + \frac{1}{9}$
9. $\frac{2}{3} + \frac{3}{4} + \frac{5}{7} + \frac{7}{12}$
10. $\frac{7}{8} + \frac{2}{3} + \frac{2}{7} + \frac{3}{5}$
11. $\frac{2}{5} + \frac{5}{9} + \frac{1}{8} + \frac{7}{20}$
12. $\frac{5}{6} + \frac{3}{14} + \frac{3}{4} + \frac{11}{12}$
13. $\frac{3}{5} + \frac{3}{7} + \frac{3}{8} + \frac{3}{10}$
14. $\frac{15}{16} + \frac{8}{3} + \frac{2}{3}$
15. $\frac{9}{28} + \frac{31}{49} + \frac{16}{21} + \frac{32}{35}$

8. Addition of Mixed Numbers

Example

Find the sum of $5\frac{3}{8}$, $6\frac{1}{4}$, $2\frac{1}{3}$, $7\frac{1}{2}$.

It is evident that the sum of the mixed numbers is equal to the sum of the whole numbers added to the sum of the fractions. The work may conveniently be arranged as shown.

SOLUTION.

$5\frac{3}{8}$	$\frac{9}{24}$
$6\frac{1}{4}$	$\frac{6}{24}$
$2\frac{1}{3}$	$\frac{8}{24}$
$7\frac{1}{2}$	$\frac{12}{24}$
$21\frac{11}{24}$	$\frac{35}{24} = 1\frac{11}{24}$

EXPLANATION. The sum of the fractions is found to be $1\frac{11}{24}$. Put down $\frac{11}{24}$ and add the 1 to the whole numbers, giving 21. The complete answer is therefore $21\frac{11}{24}$.

PROBLEMS

Find the sum of:

1. $2\frac{1}{2} + 5\frac{1}{2} + 3\frac{1}{2} + 7\frac{1}{2}$
2. $1\frac{3}{8} + 5\frac{7}{8} + 3\frac{5}{8} + 6\frac{7}{8}$
3. $3\frac{7}{16} + 4\frac{5}{16} + 2\frac{11}{16} + 8\frac{9}{16}$
4. $9\frac{3}{4} + 6\frac{1}{2} + 4\frac{3}{8}$
5. $5\frac{3}{4} + 2\frac{1}{8} + 3\frac{1}{16} + 3\frac{7}{32}$
6. $5\frac{3}{8} + 4\frac{1}{4} + 8\frac{11}{12}$
7. $2\frac{1}{5} + 8\frac{2}{3} + 10\frac{3}{4} + 4\frac{3}{7}$
8. $9\frac{5}{6} + 12\frac{3}{4} + 10\frac{15}{16} + 4\frac{11}{12}$
9. $6\frac{7}{16} + \frac{7}{8} + \frac{2}{3} + 3\frac{3}{4}$
10. $8\frac{1}{2} + 6\frac{1}{3} + 5\frac{1}{4} + 7\frac{1}{6} + 12\frac{1}{18}$
11. $4 + 11\frac{3}{5} + \frac{7}{8} + 3\frac{1}{2}$
12. $8\frac{1}{2} + 4\frac{5}{6} + 3\frac{2}{3} + 2\frac{1}{6}$
13. $14\frac{3}{10} + 8\frac{7}{15} + 6\frac{9}{20} + 2\frac{17}{30}$
14. $1\frac{11}{16} + 15 + 12 + 3\frac{7}{8}$
15. $\frac{19}{32} + \frac{11}{16} + 18 + 12$
16. $5\frac{2}{3} + \frac{49}{240} + 4\frac{7}{120} + 8\frac{21}{80}$
17. Add the following dimensions: $1\frac{7}{8}''$, $2\frac{1}{4}''$, $\frac{9}{64}''$, $\frac{11}{32}''$, $3\frac{1}{2}''$.
18. Add the following weights: $1\frac{1}{4}$ lb., $\frac{1}{2}$ lb., $\frac{7}{16}$ lb., $1\frac{5}{16}$ lb., $\frac{7}{8}$ lb.
19. Add the following dimensions: $6\frac{1}{4}''$, $1\frac{3}{64}''$, $3\frac{7}{8}''$, $2\frac{11}{32}''$, $\frac{1}{2}''$, $1\frac{9}{16}''$, $5\frac{5}{8}''$.
20. Add the following dimensions: $\frac{15}{64}''$, $\frac{13}{32}''$, $\frac{5}{8}''$, $\frac{53}{64}''$, $1\frac{7}{16}''$, $\frac{35}{64}''$, $1\frac{9}{16}''$, $1\frac{3}{4}''$.

9. Subtraction of Fractions

Example 1

From $\frac{7}{9}$ take $\frac{3}{9}$.

SOLUTION. Since the two fractions have a common denominator, we need only take the numerator of the subtrahend, 3, from the numerator of the minuend, 7, and write the result over the common denominator; because it is evident that if 3 ninths is taken from 7 ninths the remainder is 4 ninths. We therefore write, $\frac{7}{9} - \frac{3}{9} = \frac{7-3}{9} = \frac{4}{9}$; or, in a single step, $\frac{7}{9} - \frac{3}{9} = \frac{4}{9}$.

Example 2

From $\frac{7}{8}$ take $\frac{2}{3}$.

SOLUTION. Since the fractions have different denominators, we must first reduce them to fractions having a common denominator.

$\frac{7}{8}$	$\frac{21}{24}$
$\frac{2}{3}$	$\frac{16}{24}$
	$\frac{5}{24}$

The lowest common denominator of the two fractions is the least common multiple of 8 and 3, namely, 24. Hence the given fractions become $\frac{21}{24}$ and $\frac{16}{24}$. The problem is now similar to Example 1. It is evident that 21 twenty-fourths less 16 twenty-fourths leaves $\frac{5}{24}$.

PROBLEMS

1. $\frac{9}{16} - \frac{5}{16}$
2. $\frac{7}{8} - \frac{3}{8}$
3. $\frac{3}{4} - \frac{1}{2}$
4. $\frac{29}{32} - \frac{5}{8}$
5. $\frac{1}{2} - \frac{1}{3}$
6. $\frac{4}{5} - \frac{3}{4}$
7. $\frac{7}{8} - \frac{5}{16}$
8. $\frac{13}{15} - \frac{9}{20}$
9. $\frac{14}{15} - \frac{7}{12}$
10. $\frac{5}{12} - \frac{5}{18}$
11. $\frac{7}{8} - \frac{3}{10}$
12. $\frac{1}{12} - \frac{1}{144}$
13. How much is $\frac{5}{8}$ more than $\frac{1}{2}$?
14. How much is $\frac{3}{4}$ more than $\frac{2}{3}$?
15. How much is $\frac{1}{4}$ less than $\frac{1}{3}$?
16. How much is $\frac{3}{5}$ less than $\frac{5}{7}$?
17. A planer takes a $\frac{3}{32}''$ cut on a piece of steel $\frac{3}{8}''$ thick. What is the remaining thickness?
18. By how much must the diameter of a $\frac{3}{4}''$ shaft be reduced to bring it down to a diameter of $\frac{41}{64}''$?

19. What is the difference in thickness between a $\frac{9}{16}''$ plate and a $\frac{7}{8}''$ plate?

20. A steel casting $\frac{7}{8}''$ thick is finished by taking a cut $\frac{3}{64}''$ deep. What is the final thickness?

10. Subtraction of Mixed Numbers

Example 1

From $15\frac{5}{6}$ take $7\frac{3}{4}$.

$15\frac{5}{6}$	$\frac{10}{12}$
$7\frac{3}{4}$	$\frac{9}{12}$
$8\frac{1}{12}$	$\frac{1}{12}$

SOLUTION. This case is similar to that of adding mixed numbers. First find the difference between the fractions and then find the difference between the whole numbers. The complete answer consists of the two results thus obtained, written as a mixed number: $\frac{5}{6} - \frac{3}{4} = \frac{1}{12}$; $15 - 7 = 8$. Answer $8\frac{1}{12}$.

Example 2

From $8\frac{3}{8}$ take $5\frac{2}{3}$.

7	
~~8~~ $\frac{3}{8}$	$\frac{33}{24}$
$5\frac{2}{3}$	$\frac{16}{24}$
$2\frac{17}{24}$	$\frac{17}{24}$

SOLUTION. In this case, when we try to take $\frac{2}{3}$ from $\frac{3}{8}$ we find it impossible, because when reduced to a common denominator, $\frac{3}{8} = \frac{9}{24}$, while $\frac{2}{3} = \frac{16}{24}$. It is obvious we cannot take 16 from 9. Hence we borrow a unit from the whole number 8, and we change this borrowed unit to twenty-fourths, giving $\frac{24}{24}$. This we add to the $\frac{9}{24}$, giving $\frac{24+9}{24} = \frac{33}{24}$. Instead of having $8\frac{9}{24}$ in our minuend, we have its equivalent number $7\frac{33}{24}$. To take $5\frac{16}{24}$ from $7\frac{33}{24}$, we take 5 from 7, giving 2, and $\frac{16}{24}$ from $\frac{33}{24}$, giving $\frac{17}{24}$. The answer therefore is $2\frac{17}{24}$.

PROBLEMS

1. $9\frac{7}{8} - 5$	**2.** $6\frac{3}{4} - 2$	**3.** $5\frac{19}{32} - 2\frac{13}{32}$
4. $18\frac{15}{16} - 4\frac{9}{16}$	**5.** $12\frac{5}{8} - 8\frac{1}{4}$	**6.** $2 - \frac{1}{3}$
7. $4 - \frac{3}{8}$	**8.** $9 - \frac{7}{16}$	**9.** $6\frac{1}{2} - 2\frac{1}{8}$
10. $3\frac{2}{5} - 1\frac{1}{3}$	**11.** $10\frac{1}{3} - 7\frac{3}{4}$	**12.** $5\frac{2}{5} - 3\frac{7}{8}$
13. $14\frac{7}{18} - 8\frac{19}{24}$	**14.** $32\frac{5}{16} - 21\frac{11}{12}$	**15.** $5\frac{3}{8} - \frac{7}{8}$
16. $6\frac{3}{4} - \frac{15}{16}$	**17.** $8\frac{3}{7} - 6\frac{7}{9}$	**18.** $16\frac{25}{64} - 10\frac{95}{144}$
19. $4\frac{1}{4} - 2\frac{3}{5}$	**20.** $8\frac{1}{3} - 6\frac{5}{7}$	

21. Simplify $4\frac{1}{3} + 3\frac{1}{5} - 2\frac{1}{4} - 1\frac{1}{8}$.

22. Simplify $16\frac{3}{4} - 2\frac{1}{2} - 4\frac{3}{16} + 2\frac{7}{16}$.

23. Find the value of $5\frac{3}{7} + 8\frac{7}{10} - 3\frac{1}{3} - 1\frac{1}{5}$.

24. Find the value of $8\frac{3}{4} - 1\frac{7}{16} + 4\frac{1}{8} - 2\frac{1}{3}$.

25. From a bar of brass $16\frac{1}{2}''$ long the following three pieces are cut: $1\frac{1}{8}''$, $3\frac{1}{2}''$, $3\frac{3}{32}''$. What is the final length of the bar, allowing $\frac{1}{16}''$ for each cut?

26. Allowing $\frac{1}{8}''$ for each cut, what must be the length of a bar of brass to provide the following four pieces: $1\frac{7}{8}''$, $3\frac{3}{16}''$, $\frac{11}{32}''$, and $2\frac{1}{4}''$?

27. Add the following dimensions: $2\frac{7}{16}''$, $3\frac{1}{8}''$, $5\frac{9}{64}''$, $4\frac{1}{32}''$, $\frac{1}{2}''$.

28. A shaft which is required to be $\frac{11}{16}''$ in diameter measures $\frac{49}{64}''$. How much is it oversize?

29. A planer takes a $\frac{3}{32}''$ cut on a plate which is $1\frac{3}{4}''$ thick. What is the final thickness of the plate?

30. A piece of tapered stock has a diameter of $1\frac{1}{16}''$ at one end and $\frac{57}{64}''$ at the other end. Find the difference between the two diameters.

11. Multiplication of Fractions—Cancellation

Example 1

Multiply $\frac{3}{4}$ by 5.

SOLUTION. $\frac{3}{4} \times 5 = \frac{3 \times 5}{4} = \frac{15}{4} = 3\frac{3}{4}$;

or, more briefly, $\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$

EXPLANATION. In this case the multiplier is 5, a whole number. To obtain the product, multiply the numerator of the given fraction by the whole number and place the result over the denominator. Thus $3 \times 5 = 15$, the numerator of the product which is written over the denominator 4, giving $\frac{15}{4}$ or $3\frac{3}{4}$.

Example 2

Multiply $\frac{2}{3}$ by $\frac{5}{7}$.

SOLUTION. $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$;

or, in one step, $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

EXPLANATION. To multiply one fraction by another, multiply the numerators together and place the result over the product of

the denominators. In the example above, $2 \times 5 = 10$, the numerator of the product, and $3 \times 7 = 21$, the denominator of the product.

Example 3

Multiply $\frac{3}{4}$ by $\frac{8}{9}$.

This example, being similar to Example 2, may be done in the same way.

$\frac{3}{4} \times \frac{8}{9} = \frac{24}{36}$. This answer is reduced to lowest terms by dividing both numerator and denominator by 12, thus: $\frac{24}{36} = \frac{24 \div 12}{36 \div 12} = \frac{2}{3}$. The answer $\frac{2}{3}$ could be obtained more conveniently by the method of cancellation. ***Cancellation*** consists in striking out the factors that are common to both numerators and denominators of the given fractions, thus: $\frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$. In this case we can *divide* or *cancel* 3 into the 3 of the numerator, giving a quotient of 1, and 3 into the 9 of the denominator, giving a quotient of 3. Likewise we can divide 4 into 8, giving 2, and 4 into 4 of the denominator, giving 1. The product of the remaining numerators, 1 and 2, gives the numerator of the answer, and the product of the remaining denominators gives the denominator of the answer.

Example 4

Multiply $\frac{8}{11} \times \frac{5}{12}$.

SOLUTION. $\frac{\cancel{8}^{2}}{11} \times \frac{5}{\cancel{12}_{3}} = \frac{2 \times 5}{11 \times 3} = \frac{10}{33}$

EXPLANATION. In this case no whole numerator can be cancelled into a denominator as was done in Example 3, but we can cancel out 4 from the numerator 8 leaving 2, and from the denominator 12, leaving 3. The answer is obtained by multiplying all the remaining numerators for a new numerator and all the re-

maining denominators for a new denominator. Thus $2 \times 5 = 10$, the numerator of the product, and $11 \times 3 = 33$, the denominator of the product, giving the fraction $\frac{10}{33}$.

Example 5

$2\frac{1}{2} \times 3\frac{1}{4}$

SOLUTION. $2\frac{1}{2} = \frac{5}{2}$ and $3\frac{1}{4} = \frac{13}{4}$. Hence $2\frac{1}{2} \times 3\frac{1}{4} = \frac{5}{2} \times \frac{13}{4} = \frac{65}{8} = 8\frac{1}{8}$.

EXPLANATION. When the quantities to be multiplied are mixed numbers, we must first change them to improper fractions, and then proceed as in Example 4.

Example 6

$\frac{2}{3}$ of $\frac{4}{5}$

SOLUTION. $$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

EXPLANATION. The word "of" has exactly the same meaning as the multiplication sign ($\times$). Hence $\frac{2}{3}$ of $\frac{4}{5}$ means $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

Example 7

$\frac{3}{4} \times \frac{5}{8} \times \frac{2}{7}$

SOLUTION. $$\frac{3}{\cancel{4}_{2}} \times \frac{5}{8} \times \frac{\cancel{2}^{1}}{7} = \frac{3 \times 5 \times 1}{2 \times 8 \times 7} = \frac{15}{112}$$

EXPLANATION. Dividing 2 into the 2 of the numerator gives 1, and 2 into the 4 of the denominator gives 2. The product of the remaining numerators, $3 \times 5 \times 1 = 15$, the numerator of the answer; the product of the remaining denominators, $2 \times 8 \times 7 = 112$, the denominator of the answer. Hence the answer is $\frac{15}{112}$.

PROBLEMS

1. $\frac{5}{8} \times 9$	**2.** $\frac{2}{3} \times 7$	**3.** $14 \times \frac{5}{7}$
4. $36 \times \frac{2}{9}$	**5.** $\frac{1}{3} \times \frac{1}{2}$	**6.** $\frac{3}{4} \times \frac{3}{5}$
7. $\frac{7}{8} \times \frac{3}{10}$	**8.** $\frac{5}{8} \times \frac{24}{50}$	**9.** $\frac{7}{12} \times \frac{36}{49}$
10. $\frac{2}{3} \times \frac{9}{16}$	**11.** $\frac{16}{25} \times \frac{35}{56}$	**12.** $\frac{14}{27} \times \frac{18}{49}$

13. $\frac{15}{28} \times \frac{16}{21}$ 14. $1\frac{1}{3} \times 4\frac{2}{5}$ 15. $3\frac{2}{7} \times 8\frac{3}{4}$
16. $5\frac{5}{6} \times 2\frac{1}{2}$ 17. $16\frac{2}{3} \times 12$ 18. $10\frac{3}{8} \times 6$
19. $\frac{1}{2}$ of $\frac{1}{4}$ 20. $\frac{1}{2} \times \frac{1}{2}$ 21. $\frac{1}{2}$ of $\frac{1}{2}$
22. $\frac{1}{3} \times \frac{1}{3}$ 23. $\frac{1}{4}$ of $\frac{1}{4}$ 24. $\frac{2}{3}$ of $\frac{5}{8}$
25. $\frac{5}{9}$ of $\frac{18}{35}$ 26. $\frac{2}{7}$ of 28 27. $\frac{3}{4}$ of 30
28. $\frac{3}{4} \times \frac{4}{9} \times \frac{6}{11} \times 3\frac{1}{7}$
29. $2\frac{1}{8} \times 4\frac{1}{3} \times 3\frac{1}{4} \times 1\frac{1}{2}$
30. $\frac{9}{16}$ of $10\frac{1}{2} \times \frac{2}{3}$ of $12\frac{7}{8}$
31. $2\frac{2}{5} \times 7\frac{1}{2} \times 3\frac{1}{3} \times 2\frac{1}{10}$
32. What is the total length of 5 pieces of drill rod each $2\frac{9}{16}''$ long?
33. How many cubic inches in a block of steel $8\frac{1}{2}''$ long, $2\frac{1}{8}''$ wide, and $1\frac{3}{4}''$ thick?
34. To find the circumference of a circle we multiply the diameter by $3\frac{1}{7}$. Find the circumference of a circle whose diameter is $2\frac{3}{4}''$.

12. Division of Fractions

Example 1

Divide $\frac{5}{7}$ by $\frac{3}{4}$.

SOLUTION. $\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \times \frac{4}{3} = \frac{20}{21}$

EXPLANATION. The divisor $\frac{3}{4}$ is ***inverted;*** that is, the denominator is made the numerator and the numerator is made the denominator, giving the fraction $\frac{4}{3}$. We then multiply the dividend $\frac{5}{7}$ by $\frac{4}{3}$ as if it were a problem in multiplication and get the result $\frac{20}{21}$. Where the divisor is a whole number, we follow the same rule of inverting the divisor and multiplying. A whole number is inverted by writing 1 over it.

Example 2

$\frac{2}{3} \div 5$

SOLUTION. $\frac{2}{3} \div \frac{5}{1} = \frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

Example 3

Divide $4\frac{5}{8}$ by $2\frac{1}{3}$.

SOLUTION. Changing each of the fractions into an improper fraction, we get $4\frac{5}{8} = \frac{37}{8}$ and $2\frac{1}{3} = \frac{7}{3}$.

$$4\frac{5}{8} \div 2\frac{1}{3} = \frac{37}{8} \div \frac{7}{3} = \frac{37}{8} \times \frac{3}{7} = \frac{111}{56} = 1\frac{55}{56}$$

Example 4

Divide 7 by $\frac{2}{5}$.

SOLUTION. $7 \div \frac{2}{5} = \frac{7}{1} \times \frac{5}{2} = \frac{35}{2} = 17\frac{1}{2}$

PROBLEMS

Find the quotient of:

1. $\frac{18}{25} \div 3$	**2.** $\frac{12}{9} \div 4$	**3.** $\frac{7}{8} \div 5$	**4.** $\frac{3}{4} \div 7$
5. $\frac{9}{16} \div \frac{3}{8}$	**6.** $\frac{11}{12} \div \frac{5}{7}$	**7.** $7 \div \frac{1}{2}$	**8.** $4\frac{1}{2} \div \frac{1}{4}$
9. $3\frac{3}{4} \div \frac{1}{5}$	**10.** $8 \div \frac{2}{3}$	**11.** $15 \div \frac{3}{8}$	**12.** $18\frac{2}{3} \div 6$
13. $12\frac{3}{4} \div 8$	**14.** $10\frac{7}{16} \div \frac{3}{4}$	**15.** $4\frac{1}{3} \div \frac{2}{5}$	**16.** $5 \div 4\frac{2}{3}$
17. $7 \div 3\frac{1}{8}$	**18.** $6 \div 2\frac{1}{4}$	**19.** $2\frac{1}{3} \div 3\frac{1}{2}$	**20.** $4\frac{1}{16} \div 8\frac{1}{8}$
21. $12\frac{5}{6} \div 10\frac{3}{8}$	**22.** $3\frac{1}{10} \div 18\frac{1}{3}$	**23.** $8\frac{8}{9} \div 6\frac{4}{5}$	**24.** $20\frac{1}{2} \div 16\frac{2}{3}$
25. $6\frac{1}{5} \div 5\frac{1}{4}$	**26.** $3\frac{1}{16} \div 4\frac{7}{8}$	**27.** $7\frac{3}{10} \div 12\frac{3}{4}$	

28. The diameter of a circle is found by dividing the circumference by $3\frac{1}{7}$. What is the diameter of a circle whose circumference is $26\frac{3}{4}''$?

29. A space $61\frac{1}{2}''$ long is to be divided into 14 equal parts. What is the length of each space?

30. A cubic foot contains approximately $7\frac{1}{2}$ gallons. How many cubic feet are there in a tank which contains $68\frac{1}{4}$ gallons?

31. The diagonal of a square may be found by dividing the side by $\frac{707}{1000}$. How long is the diagonal of a $1\frac{3}{4}''$ square?

13. Problems Involving Multiplication and Division

Example

Find the value of $\frac{2}{3} \times \frac{7}{8} \div 3\frac{1}{2}$.

SOLUTION. $\frac{\cancel{2}^{1}}{3} \times \frac{\cancel{7}^{1}}{\cancel{8}_{\cancel{4}_{2}}} \times \frac{\cancel{2}^{1}}{7} = \frac{1}{6}$

EXPLANATION. First we change the mixed number $3\frac{1}{2}$ to the improper fraction $\frac{7}{2}$. Then, since we must divide by $\frac{7}{2}$, we invert it and multiply by $\frac{2}{7}$. We cancel the 7 from numerator and denominator, and we cancel the 2 of the numerator into the 8 of the denominator, giving 4. We then cancel the other 2 of the numerator into the 4 of the denomi-

nator, giving 2. The numerator of the answer is the product of the remaining numerators, $1 \times 1 \times 1 = 1$. The denominator of the answer is the product of the remaining denominators, $3 \times 2 \times 1 = 6$. The answer therefore is $\frac{1}{6}$.

PROBLEMS

1. $\frac{2}{5} \times \frac{1}{3} \div \frac{3}{4}$
2. $\frac{7}{8} \times \frac{1}{2} \div \frac{1}{7}$
3. $2\frac{1}{3} \times \frac{5}{8} \div 1\frac{1}{4}$
4. $3\frac{1}{5} \times 1\frac{1}{2} \times 2\frac{1}{2} \div 4\frac{2}{3}$
5. $\frac{2}{3} \times \frac{15}{16} \times \frac{7}{8} \div \frac{1}{2}$
6. $\frac{2}{7} \times \frac{5}{9} \times \frac{3}{10} \div 4$
7. $2 \times \frac{5}{6} \times 6 \div 4\frac{7}{10}$
8. $3\frac{1}{7} \times 2 \div 4$
9. $5 \times 5 \times 3\frac{1}{7} \div 144$
10. $18\frac{2}{3} \times 16\frac{1}{2} \div 144$
11. $10\frac{3}{4} \times 12\frac{1}{8} \times 14\frac{1}{2} \div 1728$
12. $2\frac{1}{8} \times 2\frac{1}{8} \times 3\frac{1}{7} \div 144$
13. $1\frac{1}{2} \times 1\frac{1}{3} \times 2\frac{1}{8} \times 450$
14. $\frac{3}{4} \times 1728 \times 490$
15. $15\frac{1}{4} \times \frac{1}{2} \div 3\frac{1}{7}$

14. Complex Fractions

Example 1

Find the value of the expression $\dfrac{\frac{2}{3}}{\frac{5}{7}}$.

An expression like this is called a complex fraction. A ***complex fraction*** is one in which the numerator or the denominator or both are fractions or mixed numbers. The student will recall that in the first section of this chapter it was pointed out that a fraction may be regarded as an indicated division, in which the numerator is the dividend and the denominator is the divisor. Hence to evaluate the given expression, we need only perform the indicated division, thus:

$$\frac{\frac{2}{3}}{\frac{5}{7}} = \frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$$

Example 2

$$\frac{2\frac{3}{4}}{16}$$

SOLUTION. $2\frac{3}{4} \div \frac{16}{1} = \frac{11}{4} \times \frac{1}{16} = \frac{11}{64}$

Example 3

$$\frac{3\frac{1}{4} - 2\frac{1}{8}}{4\frac{1}{8} - 1\frac{1}{2}}$$

SOLUTION. $3\frac{1}{4} - 2\frac{1}{8} = 1\frac{1}{8}$; $4\frac{1}{8} - 1\frac{1}{2} = 2\frac{5}{8}$;

therefore, $$\frac{3\frac{1}{4} - 2\frac{1}{8}}{4\frac{1}{8} - 1\frac{1}{2}} = \frac{1\frac{1}{8}}{2\frac{5}{8}} = \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{1}{\cancel{8}}}{\underset{7}{\cancel{21}}} = \frac{3}{7}$$

EXPLANATION. First simplify the numerator by performing the indicated subtraction, giving the mixed number $1\frac{1}{8}$. Then simplify the denominator in the same way, obtaining $2\frac{5}{8}$. Finally the resulting fraction $\frac{1\frac{1}{8}}{2\frac{5}{8}}$ is simplified by carrying out the indicated division of $1\frac{1}{8}$ by $2\frac{5}{8}$.

Example 4

$$\frac{\frac{3}{4} \text{ of } 2\frac{3}{8}}{3\frac{1}{4} \div 14\frac{1}{3}}$$

SOLUTION. $\frac{3}{4}$ of $2\frac{3}{8} = \frac{3}{4} \times \frac{19}{8} = \frac{57}{32}$

$$3\frac{1}{4} \div 14\frac{1}{3} = \frac{13}{4} \times \frac{3}{43} = \frac{39}{172}$$

$$\frac{\frac{3}{4} \text{ of } 2\frac{3}{8}}{3\frac{1}{4} \div 14\frac{1}{3}} = \frac{57}{32} \div \frac{39}{172}$$

$$\frac{57}{32} \div \frac{39}{172} = \frac{\overset{19}{\cancel{57}}}{\underset{8}{\cancel{32}}} \times \frac{\overset{43}{\cancel{172}}}{\underset{13}{\cancel{39}}} = \frac{817}{104} = 7\frac{89}{104}$$

PROBLEMS

1. $\dfrac{\frac{1}{2}}{\frac{1}{3}}$
2. $\dfrac{\frac{3}{5}}{4\frac{1}{8}}$
3. $\dfrac{6}{3\frac{2}{3}}$
4. $\dfrac{12\frac{1}{2}}{16\frac{2}{3}}$
5. $\dfrac{2\frac{5}{8}}{4}$
6. $\dfrac{4\frac{9}{16} - 3\frac{1}{8}}{2\frac{1}{4} + 1\frac{3}{16}}$
7. $\dfrac{5\frac{1}{2} - 4\frac{3}{4}}{3\frac{7}{8} - 2\frac{15}{16}}$
8. $\dfrac{3\frac{9}{16} + 5\frac{1}{4}}{4\frac{1}{3} + 2\frac{1}{2}}$
9. $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2} \text{ of } \frac{7}{8}}$
10. $\dfrac{2\frac{1}{3} \times 3\frac{1}{3}}{4\frac{3}{4} \times 5\frac{1}{8}}$
11. $\dfrac{8\frac{4}{5} \div 3\frac{1}{7}}{10\frac{1}{2} \times 6\frac{2}{3}}$
12. $\dfrac{\frac{7}{16} \text{ of } 4\frac{2}{5}}{\frac{2}{5} \text{ of } \frac{9}{14}}$

MISCELLANEOUS PROBLEMS IN COMMON FRACTIONS

1. Find the total length of the shaft in Fig. 1.

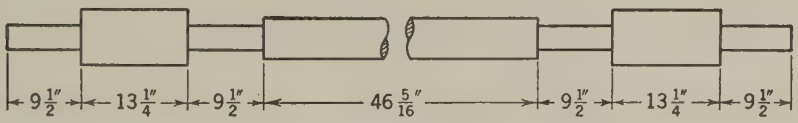


Fig. 1

2. Find the total width of the piece of boiler plate in Fig. 2.

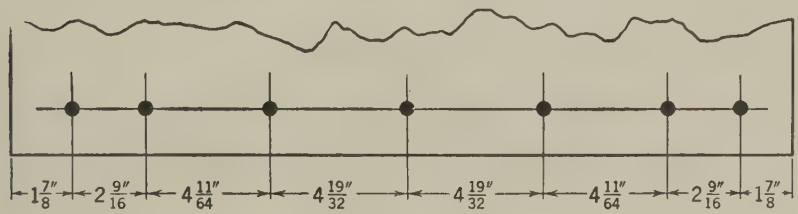


Fig. 2

3. Find the total length of the crank shaft in Fig. 3.

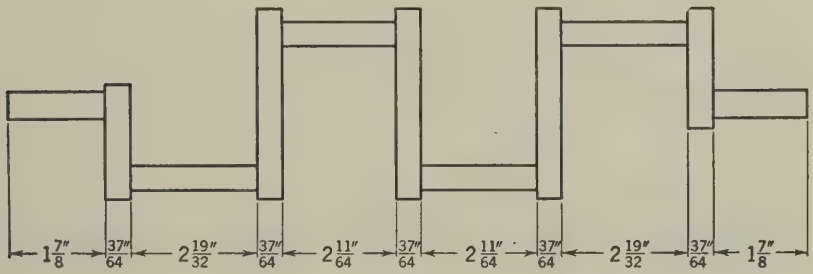


Fig. 3

4. Find the distance between end holes of the splice plate in Fig. 4.

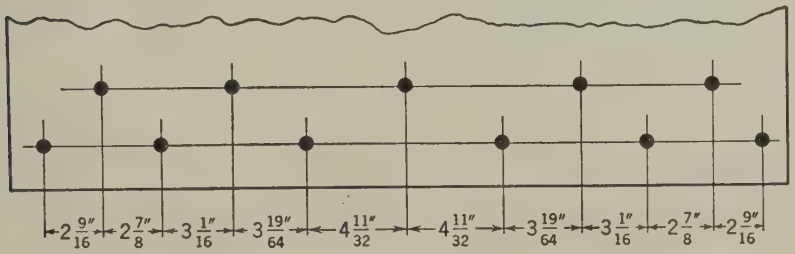


Fig. 4

5. Find the total length of the bolt in Fig. 5.

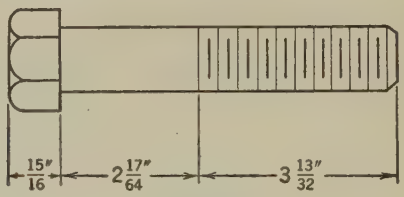


Fig. 5

6. Find the total length of the shaft in Fig. 6.

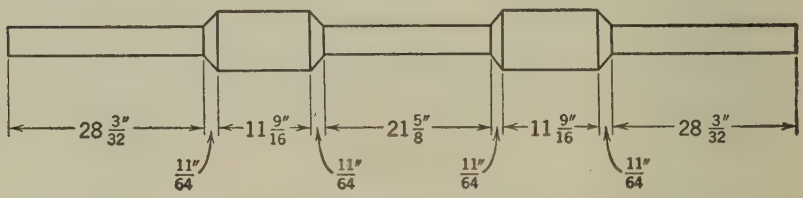


Fig. 6

7. Compute the total length of the crank shaft in Fig. 7.

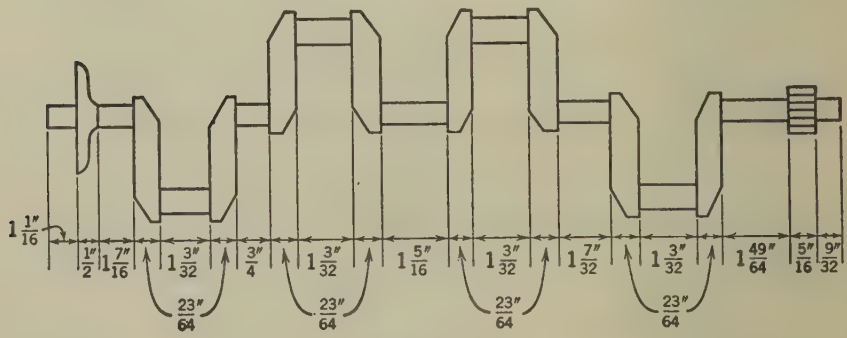


Fig. 7

8. Compute overall length of the cone pulley in Fig. 8.

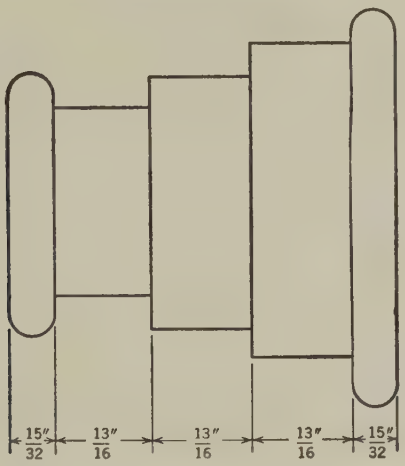


Fig. 8

9. Compute overall length of the bevel gear blank in Fig. 9.
10. Compute overall length of the gland in Fig. 10.

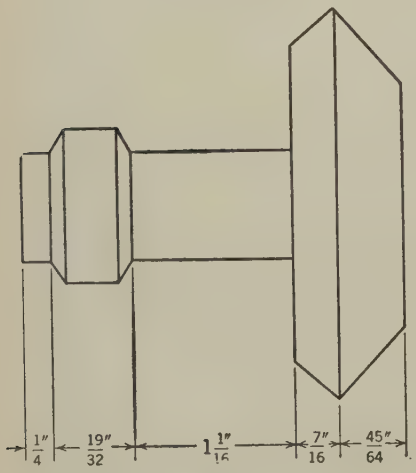


Fig. 9

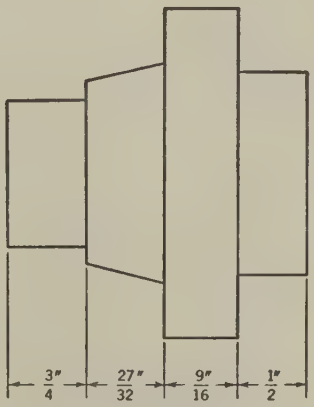


Fig. 10

11. Compute overall length of the wheel hub in Fig. 11.

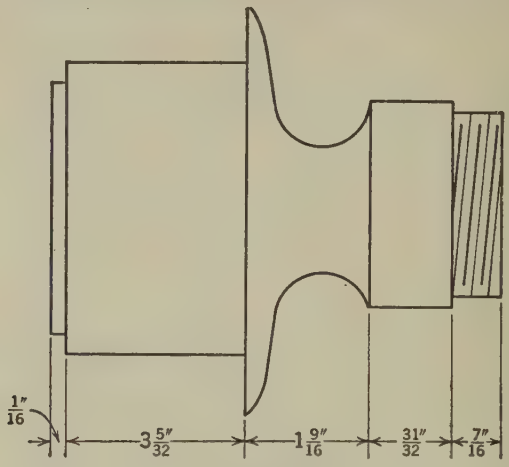


Fig. 11

12. Find the difference between the diameters at the ends of the piece in Fig. 12.

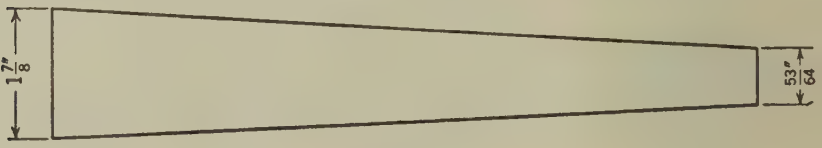


Fig. 12

13. Supply the missing dimension on the sketch in Fig. 13, if the overall length is $1\frac{5}{8}$".

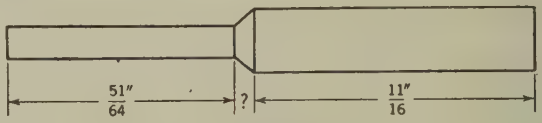


Fig. 13

14. Find the distance center to center of holes in the crank case cover in Fig. 14.

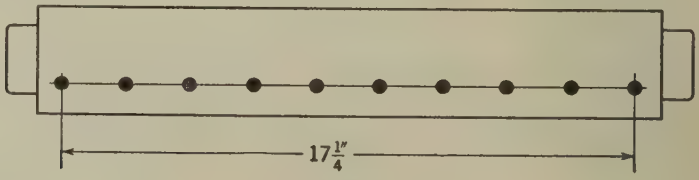


Fig. 14

15. Find the distance between end holes in the girder connection in Fig. 15.

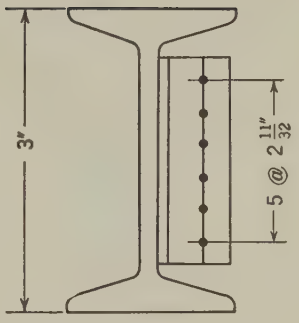


Fig. 15

16. How many binding posts $1\frac{7}{8}''$ long can be cut from a piece of brass rod $27\frac{1}{2}''$ long? Allow $\frac{1}{8}''$ waste for each cut.

17. How long a piece of drill rod is required to make 15 drills $3\frac{1}{16}''$ long? Allow $\frac{1}{8}''$ waste for each cut.

18. Four standard distance blocks are used to measure a certain space. The blocks are $\frac{5}{16}''$, $\frac{7}{64}''$, $\frac{3}{8}''$, $\frac{1}{32}''$. How wide is the space?

19. How many holes spaced $1\frac{7}{16}''$ center to center can be drilled in an angle iron $22\frac{5}{8}''$ long, allowing $1\frac{1}{4}''$ end distances?

20. Find the length of the steel plate required to make the drill jig shown in Fig. 16.

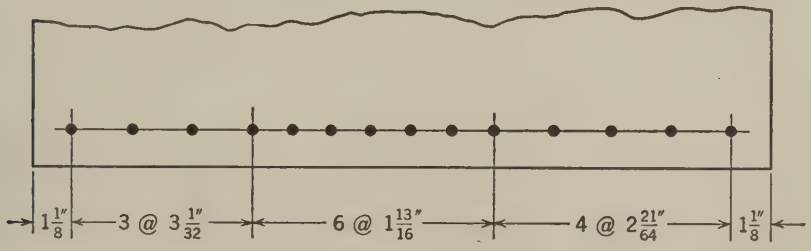


Fig. 16

21. Seven taper pins each $6\frac{7}{8}''$ long were cut from a piece of drill rod $72''$ long. Allowing $\frac{3}{32}''$ waste for each cut, what was the length of the piece left?

22. How many washers each $\frac{3}{32}''$ thick can be made from a piece of stock $25\frac{1}{2}''$ long, allowing $\frac{1}{16}''$ waste for each cut?

23. Allowing $1\frac{1}{4}$ minutes for putting away the finished work and placing the stock in the lathe, how long will it take to machine 25 pins if each pin requires $10\frac{1}{2}$ minutes?

24. Find length of piece of stock required for 8 taper keys, each $6\frac{1}{2}''$ long, allowing $\frac{1}{8}''$ waste for each cut.

25. A machinist spends $2\frac{1}{4}$ hours at a lathe, $4\frac{1}{2}$ hours at the planer, and the rest of the day at the shaper. If the working day is $8\frac{1}{2}$ hours, how long does he work at the shaper? If his pay is \$2.10 per hour, how much should he charge for the time spent at each machine?

26. Find the missing dimensions in the sketch in Fig. 17.

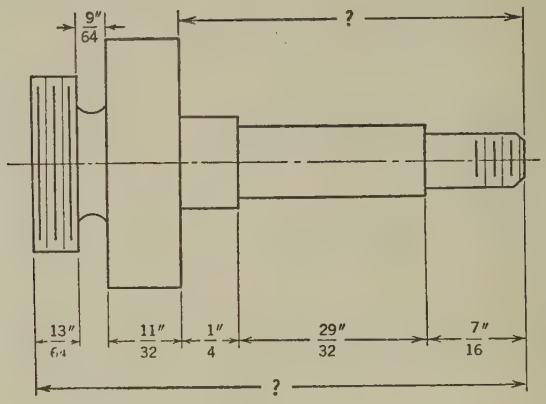


Fig. 17

27. Find the missing dimensions in the sketch in Fig. 18.

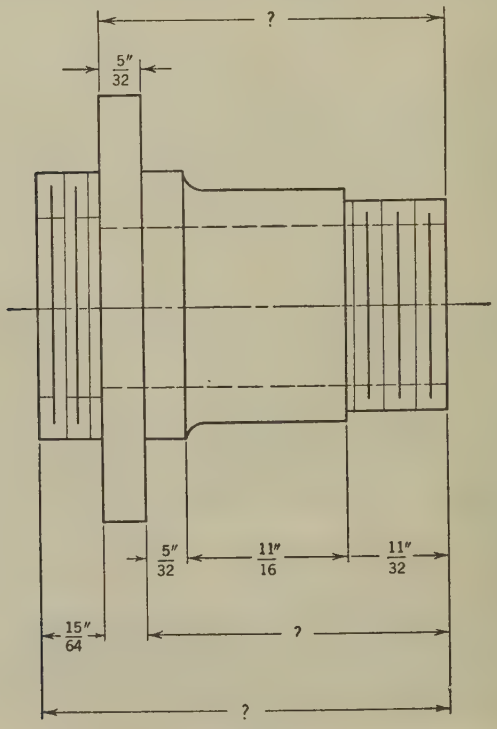


Fig. 18

28. Find the inside diameter of the pipe shown in cross-section in Fig. 19.

29. Find the missing dimension in Fig. 20.

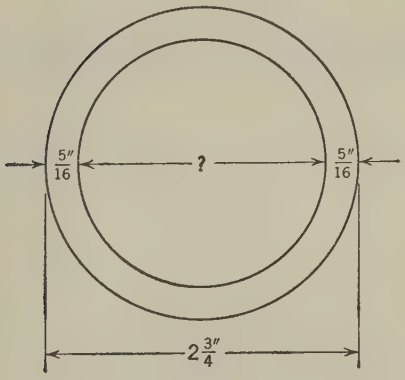


Fig. 19

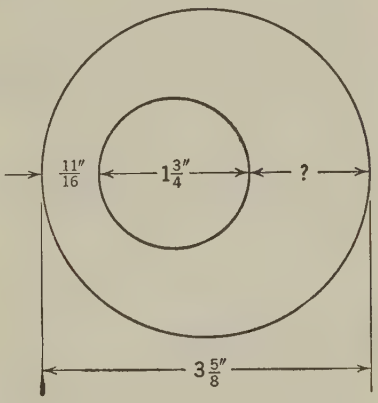


Fig. 20

30. Compute the overall dimension in Fig. 21.

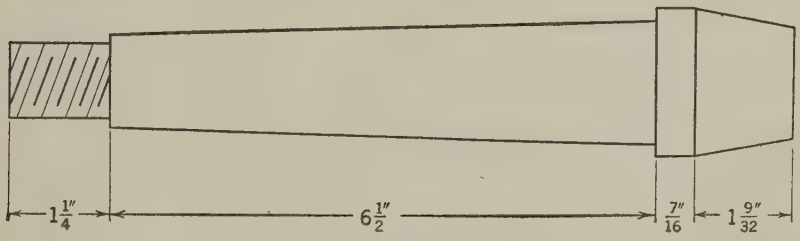


Fig. 21

2 Decimal Fractions

15. Definitions

A ***decimal fraction*** is a fraction whose denominator is 10 or some power of 10, as 100, 1000, 10,000, etc. Thus $\frac{3}{10}$, $\frac{25}{100}$, $\frac{625}{1000}$ are decimal fractions. Since the denominator of a decimal fraction is always 10 or a power of 10, that is, since the denominator is always a 1 with ciphers, we write a decimal fraction more compactly, and therefore more conveniently, by omitting the denominator entirely. Thus $\frac{3}{10}$ is written .3; $\frac{25}{100}$ is written .25; $\frac{625}{1000}$ is written .625. To distinguish the decimal fraction .3 from the whole number 3, we place a period (.) in front of the numerator. This period is called the ***decimal point.*** Any number, therefore with a decimal point in front of it is a fraction whose numerator is the number after the point, and whose denominator is a 1 with as many ciphers after it as there are figures in the number to the right of this point. Thus .3 means $\frac{3}{10}$; .35 means $\frac{35}{100}$; .03 means $\frac{3}{100}$; .003 means $\frac{3}{1000}$, etc.

100.	= One hundred
10.	= Ten
1.	= One
.1	= One tenth
.01	= One hundredth
.001	= One thousandth
.0001	= One ten-thousandth
.00001	= One hundred-thousandth

The relative values of the figures in any number are shown by the accompanying table.

PROBLEMS

I. Read the following numbers.

1. 0.1	**2.** 0.01	**3.** 0.001	**4.** 0.101
5. 0.1001	**6.** 0.63	**7.** 0.047	**8.** 0.000625
9. 0.5008	**10.** 0.0806	**11.** 2.875	**12.** 3.3125

II. Write the following numbers.

1. Six tenths.
2. Two and five tenths.
3. Three hundredths.
4. Twenty-five hundredths.
5. One and thirty-three hundredths.
6. Three thousandths.
7. Three hundred two thousandths.
8. Two hundred five and sixty-four thousandths.
9. One thousand one ten-thousandths.
10. Four and one hundred twenty-five ten-thousandths.
11. Ten and one tenth.
12. One hundred one and one hundred one ten-thousandths.
13. Two and six hundred twenty-five ten-thousandths.
14. One and three thousand one hundred twenty-five hundred-thousandths.
15. Five hundred twelve and seventy-five ten-thousandths.
16. Sixty-five ten-thousandths.
17. Five and forty-two millionths.
18. One hundred twenty-five thousand and one hundred twenty-five thousandths.
19. Ten and one one-hundredth.
20. One and one one-thousandth.

16. To Reduce a Decimal Fraction to a Common Fraction

Example 1

Reduce 0.4 to a common fraction.

SOLUTION. $0.4 = \frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

EXPLANATION. The number after the decimal point is the numerator of the fraction, while the denominator is a 1 with 1 zero, because there is one figure to the right of the decimal point. Thus,

$0.4 = \frac{4}{10}$. This is reduced to the lowest terms by dividing both numerator and denominator by 2.

Example 2

Reduce 0.75 to a common fraction.

SOLUTION. $0.75 = \dfrac{75}{100} = \dfrac{75 \div 5}{100 \div 5} = \dfrac{15}{20}$; $\dfrac{15 \div 5}{20 \div 5} = \dfrac{3}{4}$

Or

$$\frac{\begin{array}{c}3\\ \cancel{15}\\ \cancel{75}\end{array}}{\begin{array}{c}\cancel{100}\\ \cancel{20}\\ 4\end{array}} = \frac{3}{4} \quad \therefore 0.75 = \frac{3}{4}.$$

EXPLANATION. The numerator of the common fraction is 75 and the denominator is 1 with 2 zeros because there are two figures after the decimal point. Thus, $0.75 = \frac{75}{100}$. To reduce $\frac{75}{100}$ to its lowest terms, we first divide both terms by 5 and get $\frac{15}{20}$; dividing each term of $\frac{15}{20}$ by 5 gives $\frac{3}{4}$. Hence, $0.75 = \frac{3}{4}$. We can also obtain the desired result in one step by dividing both terms of the fraction $\frac{75}{100}$ by 25. Thus, $\dfrac{75 \div 25}{100 \div 25} = \dfrac{3}{4}$.

Example 3

Reduce 0.024 to a common fraction.

SOLUTION. $0.024 = \dfrac{24}{1000} = \dfrac{24 \div 2}{1000 \div 2} = \dfrac{12}{500}$;

$\dfrac{12 \div 2}{500 \div 2} = \dfrac{6}{250}$; $\dfrac{6 \div 2}{250 \div 2} = \dfrac{3}{125}$

These steps can be condensed as follows:

$$\frac{\begin{array}{c}3\\ \cancel{6}\\ \cancel{12}\\ \cancel{24}\end{array}}{\begin{array}{c}\cancel{1000}\\ \cancel{500}\\ \cancel{250}\\ 125\end{array}} = \frac{3}{125}$$

EXPLANATION. The number 24 after the decimal point is the numerator of the common fraction and the denominator is a 1 with 3 zeros because there are three figures after the decimal point. Thus, $0.024 = \frac{24}{1000}$. To reduce $\frac{24}{1000}$ to the lowest terms, we divide both terms by 2 and get $\frac{12}{500}$; dividing each term of $\frac{12}{500}$ by 2 we get $\frac{6}{250}$; and finally, dividing each term of $\frac{6}{250}$ by 2 we get $\frac{3}{125}$. The same result can be obtained in one step by dividing both terms of $\frac{24}{1000}$ by 8: thus, $\frac{24 \div 8}{1000 \div 8} = \frac{3}{125}$.

PROBLEMS

Reduce the following decimal fractions to common fractions and state each answer in its lowest terms.

1. 0.5	**2.** 0.8	**3.** 0.25	**4.** 0.75
5. 0.125	**6.** 0.188	**7.** 0.625	**8.** 0.21875
9. 0.109375	**10.** 0.7854	**11.** 0.1416	**12.** 0.333
13. 0.0875	**14.** 0.005	**15.** 0.0008	**16.** 0.8125
17. 0.072	**18.** 0.433	**19.** 0.015625	**20.** 0.96875
21. 0.0125	**22.** 0.0075	**23.** 0.0625	**24.** 0.002

17. To Change a Common Fraction to a Decimal Fraction

Example 1

Change $\frac{7}{8}$ to a decimal fraction.

SOLUTION.
$$\begin{array}{r} .875 \\ 8\overline{)7.000} \end{array}$$

EXPLANATION. Since a fraction is an indicated division, we divide 7 by 8. Since 8 does not go into 7, we place the decimal point after the 7, annex ciphers, and carry out the division as shown. Hence, $\frac{7}{8} = .875$.

Example 2

Change $\frac{2}{3}$ to a decimal fraction.

SOLUTION.
$$\begin{array}{r} .66\frac{2}{3} \\ 3\overline{)2.00} \end{array}$$

EXPLANATION. Proceeding as in Example 1, we find that, no matter how many ciphers we add after the decimal point, the quotient will not come out even; there is always a remainder of 2. In such a case we must decide how far we want to carry out our answer. If we want to

state our answer to the nearest hundredth, which means two places to the right of the decimal point, we would have $\frac{2}{3} = .66\frac{2}{3}$; and since the remainder $\frac{2}{3}$ after the second decimal place is more than $\frac{1}{2}$, it is more correct to say $\frac{2}{3} = .67$ than to say $\frac{2}{3} = .66$. Hence $\frac{2}{3} = .67$.

Example 3

Change $\frac{3}{16}$ to a decimal of three places.

SOLUTION.

$$
\begin{array}{r}
3 \quad .187\frac{8}{16} \\
\hline
16)3.000 \\
1\,6xx \\
\hline
1\,40 \\
1\,28 \\
\hline
120 \\
112 \\
\hline
8
\end{array}
$$

Carrying out the indicated division, we find the remainder after the third place in the quotient to be $\frac{8}{16}$ or $\frac{1}{2}$. Hence we add 1 to the third place in the quotient, making the answer 0.188.

PROBLEMS

Change the following common fractions to decimal fractions of three places.

NOTE. *If the figure in the fourth place is a number greater than 5, add 1 to the third figure; thus 8.7236 would read 8.724 to 3 decimal places. If the fourth figure is 5, leave the third figure unchanged if it is even and add 1 if it is odd. For example, 12.4865 to 3 decimal places would read 12.486; 10.2375 to 3 decimal places would read 10.238.*

1. $\frac{1}{4}$	**2.** $\frac{3}{8}$	**3.** $\frac{7}{16}$	**4.** $\frac{1}{3}$	**5.** $\frac{2}{3}$
6. $\frac{3}{5}$	**7.** $\frac{1}{80}$	**8.** $\frac{3}{20}$	**9.** $\frac{11}{64}$	**10.** $\frac{3}{32}$
11. $\frac{7}{8}$	**12.** $\frac{13}{16}$	**13.** $\frac{5}{6}$	**14.** $\frac{1}{11}$	**15.** $\frac{1}{12}$
16. $\frac{1}{25}$	**17.** $\frac{1}{250}$	**18.** $\frac{1}{7}$	**19.** $\frac{1}{9}$	**20.** $\frac{9}{10}$
21. $\frac{19}{144}$	**22.** $\frac{450}{1728}$	**23.** $\frac{490}{1728}$	**24.** $\frac{11}{27}$	**25.** $\frac{631}{5280}$

26. What decimal part of a dollar is 42 cents?

27. What decimal fraction of a pound is 13 oz.?

28. What decimal fraction of a foot is 8 in.?

29. Change 100 sq. in. into a decimal fraction of a square foot.

30. Change 500 cu. in. into a decimal fraction of a cubic foot.

18. Table of Decimal Equivalents

The answer to a problem may often be some such fraction as $\frac{5}{7}$ in., $\frac{3}{19}$ in., etc. To lay off such a dimension with the ordinary scale, we must change the answer to the nearest eighth, sixteenth, thirty-second, or sixty-fourth. This is done most conveniently by means of a ***table of decimal equivalents.*** See Table IX, page 568, at the back of this book.

Example 1

On a scale divided into sixty-fourths of an inch, what is the dimension nearest to $\frac{13}{37}$ in.?

SOLUTION. $\frac{13}{37} = .35135$ First change the common fraction $\frac{13}{37}$ to the decimal .35135. Looking down the table of decimal equivalents, we see that .35135 lies between .34375 and .359375. Now .35135 − .34375 = .0076, while .359375 − .35135 = .008025. Therefore .35135 is nearer to .34375, and its *nearest* equivalent in sixty-fourths of an inch is $\frac{22}{64}$ or $\frac{11}{32}$.

To find the equivalent of $\frac{13}{37}$ in sixteenths: $\frac{5}{16} = .3125$; $\frac{6}{16}$ or $\frac{3}{8} = .375$. Since .35135 is nearer to .375, the equivalent of $\frac{13}{37}$ in. is $\frac{6}{16}$ or $\frac{3}{8}$ in.

Where great accuracy is not required, it is sufficient to convert the given fraction into a decimal of 3 places and use only the first three figures of the table.

If a table of decimal equivalents is not available, we can convert any fraction into any other fraction with a desired denominator as follows:

Example 2

Change $\frac{13}{37}$ to sixty-fourths.

SOLUTION. $\frac{13}{37} \times 64 = \frac{832}{37} = 22\frac{18}{37}$ or 22

Hence $\frac{13}{37} = \frac{22}{64}$.

EXPLANATION. Since in one unit there are 64 sixty-fourths, in $\frac{13}{37}$ of a unit there will be $\frac{13}{37}$ of 64, or 22 sixty-fourths.

Example 3

Change $\frac{2}{7}$ to sixteenths.

SOLUTION. $\frac{2}{7} \times 16 = \frac{32}{7} = 4\frac{4}{7}$ or 5

Hence $\frac{2}{7} = \frac{5}{16}$.

EXPLANATION. Since in one unit there are 16 sixteenths, in $\frac{2}{7}$ of a unit there will be $\frac{2}{7}$ of 16 or 5 sixteenths.

PROBLEMS

By means of Table IX get the equivalent of each of the following fractions to the nearest sixty-fourth of an inch.

1. $\frac{2}{3}''$	2. $\frac{1}{5}''$	3. $\frac{2}{9}''$	4. $\frac{3}{11}''$	5. $\frac{15}{17}''$
6. $\frac{1}{7}''$	7. $\frac{1}{10}''$	8. $\frac{5}{12}''$	9. $\frac{5}{14}''$	10. $\frac{6}{19}''$
11. $\frac{2}{21}''$	12. $\frac{7}{36}''$	13. $\frac{13}{30}''$	14. $\frac{17}{18}''$	15. $\frac{5}{9}''$
16. $1\frac{1}{3}''$	17. $2\frac{5}{9}''$	18. $\frac{11}{19}''$	19. $1\frac{7}{11}''$	20. $5\frac{5}{13}''$

19. Conversion of Dimensions

It is often necessary in computation to convert inches and fractions of an inch into feet and decimal fractions of a foot. The following examples will illustrate the procedure in problems of this type.

Example 1

Convert $3\frac{3}{4}$ in. into a decimal fraction of a foot. State the answer to 3 decimal places.

SOLUTION. $3\frac{3}{4}$ in. $= 3.75$ in.; $\frac{3.75}{12} = 0.313$

Hence $3\frac{3}{4}$ in. $= 0.313$ ft.

Example 2

Convert $46\frac{1}{8}$ in. into feet and a decimal fraction of a foot (3 decimal places).

SOLUTION. $46\frac{1}{8}$ in. $= 46.125$ in.; $\frac{46.125}{12} = 3.844$

Hence $46\frac{1}{8}$ in. $= 3.844$ ft.

PROBLEMS

Convert the following dimensions into decimal fractions of a foot. State the answers to 3 decimal places.

1. $7\frac{1}{2}''$	**2.** $9\frac{3}{4}''$	**3.** $8\frac{7}{16}''$	**4.** $9\frac{5}{8}''$
5. $\frac{7}{8}''$	**6.** $6\frac{9}{16}''$	**7.** $71\frac{3}{4}''$	**8.** $5\frac{11}{16}''$
9. $2\frac{1}{4}''$	**10.** $53\frac{1}{2}''$	**11.** $75\frac{5}{8}''$	**12.** $96\frac{1}{8}''$

Example 3

Convert 43.172 in. into feet and inches and the nearest sixty-fourth of an inch.

SOLUTION. Dividing 43 in. by 12, we get: 43 in. = 3 ft. 7 in. The figure in the table of decimal equivalents nearest to 0.172 is 0.171875. This is equivalent to $\frac{11}{64}$ in.

Hence 43.172 in. = 3 ft. $7\frac{11}{64}$ in.

PROBLEMS

Convert the following dimensions into feet, inches, and the nearest sixty-fourth of an inch (use table of decimal equivalents).

1. 219.31″	**2.** 28.19″	**3.** 39.11″	**4.** 165.3″
5. 73.26″	**6.** 326.83″	**7.** 172.8″	**8.** 91.3″
9. 53.17″	**10.** 220.94″	**11.** 14.116″	**12.** 19.1″

Example 4

Convert 0.319 ft. into inches and a fraction of an inch (use table of decimal equivalents).

SOLUTION. 0.319 ft. = 0.319 × 12 in. = 3.828 in. The figure in the table of decimal equivalents nearest to 0.828 is 0.828125, which is the decimal equivalent of $\frac{53}{64}$.

Hence 0.319 ft. = $3\frac{53}{64}$ in.

PROBLEMS

Convert the following dimensions into inches and the nearest sixty-fourth of an inch.

1. 0.91′	**2.** 0.81′	**3.** 0.864′	**4.** 0.395′
5. 0.217′	**6.** 0.09′	**7.** 0.186′	**8.** 0.78′
9. 0.146′	**10.** 0.391′	**11.** 0.82′	**12.** 0.211′

20. Addition of Decimals

Example 1

Add 3.25, 72.004, 864.0725, 647, and 0.875.

SOLUTION.

$$\begin{array}{r} 3.25 \\ 72.004 \\ 864.0725 \\ 647. \\ .875 \\ \hline 1587.2015 \end{array}$$

EXPLANATION. To add the given numbers, write them so that the decimal points are in the same column. Then proceed as in the addition of ordinary whole numbers and place the decimal point in the sum in the same column as the other decimal points.

$$\begin{array}{r} 3.2500 \\ 72.0040 \\ 864.0725 \\ 647.0000 \\ .8750 \\ \hline 1587.2015 \end{array}$$

To facilitate the work and to avoid errors, it is advisable to add ciphers to the numbers with the fewer decimal places, so that all the numbers will have an equal number of places after the decimal point. The work then appears as shown.

Example 2

Find the sum of $\frac{1}{2} + .662 + \frac{7}{8}$.

SOLUTION. $\frac{1}{2} = .5$, $\frac{7}{8} = .875$

$$\begin{array}{r} .500 \\ .662 \\ .875 \\ \hline 2.037 \end{array}$$

EXPLANATION. First change $\frac{1}{2}$ and $\frac{7}{8}$ to decimals, getting .5 and .875, and then add the numbers as in Example 1.

PROBLEMS

Find the sum of:

1. $235 + 49.2$
2. $321.42 + 69.73$
3. $1634.08 + 12.236$
4. $1.728 + 0.0084 + 6.52$
5. $932.04 + 93.204 + 9.3204 + 0.93204$
6. $0.732 + 4.896 + 0.153 + 18.654 + 2.404$

7. $1.335 + \frac{5}{8} + 1\frac{1}{2}$
8. $2\frac{1}{3} + \frac{3}{4} + .625$
9. $3.482 + \frac{1}{6} + .025$
10. $4.603 + 2.136 + \frac{2}{3}$
11. $\frac{3}{8} + .509 + \frac{1}{7}$

12.	13.	14.	15.
264.	1.508	0.8435	0.5957
0.065	4.391	2.6264	0.4056
30.832	0.484	5.1706	2.9540
716.45	0.745	0.2035	3.3470
9.7	0.656	8.6302	0.4921

21. Subtraction of Decimals

Example 1

From 42.63 take 18.275.

Solution.	
(Minuend)	42.630
(Subtrahend)	18.275
(Difference)	24.355

Explanation. Write the numbers so that the decimal points are under each other. If the minuend contains fewer figures after the decimal point than the subtrahend, annex ciphers and proceed as in subtraction of whole numbers. Place the decimal point in the difference under the other decimal points.

Example 2

From $2\frac{1}{3}$ take .675.

Solution.	
$2\frac{1}{3} = 2.333$	(Minuend)
.675	(Subtrahend)
1.658	(Difference)

Explanation. Changing $\frac{1}{3}$ to a decimal fraction, the number $2\frac{1}{3}$ becomes 2.333. We then perform the subtraction as in Example 1.

PROBLEMS

1. From 625.75 take 234.
2. From 436.62 take 148.35.
3. From 125 take 82.875.
4. From 218.08 take 139.25.
5. From 0.4929 take 0.4025.
6. From 1 take 0.6535.
7. From 2.856 take 0.40875.
8. From 5.9205 take 0.545.
9. Take 0.6346 from 0.8964.
10. Take 0.4058 from 0.4948.

11. From $4\frac{5}{8}$ take 2.326.
12. From 5.604 take $1\frac{2}{3}$.
13. From 3.862 take $2\frac{1}{6}$.
14. From $5\frac{2}{7}$ take 3.078.
15. From $6\frac{5}{9}$ take 4.286.

22. Multiplication of Decimals

Example 1

Multiply 43.286 by 6.04.

SOLUTION.

43.286	(Multiplicand)
6.04	(Multiplier)
173144	
2597160	
261.44744	(Product)

EXPLANATION. Multiply as in whole numbers; then, beginning at the right, point off in the product as many decimal places as there are decimal places in both the multiplier and the multiplicand. Thus in the given example the multiplicand has 3 places after the decimal point and the multiplier has 2, making a total of 5 places. Hence the product must have 5 decimal places.

The decimal part of this product denotes hundred-thousandths. But neither the machinist nor the cabinet maker nor the electrician is interested in such small fractions of a unit. We must therefore rewrite the result so that it will be of practical value to the man in the shop or factory. The machinist may want his measurements given to the nearest thousandth of an inch; therefore the answer to a machinist's problem will be stated with 3 decimal places. The answer to Example 1 would then read 261.447. The figures after the third decimal place are dropped because their value is less than $\frac{1}{2}$ of a thousandth.

Likewise the answer to Example 1, to 2 decimal places, would be 261.45; to 1 decimal place, it would read 261.4.

It is therefore important to decide in advance the degree of accuracy required in the answer to any given problem. After performing the indicated operations and obtaining the exact answer, we must rewrite the answer with as many decimal places as are required to bring it to the degree of accuracy previously determined upon for the given problem.

Example 2

Multiply 0.36 by 0.24.

SOLUTION.

$$\begin{array}{r} .36 \\ .24 \\ \hline 144 \\ 72 \\ \hline .0864 \end{array}$$

EXPLANATION. Multiplying as in whole numbers, we find there are only 3 figures in the product while, according to the rule given above, we must have 4 places after the decimal point. The fourth place is supplied by a cipher written in front of the 3 figures, thus, 0.0864.

Example 3

Multiply .85 by $1\frac{3}{4}$.

SOLUTION.

$$\begin{array}{r} 1.75 \\ .85 \\ \hline 875 \\ 1400 \\ \hline 1.4875 \end{array}$$

EXPLANATION. Changing the common fraction $\frac{3}{4}$ to a decimal fraction, we get $1\frac{3}{4}$ = 1.75; then multiply .85 by 1.75 as in Example 2.

To multiply a number by 10, 100, 1000, 10,000, etc., move the decimal point in the multiplicand as many places to the right as there are zeros in the multiplier; if necessary, annex zeros. Thus $10 \times 3.25 = 32.5$; $100 \times 3.25 = 325$; $10{,}000 \times 3.25 = 32{,}500$, etc.

PROBLEMS

Find the product of:

1. 8×0.5346	**2.** 16×0.1875	**3.** 0.2465×12
4. 8.75×3.5	**5.** 0.125×0.36	**6.** 2.457×2.568
7. 8.94808×0.6	**8.** 0.008×0.072	**9.** 0.457×12.36
10. 86.4×0.0458	**11.** 5.84×0.0059	**12.** 0.008×24.8635
13. 1.125×3.375	**14.** 2.25×0.0125	**15.** $3.1416 \times 2.5 \times 2.5$
16. $0.7854 \times 3.25 \times 3.25$	**17.** 1728×0.621	**18.** 144×2.368
19. 5280×0.876	**20.** 231×3.125	

21. Restate the answers to problems 1 to 5 with 3 decimal places.
22. Restate the answers to problems 6 to 10 with 2 decimal places.
23. Restate the answers to problems 11 to 15 with 4 decimal places
24. Restate the answers to problems 16 to 20 with 2 decimal places.

25. $.062 \times 2\frac{5}{8}$ **26.** $5\frac{2}{3} \times .425$ **27.** $3\frac{5}{6} \times 2.083$

28. $2.246 \times 4\frac{1}{3}$ **29.** $4\frac{2}{9} \times 3.065$

30. (*a*) 72×10 (*b*) 6.86×10 (*c*) 0.721×10

31. (*a*) 18×100 (*b*) 7.2×100 (*c*) 0.063×100

32. (*a*) 92×1000 (*b*) 1.37×1000 (*c*) 0.375×1000

23. Division of Decimals

Example 1

Divide 76.125 by 24. State the answer to 3 decimal places.

SOLUTION.

(QUOTIENT) $3.171\frac{21}{24}$ or 3.172

(DIVISOR) 24)76.125 (DIVIDEND)

72 xxx

4 1

2 4

1 72

1 68

45

24

21

EXPLANATION. Divide as in whole numbers and place the decimal point in the quotient above the point in the dividend. Since the remainder after the third place is $\frac{21}{24}$ or more than $\frac{1}{2}$, we increase the third place by 1, giving the quotient 3.172.

Example 2

Divide 432 by 0.625.

SOLUTION.

691.2

x625.)432x000.0

375 0xx x

57 00

56 25

750

625

1250

1250

EXPLANATION. When the dividend is a whole number and the divisor a decimal fraction, we first convert the divisor into a whole number by moving the decimal point to the right of it; in this case 3 places to the right. We then move the decimal point in the dividend also 3 places to the right. To do this, we annex 3 ciphers after 432, giving the number 432000, with the decimal point after the third cipher. We then proceed to divide as in ordinary division. Since the quotient does not come out an even whole number, we place the decimal point in the

quotient directly above the decimal point in the dividend, annex ciphers after the decimal point in the dividend and continue the division to as many decimal places as is required. In this case there is no remainder after the first decimal place.

Example 3

Divide 0.78 by 0.964.

SOLUTION.

$$
\begin{array}{r}
.809 \\
964.\overline{)780.000} \\
771\ 2xx \\
\hline
8\ 800 \\
8\ 676 \\
\hline
124
\end{array}
$$

EXPLANATION. First convert the divisor into a whole number by moving the decimal point to the right 3 places. Then move the decimal point in the dividend an equal number of places to the right. If, as in this case, the dividend has fewer figures than the divisor, annex ciphers. Then proceed to divide as explained in the preceding examples.

Example 4

Divide 0.0075 by 2.31 State answer to three significant figures.

SOLUTION.

$$
\begin{array}{r}
.00324\frac{156}{231} \\
2.31\overline{)00.75000} \\
693xx \\
\hline
570 \\
462 \\
\hline
1080 \\
924 \\
\hline
156
\end{array}
$$

EXPLANATION. First convert the divisor into a whole number by moving the decimal point to the right 2 places. Then move the decimal point in the dividend also 2 places to the right. This brings the decimal point before the 7. Place the point in the answer directly above the point in the dividend and proceed to carry out the division as shown in the example. It will be noticed that since the remainder after the third significant figure is more than $\frac{1}{2}$, we increase the third significant figure by 1, giving the answer 0.00325.

Example 5

Divide $\frac{9}{11}$ by 1.86. State answer to two significant figures.

SOLUTION. $\frac{9}{11} = .8182$

$$\begin{array}{r} .43 \text{ or } .44 \\ 1_{x}86.\overline{)_{x}81.82} \\ 74\ 4 \\ \hline 7\ 42 \\ 5\ 58 \\ \hline 1\ 84 \end{array}$$

EXPLANATION. First change the common fraction $\frac{9}{11}$ to a decimal fraction, then perform the division as in Example 4.

To divide a number by 10, 100, 1000, 10,000, etc., move the decimal point in the dividend as many places to the left as there are zeros in the divisor; if necessary, prefix zeros. Thus $56.4 \div 10 = 5.64$; $56.4 \div 100 = .564$; $56.4 \div 1000 = .0564$.

PROBLEMS

Give the answers to the following problems with three significant figures after the decimal point.

1. $124.625 \div 48$ **2.** $0.2086 \div 32$ **3.** $81.63 \div 0.09$

4. $0.875 \div 0.634$ **5.** $0.454 \div 6.05$ **6.** $72 \div 0.625$

7. $80 \div 4.05$ **8.** $2.36 \div 5.48$ **9.** $6.455 \div 0.008$

10. $0.565 \div 8.673$ **11.** $0.11 \div 1848.43$ **12.** $42 \div 0.0075$

13. $0.762 \div 888$ **14.** $1 \div 1728$ **15.** $7 \div 144$

16. $1 \div 3.1416$ **17.** $1 \div 0.7854$ **18.** $450 \div 1728$

19. $233 \div 5280$ **20.** $0.0406 \div 40.08$ **21.** $\frac{4}{7} \div 3.45$

22. $\frac{5}{9} \div 2.083$ **23.** $\frac{5}{12} \div 4.96$ **24.** $6.38 \div \frac{2}{3}$

25. $5.09 \div \frac{3}{7}$

26. (*a*) $766 \div 10$ (*b*) $1.28 \div 10$ (*c*) $0.75 \div 10$

27. (*a*) $13 \div 100$ (*b*) $42.6 \div 100$ (*c*) $0.153 \div 100$

28. (*a*) $7216 \div 1000$ (*b*) $11.9 \div 1000$ (*c*) $0.4 \div 1000$

MISCELLANEOUS PROBLEMS

NOTE. *Where the number of decimal places desired in the answer is not stated, decide what degree of accuracy is required in the answer to any given problem. After obtaining the exact answer, rewrite it with the required number of decimal places.*

1. Cast iron weighs 450 lb. per cubic foot; wrought iron, 480 lb. per cubic foot; and steel, 490 lb. per cubic foot. Find the weight of each per cubic inch (1 cu. ft. = 1728 cu. in.). State the answers as decimal fractions of 3 places.

2. What will be the weight of a wrought iron rod which contains 62.25 cu. in.?

3. What part of a cubic foot is contained in a steel shaft weighing 215.75 lb.? State answer as a decimal fraction, 3 places.

4. What will be the weight of 312 iron castings if each contains 86.5 cu. in.?

5. The actual inside diameter of a "one-inch pipe" is 1.04″, while the actual outside diameter is 1.315″. Find the thickness of the pipe, Fig. 22.

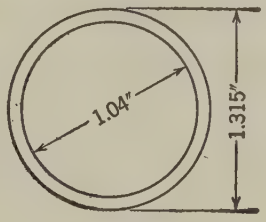


Fig. 22

6. A $\frac{7}{8}$″ bolt (without nut) weighs 0.664 lb. How many such bolts are there in a keg of 250 lb.?

7. $3 \times 3 \times \frac{3}{8}$ angle iron weighs 7.2 lb. per foot. Find the weight of a piece 12′ 3″ long.

8. Find the difference in diameters of the piece of tapered work in Fig. 23.

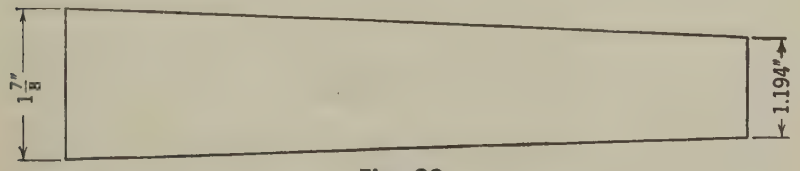


Fig. 23

9. The circumference of a circle is found by multiplying its diameter by 3.1416. Find the circumference of a circle whose diameter is 2.35″. State answer to 2 decimal places.

10. To find the diameter of a circle when the circumference is known, divide the circumference by 3.1416. Find the diameter of a circle having a circumference of 8.7865″. State answer to 1 decimal place.

11. Find the whole depth of the gear tooth shown in Fig. 24. State answer to the nearest ten-thousandth of an inch.

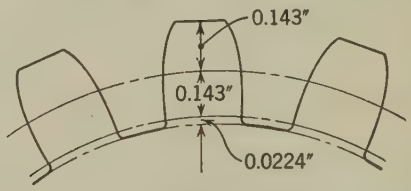


Fig. 24

12. How many pounds of water are there in a tank which contains 904.7 cu. ft.? A cubic foot of water weighs 62.5 lb.

13. How many gallons of water will the tank in Problem 12 contain if there are 231 cu. in. in a gallon?

14. Find: (*a*) the weight of a cubic inch of water; (*b*) the weight of a gallon of water.

15. With gasoline at $0.23 per gallon, what will it cost to fill a tank whose capacity is 1.75 cu. ft.?

16. Find the diameter at *A* of the taper drill shank in Fig. 25 if the difference between the small diameter and the diameter at *A* is 0.392″.

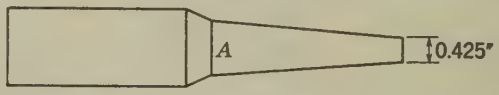


Fig. 25

17. A machinist's helper gets $1.32 per hour. How much will he earn in a week of 44 hours?

18. A welder gets $2.15 per hour for a 40-hour week. During the week he works 6 hours overtime at time-and-a-half. How much does he earn?

19. A sheet metal worker works a 40-hour week at $1.95 per hour. What is his take-home pay if $1.56 is deducted for social security tax and $14.00 for federal income tax?

20. If the basic week is 42 hours at $2.35 per hour and a man works 49 hours, how much does he earn during the week, allowing time-and-a-half for overtime?

21. The rim or cutting speed on cylindrical work is found by multiplying the circumference of the work by the number of revolutions per min-

ute. Find the cutting speed on a piece of work whose circumference is 4.273″. The work is making 120 revolutions per minute. Express the answer in feet per minute.

22. At 6 cents per pound find the cost of 125 castings each weighing 8.68 lb.

23. Compute the depth of tooth in Fig. 26.

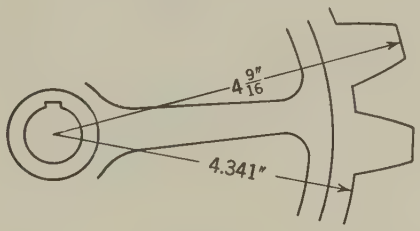


Fig. 26

24. Compute the root diameter of the screw in Fig. 27.

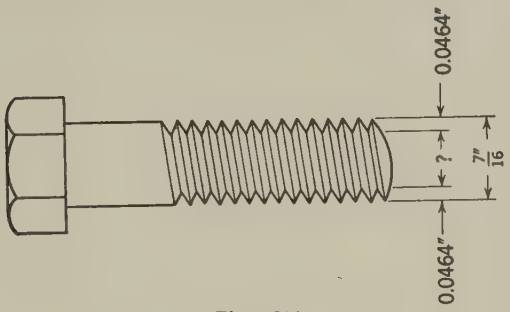


Fig. 27

25. Find the corner measurement for laying out an octagonal end on the square bar in Fig. 28.

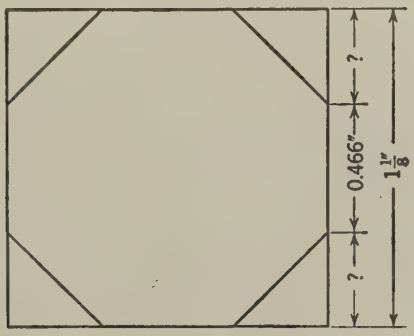


Fig. 28

26. Find the depth of cut required to mill a square end on the round shaft in Fig. 29.

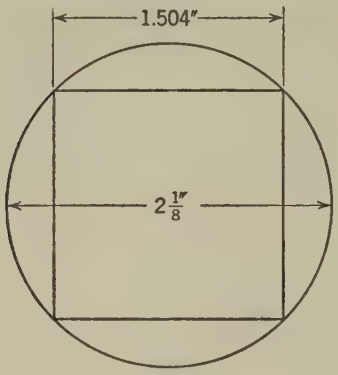


Fig. 29

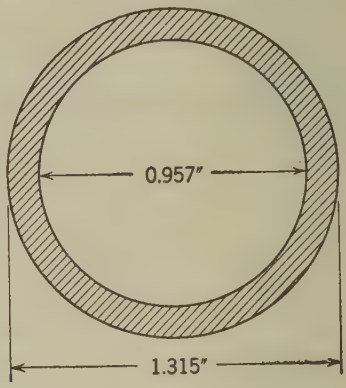


Fig. 30

27. What is the thickness of metal in the pipe cross-section in Fig. 30?

28. (*a*) Find the diameter at the large end of the tapered portion in Fig. 31. (*b*) Find the difference between the diameters at the large and small ends of the tapered part. (*c*) Compute the missing dimension.

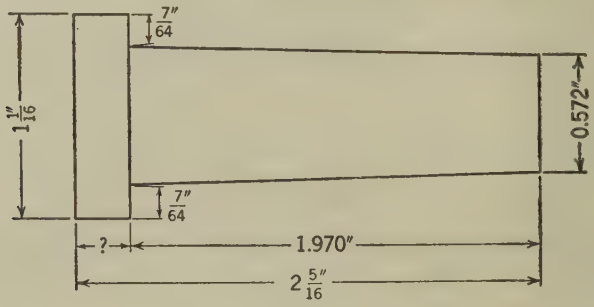


Fig. 31

29. A shaft which should be $1\frac{3}{16}''$ in diameter measures 1.317″. How much is it oversize?

30. A round piece of work is $1\frac{5}{8}''$ in diameter. How deep a cut should be taken to bring the diameter down to 1.594″?

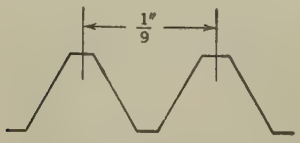


Fig. 32

31. Figure 32 shows an American Standard thread of $\frac{1}{9}''$ pitch. Find the flat at top and bottom if the flat is $\frac{1}{8}$ of the pitch. State the answer to the nearest ten-thousandth of an inch.

32. Find the value of the following expressions. State the answers as decimals of 3 places.

(*a*) $2.689 + 1\frac{7}{12}$ (*b*) $1\frac{2}{7} - 0.875$ (*c*) $3.256 \times 4\frac{1}{8}$

(*d*) $4.586 \div \frac{9}{16}$ (*e*) $\dfrac{\frac{1}{3} \text{ of } \frac{7}{8}}{\frac{1}{2} \text{ of } \frac{3}{4}}$ (*f*) $\dfrac{\frac{3}{4} \div 2\frac{1}{2}}{\frac{5}{8} \times 4\frac{1}{4}}$

33. Fourteen holes, equally spaced, are to be drilled as shown in Fig. 33. Find the distance center to center of adjacent holes. Give the answer to the nearest sixty-fourth of an inch.

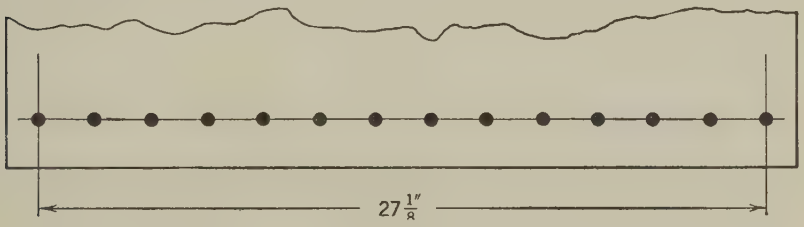
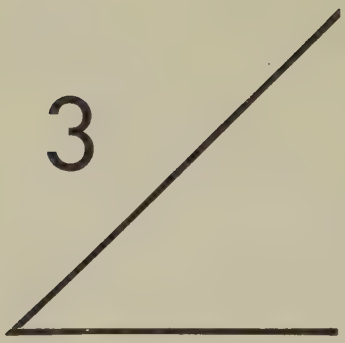


Fig. 33



3 Percentage

24. Definitions

One *per cent* of a number means one-*hundredth* of a number; six per cent means six-*hundredths*, and so on.

The symbol % stands for the words per cent. Thus 4 per cent is written 4%, 6 per cent is written 6%. The symbol % does the work of 2 decimal places. For example, 6% = .06; 25% = .25; 110% = 1.10; .5% = .005; $2\frac{1}{4}\%$ = .0225 or $.02\frac{1}{4}$; 5.7% = .057.

Since per cent means hundredths, the whole of any number contains 100% of itself. Thus 100% of 75 is 75.

Every problem in percentage contains three elements, the ***base,*** the ***rate,*** and the ***percentage.*** When we say 6% of \$100 is \$6, the base is \$100, the rate per cent is 6, and the percentage is \$6. If any two of these three elements are known, the third may be found.

The student should memorize the following table, which gives the percentage equivalents of the common fractions most frequently used in practice.

$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{1}{6} = 16\frac{2}{3}\%$
$\frac{1}{4} = 25\%$	$\frac{3}{4} = 75\%$	$\frac{1}{3} = 33\frac{1}{3}\%$
$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$	$\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{1}{2} = 50\%$		$\frac{5}{6} = 83\frac{1}{3}\%$

25. Given the Base and the Rate, To Find the Percentage

Example

Find 4% of 683.

SOLUTION. $4\% = .04$; $.04 \times 683 = 27.32$.

EXPLANATION. Since per cent means hundredths, 4% of a number means .04 of a number. Hence to obtain 4% of 683, we multiply 683 by .04 and get the percentage 27.32.

Rule

To find the percentage, multiply the base by the rate per cent.

PROBLEMS

Find the following.

1. 8% of 250	**2.** 15% of 462	**3.** 25% of 9.6
4. $33\frac{1}{3}\%$ of 1236	**5.** 110% of 8.4	**6.** 2.3% of 48.72
7. 345% of 324	**8.** 0.75% of 240	**9.** 0.05% of 612
10. $3\frac{1}{4}\%$ of 112.3	**11.** $62\frac{1}{2}\%$ of 16.8	**12.** 3.08% of 832
13. 1% of 6519	**14.** 2% of 7	**15.** $12\frac{1}{2}\%$ of 786
16. $16\frac{2}{3}\%$ of 150	**17.** $83\frac{1}{3}\%$ of 240	**18.** 50% of 48

19. White metal is made up of 3.7% copper, 88.8% tin, and 7.5% antimony. How many pounds of each metal are there in 465 lb. of the alloy?

20. A machinist earning \$105 per week had his wages increased 15%. What did he get after the increase?

21. If a man is allowed a discount of 2% from a bill amounting to \$242.85, what must he pay?

22. A dealer in electrical supplies allows discounts of 15% and 5% from the list price, and a 2% discount for cash. How much must be paid for a bill of goods whose list price amounts to \$162.50? (Deduct 15% from the original amount, then 5% from the remainder, and 2% from the second remainder.)

23. Muntz metal is composed of 59.5% copper, 39.9% zinc, and .6% lead. How many pounds of each are there in 432 lb. of the alloy?

24. U. S. Navy specifications for phosphor bronze call for 85% copper, 7% tin, .06% iron, .2% lead, .3% phosphorus, and the remainder of zinc. How many pounds of each element are required to make 500 lb. of phosphor bronze?

25. On a \$15,000 contract, 52% was paid for labor, 6% for supervision, and the remainder for materials. How much was paid for each item?

26. The manufacturer of a certain automobile estimates the following costs: materials, $38\frac{1}{2}\%$; labor, $41\frac{1}{4}\%$; overhead, $6\frac{1}{2}\%$; and profit, $13\frac{3}{4}\%$. Find the cost of each item in an automobile which sells for \$2445.

27. A standard formula for type metal calls for $77\frac{1}{2}\%$ lead, $6\frac{1}{2}\%$ tin, and 16% antimony. How many pounds of each metal are there in 250 lb. of the type metal?

28. A certain iron ore yields $4\frac{1}{4}\%$ of iron. How many pounds of iron are there in a ton of this ore?

29. A man's weekly pay is \$77.30. At the rate of 2%, how much does he pay in social security taxes per week?

30. A man works a 42-hour week at the rate of \$2.15 per hour. How much should he find in his pay envelope at the end of the week if 18% is deducted for income tax and 2% for social security tax?

26. Given the Base and the Percentage, To Find the Rate

Example 1

What per cent of 8 is 7?

SOLUTION. 7 is $\frac{7}{8}$ of 8, and $\frac{7}{8} = .875 = 87.5\%$

Example 2

What per cent of 64.8 is 16.5?

SOLUTION. $\dfrac{16.5}{64.8} = .2546 = 25.46\%$

Rule

To find the rate per cent write a fraction with the percentage as the numerator and the base as the denominator and change this fraction to a decimal with four figures after the decimal point. Multiply the quotient by 100 to get the final answer to 2 decimal places.

PROBLEMS

1. What per cent of 12 is 6?
2. What per cent of 25 is 75?
3. What per cent of 9.8 is 4.6?
4. What per cent of 7.2 is 18.5?
5. What per cent of $\frac{3}{4}$ is $\frac{1}{3}$?
6. What per cent of $5\frac{1}{2}$ is $3\frac{1}{4}$?

7. A motor receiving 8 H.P. delivers 6.8 H.P. of work. What per cent of the input is the output?

8. A ton of ore yields 80 lb. of iron. What per cent of the ore is iron?

9. A man pays \$5.75 for an article and sells it for \$6.50. What per cent profit does he make?

10. The usual allowance made for shrinkage when casting iron pipe is $\frac{1}{8}$ in. per foot. What per cent is this?

11. In making 95 lb. of solder, we used $11\frac{1}{2}$ lb. of lead and $83\frac{1}{2}$ lb. of tin. What per cent of each was used?

12. The indicated horsepower of a steam engine is 9.4 and the effective horsepower is 8.1. What per cent of the indicated horsepower is the effective horsepower?

13. What per cent of a foot is $\frac{5}{8}$ of an inch?

14. What per cent of a mile is 100 ft.?

15. Out of a total production of 2715 ball bearings manufactured during a day, 107 were rejected by the inspectors as imperfect. What per cent of the total was rejected?

16. An alloy of common yellow brass is made of the following ingredients: copper, 170.5 lb.; lead, 7.7 lb.; tin, 0.55 lb.; and zinc, 96.25 lb. What per cent of the entire alloy does each of the metals represent?

17. A machine shop job required 42 hours on the lathe, $7\frac{1}{2}$ hours on the milling machine, and $11\frac{1}{4}$ hours on the planer. What per cent of the total time should be charged to each machine?

27. Given the Percentage and the Rate, To Find the Base

Example 1

1022 is 28% of what number?

SOLUTION.

```
1022      36 50
x28.)1022x00.
      84
      ---
      182
      168
      ---
       140
       140
       ---
```

EXPLANATION. Dividing by 28 gives 1% of the number. To get 100%, that is, the whole number, multiply by 100. The result is obtained in one step by dividing 1022 by .28.

Example 2

300 is 20% more than what number?

SOLUTION.

```
300      2 50
1x20.)300x00.
      240
       600
       600
```

EXPLANATION. Since 300 is 120% of the number, divide by 120 to get 1% of the number. To get 100%, or the whole number, multiply by 100. The result is obtained in one step by dividing 300 by 1.20.

Example 3

210 is 30% less than what number?

SOLUTION.

```
210     3 00
x70.)210x00.
     210
```

EXPLANATION. Since 210 is 70% of the number, divide by 70 to get 1% of the number. To get 100%, or the whole number, multiply by 100. The result is obtained in one step by dividing 210 by .70.

Rule

To find the base, divide the percentage by the rate per cent expressed as hundredths.

PROBLEMS

1. 3 is 75% of what number?

2. 22.4 is 65% of what number?

3. $\frac{2}{3}$ is 25% of what number?

4. 30 is $16\frac{2}{3}$% more than what number?

5. 48 is 20% less than what number?

6. After a man's wages were reduced 15%, he got $62.40. How much did he get before the reduction?

7. A dealer sells coal at $17.75 per ton. If his profit is 12%, what does the coal cost him?

8. The efficiency of a motor is 90%; that is, the output is 90% of the input. If the motor delivers 8 H.P., what is the input?

9. A man sells an article for $3.60, thereby losing 10%. What did the article cost him?

10. A certain ore yields 5% of iron. How many tons of ore are required to produce $2\frac{1}{2}$ tons of iron?

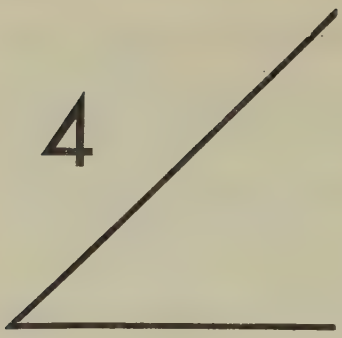
11. An article loses $3\frac{1}{2}\%$ by drying. If it weighs $8\frac{1}{2}$ lb. when dry, what was its weight when moist?

12. The inspectors in a factory rejected 33 pieces as imperfect. This represented $1\frac{1}{2}\%$ of the daily production. How many pieces were produced?

13. A motor whose efficiency is 86% delivers $10\frac{3}{4}$ H.P. What horsepower does it receive?

14. What must be the length of a pattern for a casting $18\frac{1}{2}''$ long if the allowance for shrinkage is $\frac{3}{4}$ of 1%?

15. A mechanic gets an increase of \$7.80 per week, which represents an increase of 10%. What is his new salary?



4 Ratio and Proportion

28. Ratio

Ratio is the comparison of two *like numbers*. It is always expressed by the quotient obtained by dividing the first number by the second. For example, if we wish to express the relation between 10 and 5, we say the ratio of 10 to 5 is 2; that is, $\frac{10}{5} = 2$. A ratio can be stated in one of four ways: the ratio of 10 to 5, or $10 \div 5$, or 10:5, or $\frac{10}{5}$. In each case the expression is read "the ratio of 10 to 5" and the value of the ratio is obtained by dividing 10 by 5, giving 2.

The two numbers compared are the ***terms*** of the ratio.

The ***antecedent*** is the first term, or the dividend.

The ***consequent*** is the second term, or the divisor. In the ratio 10:5, 10 is the antecedent and 5 is the consequent.

Every common fraction may be regarded as a ratio in which the numerator is the antecedent or dividend and the denominator is the consequent or divisor. Thus the fraction $\frac{2}{3}$ is the ratio of 2 to 3.

If we invert the terms of a ratio, we get an ***inverse ratio.*** Thus 3:2 is the inverse of 2:3.

Ratios can be expressed only by numbers of like denomination. For example, we cannot compare 6 pencils with 3 books or 1 mile with 100 yards. We can compare 6 pencils with 3 pencils or 1760 yards with 100 yards, etc.

29. Reduction of Ratios to Lowest Terms

To reduce a ratio to lowest terms, we proceed as in common fractions.

Example 1

Express in lowest terms the ratio of 6 to 30.

SOLUTION. $\frac{6}{30} = \frac{6 \div 6}{30 \div 6} = \frac{1}{5}$ EXPLANATION. Express the ratio as a fraction $\frac{6}{30}$ and divide both terms by the highest common divisor, 6, giving the fraction $\frac{1}{5}$. Hence the relation between 6 and 30 is the same as between 1 and 5.

Example 2

Express in lowest terms the ratio $\frac{2}{3}$ to $\frac{4}{5}$.

SOLUTION. Ratio of $\frac{2}{3}$ to $\frac{4}{5} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$

EXPLANATION. Since the value of a ratio is the quotient obtained by dividing the first term by the second, we divide $\frac{2}{3}$ by $\frac{4}{5}$, getting as a quotient $\frac{5}{6}$. That is, the ratio of $\frac{2}{3}$ to $\frac{4}{5}$ is the same as the ratio of 5 to 6.

Example 3

Divide \$28 between two men in the ratio of 2 to 5.

SOLUTION. $\frac{2}{7}$ of 28 = \$8
$\frac{5}{7}$ of 28 = \$20

EXPLANATION. To divide in the ratio of 2 to 5 means that for every \$2 given to one man, \$5 must be given to the other man. In other words, out of every \$7, one gets $\frac{2}{7}$ or \$2 while the other gets $\frac{5}{7}$ or \$5. In general, one man gets $\frac{2}{7}$ of the total amount and the other gets $\frac{5}{7}$.

PROBLEMS

1. Express the following ratios in lowest terms: 12 to 3; 8 to 16; 12 to 18; $\frac{7}{8}$ to $\frac{9}{16}$; $\frac{1}{2}$ to $\frac{1}{3}$; $2\frac{1}{2}$ to 10; $3\frac{1}{2}$ to $16\frac{2}{3}$.

2. Find the inverse of the following ratios: 5:3; 7:8; $4\frac{1}{2}$:15; $2\frac{2}{3}$:$3\frac{1}{4}$; $\frac{6}{7}$:$\frac{9}{10}$.

3. The smaller of two belted pulleys makes 240 revolutions per minute and the larger one makes 80. What is the ratio of their speeds?

4. Of two gears in mesh, the smaller gear makes 75 r.p.m. and the larger gear makes 50 r.p.m. What is the ratio of their speeds?

5. An express train runs at the rate of 60 miles per hour, and an aeroplane flies at the rate of 240 miles per hour. Find the ratio of their speeds.

6. Tool steel may be worked at a cutting speed of 20 ft. per minute in a lathe while cast iron may be worked at a cutting speed of 45 ft. per minute. Find the ratio of the cutting speeds.

7. A high-speed drill $\frac{3}{4}''$ in diameter drills through 240 castings while a carbon steel drill of the same diameter drills only 65 castings. Find the ratio of their speeds.

8. A foot of copper wire .001″ in diameter has a resistance of 10.4 ohms, while a foot of aluminum wire of the same diameter has a resistance of 18.7 ohms. What is the ratio of the two resistances?

9. One man earns $72 per week and another earns $64. What is the ratio of their earnings?

10. Bell metal is made of 4 parts of copper and 1 part of tin. Find the amount of each in a bell weighing 8.5 lb.

11. Divide $40 between two men in the ratio of 3 to 5.

12. Britannia metal consists of 2 parts antimony, 1 part bismuth, and 1 part tin. How many pounds of each are there in a casting weighing 24 lb.?

13. White pine weighs 25 lb. per cubic foot; steel, 490 lb. per cubic foot. Find the ratio of their weights.

14. A 3″ square has an area of 9 sq. in. A circle 3″ in diameter has an area of 7.0686 sq. in. Find the ratio of these areas. Express the answer as a decimal of four places.

15. The circumference of a $2\frac{3}{4}''$ circle is 8.6394″. Find the ratio between the circumference and the diameter. State the answer as a decimal of four places.

30. Proportion

If we express the fact that one ratio is equal to another ratio we form a ***proportion.*** For example, if we wish to state that the ratio of 2 to 5 is equal to the ratio of 4 to 10, we form the proportion, $2:5 = 4:10$. This equality of ratios can also be expressed in the following ways:

$$2:5::4:10, \quad \text{or} \quad \frac{2}{5} = \frac{4}{10}$$

In the proportion $2:5::4:10$, the *outside* terms 2 and 10 are called the ***extremes,*** and the *inside* terms 4 and 5 are

called the ***means.*** In the proportion 2:5::4:10, we notice the following:

(1) $$5 \times 4 = 2 \times 10;$$

hence *the product of the means is equal to the product of the extremes.*

(2) $$2 = \frac{5 \times 4}{10}, \quad \text{and} \quad 10 = \frac{5 \times 4}{2};$$

hence *either extreme can be found by multiplying the means and dividing the product by the other extreme.*

(3) $$5 = \frac{2 \times 10}{4}, \quad \text{and} \quad 4 = \frac{2 \times 10}{5};$$

hence *either mean can be found by multiplying the extremes and dividing by the other mean.*

Example 1

A man can machine 12 studs in 15 minutes. How long will it take him to turn out 250 studs?

SOLUTION.

$$12:250::15:x \quad \text{(studs:studs::minutes:minutes)}$$

$$\therefore x = \frac{250 \times 15}{12} = 312\tfrac{1}{2} \text{ minutes} = 5 \text{ hours } 12\tfrac{1}{2} \text{ minutes}$$

EXPLANATION. Let x stand for the number of minutes in the answer and make it the right-hand extreme of the proportion. The third term must be of the same kind or denomination as x. That is, since x is to be minutes, the third term must also be minutes, and hence we make 15 the third term, giving the ratio $15:x$. Since it will take longer to machine 250 studs than 12 studs, we know that x must be greater than 15. That is, the antecedent 15 is smaller than the consequent x. Hence the antecedent in the first ratio must be correspondingly smaller than its consequent.

The first ratio must therefore be written 12:250, giving the proportion, $12:250::15:x$. The ratio of the first item to the second is the same as the ratio of the third to the fourth. To find x, we multiply the two means 250 and 15 and divide by 12, the other extreme.

Example 2

An 18″ gear meshes with a 6″ gear. If the large gear has 72 teeth, how many teeth will the small gear have?

SOLUTION. $18:6::72:x \quad \therefore x = \dfrac{72 \times 6}{18} = 24$ teeth

EXPLANATION. Let x represent the number of teeth on the small gear and put it in the fourth place. The 72 must be in the third place. We know that the small gear will have fewer teeth; therefore x will be smaller than 72. Since the fourth term is less than the third term, the second term will be less than the first term. Hence we put 18 in the first place and 6 in the second place, giving the proportion $18:6::72:x$. Multiplying the two means, 6 and 72, and dividing by 18, the extreme, we get $x = 24$ teeth.

Example 3

If a man earns \$65 per week, how long must he work to earn \$780?

SOLUTION. $65:780::1:x \quad \therefore x = \dfrac{780 \times 1}{65} = 12$ weeks.

EXPLANATION. Since the same relation must exist between the lengths of time as between the amounts earned, the ratio 65:780 must equal the ratio $1:x$. Hence the proportion $65:780::1:x$; whence, $x = \dfrac{780 \times 1}{65} = 12$ weeks.

PROBLEMS

1. If a 14 lb. casting costs \$1.04, what would a 30 lb. casting cost?
2. A pattern made of white pine weighs 3.74 lb. What will a brass casting made from this pattern weigh, if white pine weighs 25 lb. per cubic foot and brass weighs 520 lb. per cubic foot?

3. The lengths of the two rectangles shown in Fig. 34 are proportional to their widths. What is the length of the smaller rectangle?

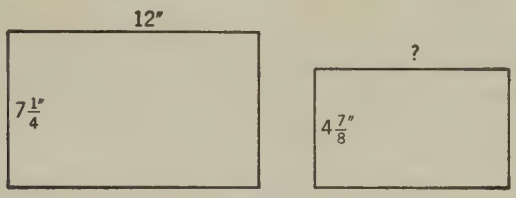


Fig. 34

4. A copper wire 400 ft. long has a resistance of 1.084 ohms. What is the resistance of 2500 ft.?

5. Cast iron weighs 450 lb. per cubic foot; white pine, 25 lb. per cubic foot. If a certain pattern made of white pine weighs $2\frac{1}{4}$ lb., what will the casting weigh?

6. A 24″ pulley running at 180 r.p.m. drives a 14″ pulley. How many revolutions per minute will the smaller pulley make?

NOTE. *The ratio of their speeds is the inverse of the ratio of their sizes.*

7. A 14″ pulley makes 240 r.p.m. and drives a larger pulley making 210 r.p.m. What is the diameter of the larger pulley?

8. A 16″ grinding wheel makes 1000 r.p.m. and the driving pulley is 10″ in diameter. If the driven pulley is 6″ in diameter, how many revolutions per minute does the driving pulley make?

9. A 15″ shaft is found to have a taper of .204 in. in 4 in. Find the taper in the entire length of the shaft.

10. If the corresponding sides of the two triangles in Fig. 35 are to be proportional, find the missing dimensions of the small triangle.

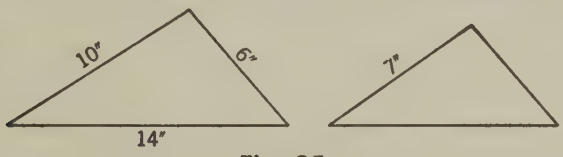


Fig. 35

11. In the two triangles shown in Fig. 36 the heights are proportional to the bases. Find the height of the smaller triangle.

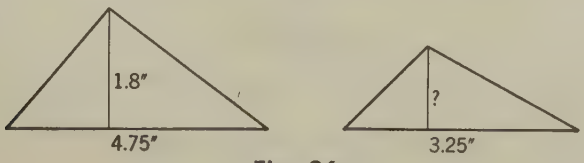


Fig. 36

12. The circumferences of circles are proportional to their diameters. Find the circumference of a 7″ circle if the circumference of a 3″ circle is 9.4248 in.

13. The areas of circles are proportional to the squares of their diameters. Find the area of a 9″ circle if the area of a 5″ circle is 19.635 sq. in.

14. The pressure of water increases with the depth. If the pressure is 6.51 lb. per square inch at a depth of 15 ft., what is the pressure at a depth of 80 ft.?

15. The power of a gas engine increases with the area of the piston. If an engine with a piston area of 8.30 sq. in. develops 25.5 H.P., how many horsepower will be developed by an engine with a piston area of 7:07 sq. in.?

31. Averages

If three men have \$5, \$8, and \$11 respectively, the sum possessed by the three men is \$24. This sum would be the same if each had an equal amount, that is, $\frac{1}{3}$ of \$24 or \$8. Eight dollars is the *average amount* of money possessed by the three men.

Example

Find the average length of four rods whose respective lengths are 5′, 7′, 12′, and 16′.

SOLUTION. Total length $= 5 + 7 + 12 + 16 = 40'$

Average length $= \frac{40}{4} = 10'$

EXPLANATION. Divide the total length of the rods by the number of rods.

Rule

To find the average or mean of two or more quantities, divide their sum by the number of quantities.

PROBLEMS

1. Find the average weight of the following five castings: 8 lb., 10 lb., 14 lb., 9 lb., 19 lb.

2. Find the average length of the following rods: 6′ 8″, 5′ 4″, 3′ 10″, 4′ 6″, 5′ 2″, 4′ 9″.

3. One of three apprentices turns out 184 cotter pins, and the other two turn out 206 and 195 respectively. What is the average output per man?

4. A machinist works $8\frac{1}{2}$ hours on Monday, $8\frac{1}{4}$ hours on Tuesday, 8 hours on Wednesday, $9\frac{1}{4}$ hours on Thursday, 9 hours on Friday, and $4\frac{1}{4}$ hours on Saturday. What is the average number of hours per day?

5. What would you consider the correct diameter of a ball bearing if you measure it three times with a vernier caliper and find a different reading each time as follows: .214″, .212″, .213″?

6. The measurements of the diameter of a piece of work are found to be as follows: .4206″, .4203″, .4209″, .4204″. What is the most probable diameter?

7. Find the mean of the following dimensions: 1.6435″, 1.6440″, 1.6438″, 1.6429″, 1.6432″, 1.6426″.

8. Which of the dimensions in Problem 7 differed most from the mean and how much?

9. The average weight of 24 castings is 16.5 lb. What does the lot weigh?

10. Several mechanics in a shop require different amounts of time for a certain job. A can do it in 3 hours 20 minutes, B in 2 hours 50 minutes, C in 2 hours 45 minutes, D in 3 hours 35 minutes. What is the average amount of time required in the shop for that job?

11. In a shop where a careful inspection is maintained, the following number of pieces were rejected during a certain week: Monday 140, Tuesday 166, Wednesday 161, Thursday 171, Friday 155, and Saturday 93. What is the average number of rejects per day?

12. Several samples of steel wire taken from the same lot are tested for tensile strength with the following results: 278 lb., 276 lb., 285 lb., 270 lb., 281 lb., and 275 lb. What is the average strength of the wire?

5 Mensuration

32. Area of Surfaces. Units of Area

The most common units of area are the square inch, the square foot, the square yard, the acre, and the square mile.

A square inch is the area contained in a square 1 in. on each side (Fig. 37).

A square foot is the area contained in a square 1 ft. on each side.

A square yard is the area contained in a square 1 yd. on each side.

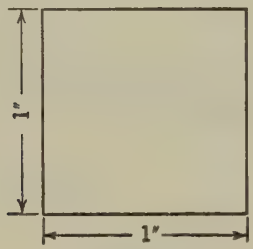


Fig. 37

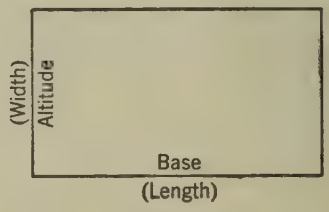


Fig. 38

Since there are 12 in. in a foot, a square foot contains 12×12, or 144 sq. in.

Likewise, since there are 3 ft. in a yard, a square yard contains 3×3, or 9 sq. ft.

The figure most frequently encountered is the rectangle, popularly known as the oblong.

A rectangle is a four-sided figure all of whose angles are right angles (Fig. 38).

Rule

The area of a rectangle is found by multiplying the length by the width, or the base by the altitude.

If b = base and a = altitude, the rule may be expressed by the following formula:

$$\text{Area} = b \times a$$

Example 1

Find the area of a rectangle whose base is 5″ and altitude 3″.

SOLUTION. $\text{Area} = b \times a$

$$= 5 \times 3 = 15 \text{ sq. in.}$$

EXPLANATION. Since the length of the rectangle is 5″ and the width 3″, to get the area, multiply the length 5″ by the width 3″, obtaining the area 15 sq. in.

Example 2

Find the area of a floor 18′ 5″ × 12′ 9″ (that is, 18′ 5″ long and 12′ 9″ wide).

SOLUTION. Change 18′ 5″ and 12′ 9″ to inches.

$$18' 5'' = (18 \times 12) + 5 = 216'' + 5'' = 221''$$

$$12' 9'' = (12 \times 12) + 9 = 144'' + 9'' = 153''$$

$$\text{Area} = b \times a = 221 \times 153 = 33{,}813 \text{ sq. in.}$$

$$33{,}813 \div 144 = 234.8 \text{ sq. ft.}$$

EXPLANATION. To get the area in square feet, we divide by 144, the number of square inches in a square foot. Since 1 decimal place is sufficiently accurate for this type of work, we state the answer as 234.8 sq. ft.

PROBLEMS

1. Find the area of a page of a book $7\frac{1}{2}$″ long and 5″ wide.

2. Find the area of the floor of a room 16′ 4″ long and 10′ 8″ wide. Give the answer in square feet, 1 decimal place.

3. How many square feet of pavement are there in a sidewalk 198′ long and 18′ wide?

4. Find the area of the walls of a room 9′ 2″ high if the room is 19′ 4″ long and 12′ 3″ wide. Make no allowance for doors or windows.

5. How many square feet are there in the top of a desk 52″ long and 28″ wide?

6. How many square inches are there in a pane of glass $15\frac{1}{2}'' \times 19\frac{1}{2}''$?

7. A parking space is 186′ × 91′. How many square yards does it contain? Find the cost of paving this area with concrete at $3.40 per square yard.

8. How many square feet are there in a lot 88′ 4″ deep with a frontage of 24′ 2″?

9. Allowing 25 sq. ft. per pupil, how many children may safely play in a playground 200′ long and 140′ wide?

10. How many square inches are there in the surface of a cubical box which measures 8″ on a side?

11. How many square feet are there in the six sides of a box $8\frac{1}{2}'' \times 6'' \times 3\frac{1}{2}''$?

12. How many square feet of corrugated iron are required for the walls of a garage 15′ 6″ × 8′ 9″ and 8′ 3″ high? Allow 64 sq. ft. for the door and window.

13. Find the areas of the figures shown in Fig. 39 (*a*), (*b*), and (*c*).

NOTE. *Divide the figures into rectangles.*

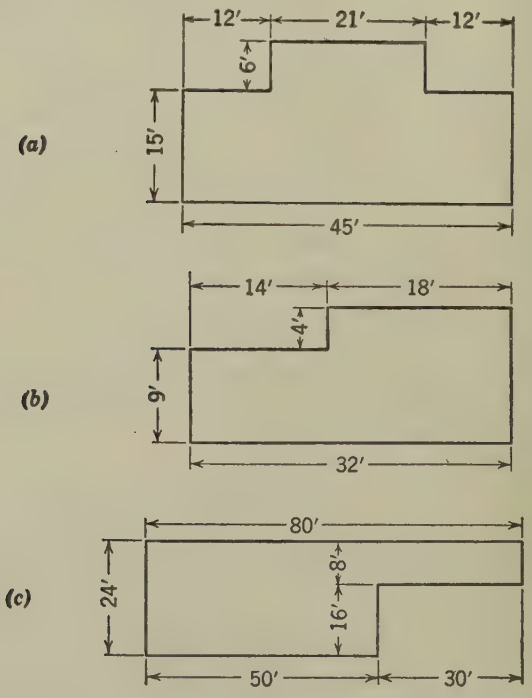


Fig. 39

14. Find the areas of the figures shown in Fig. 40 (*a*), (*b*), (*c*), and (*d*). Give all answers to the nearest tenth of a square foot.

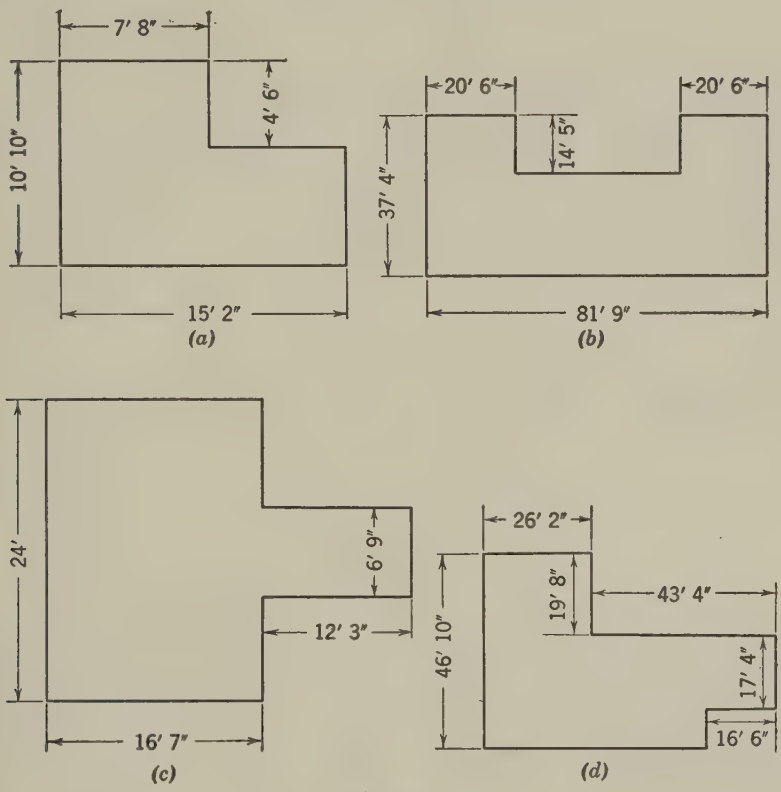


Fig. 40

15. Find the cost of covering the floor of a room 13′ 9″ × 21′ 6″ with carpet costing $7.85 a square yard.

33. To Find the Altitude of a Rectangle

If the area and the base of a rectangle are known, divide the area by the base to find the altitude. The formula is:

$$a = \frac{\text{Area}}{b}$$

Example

The area of a rectangle is 72 sq. in. and the base is 12″ long. Find the altitude.

SOLUTION. $\text{Altitude} = \dfrac{\text{Area}}{b}$

$$= \tfrac{72}{12} = 6''$$

34. To Find the Base of a Rectangle

If the area and the altitude of a rectangle are known, divide the area by the altitude to find the base. The formula is:

$$b = \frac{\text{Area}}{a}$$

Example

Find the base of a rectangle whose area is 135 sq. in. and width 7.5″.

SOLUTION. $\text{Base} = \dfrac{\text{Area}}{a}$

$$= \frac{135}{7.5} = 18''$$

PROBLEMS

1. A floor contains 360 sq. ft. and is 24′ long. How wide is it?

2. What must be the length of a parking space 29,100 sq. ft. in area if it is 145′ 6″ wide?

3. A building lot has an area of 3125 sq. ft. If it is 25′ 0″ wide, how deep is the lot?

4. The floor of a room has an area of 308 sq. ft. If the room is 22′ long, how wide is it?

5. A rug 14′ 6″ long has an area of 152.25 sq. ft. How wide is the rug?

6. A playground covers an area of 11,250 sq. ft. If it is 62′ 6″ wide, how long is it?

7. Find the length of a factory floor 49′ 6″ wide if its area is 6162.75 sq. ft.

8. What is the width of a loft 82′ 9″ long if its area is 2027.38 sq. ft.?

9. A rectangle 2′ 4″ wide has an area of 1456 sq. in. How long is it?

10. The floor area of a garage 65′ × 85′ is to be increased by 1000 sq. ft. If it is made 6′ longer, how much will the width have to be increased?

35. Squares and Square Roots

In Fig. 41, which for convenience is drawn one-fourth size, the 1″ square, (*a*), contains 1 sq. in.; the 2″ square, (*b*),

contains 4 sq. in.; the 5″ square, (*c*), contains 25 sq. in. In each case the area of the square is obtained by multiplying the length of a side of the square by itself.

$$\text{Area of } 1'' \text{ square} = 1 \times 1 = 1 \text{ sq. in.}$$
$$\text{Area of } 2'' \text{ square} = 2 \times 2 = 4 \text{ sq. in.}$$
$$\text{Area of } 5'' \text{ square} = 5 \times 5 = 25 \text{ sq. in.}$$

In mathematics we use short ways of saying and writing certain expressions. Instead of saying, 3 multiplied by itself

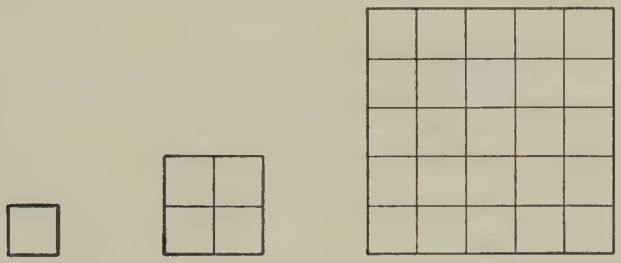


Fig. 41

equals 9, or, the square of 3 equals 9, we say, 3 squared equals 9. On paper we show this as $3^2 = 9$. The small figure 2, written above and to the right of the 3, means 3 squared or 3 multiplied by itself. The number 2 used in this way is called an ***exponent.***

$$\text{The square of } 3 = 9, \text{ or } 3^2 = 9.$$
$$\text{The square of } 7 = 49, \text{ or } 7^2 = 49.$$
$$\text{The square of } 5 = 25, \text{ or } 5^2 = 25.$$

The right-hand column shows the proper method of expression and should always be used.

Very often it is necessary to reverse the above operation so as to find the length of side of a square which contains a certain number of square inches, or the number that must be multiplied by itself to get a certain number. For instance, what number multiplied by itself equals 9? Or how long is the side of a square which contains 9 sq. in.? We know that

the square which contains 9 sq. in. is a 3″ square. Since $3 \times 3 = 9$, we say that 3 is the ***square root*** of 9. We write $\sqrt{9} = 3$. The sign $\sqrt{\ }$ takes the place of the four words "the square root of"; so that, $\sqrt{9} = 3$, means *the square root of 9 equals 3.*

The following list of the squares and square roots of numbers should be memorized by the student.

Squares	Square Roots
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$

36. To Find the Square Root of a Whole Number

Since the numbers shown in the table above are all exact squares within the multiplication table, we have no difficulty in finding their square roots. However, when we are required to find the square root of a number like 5625, the problem is not quite so simple, and we make use of the following method.

Example

Find the square root of 5625.

Solution.

$$\begin{array}{r} 7\ \ 5 \\ \overline{56}\ \overline{25} \\ 49 \\ 145\overline{)\ 7\ 25} \\ \underline{7\ 25} \\ 0 \end{array}$$

Explanation. Write the number and divide it into pairs of figures from right to left, $\overline{56}\ \overline{25}$. The first pair of figures is 56. The largest square equal to or less than 56 is 49. The square root of 49 is 7. Write the 49 under the 56 and the 7 over the 56. Subtracting 49 from 56 leaves 7. Bring

down the next pair of figures, 25, with the 7, making 725. Now double the answer already obtained: $2 \times 7 = 14$. Put this in front of the remainder 725 as a *trial divisor*. Disregard the last figure of the 725 and divide 14 into 72. This gives 5 for the second figure of the answer. Write the new figure 5 above the second pair, 25, and after the trial divisor, 14, making the complete divisor, 145. Multiply the divisor 145 by the new figure 5; $5 \times 145 = 725$. Subtracting 725 from the divisor leaves no remainder. This shows that the exact square root of 5625 is 75.

PROOF. $75 \times 75 = 5625$

37. The Square Root of Mixed Numbers

To obtain the square root of 56.25 or $56\frac{1}{4}$, we proceed in exactly the same manner. We divide the number 56.25 into pairs of figures *to the right and left of the decimal point* and place the decimal point of the square root directly above the decimal point of the square.

$$\frac{7.5}{56.25}; \quad \sqrt{56.25} = 7.5 \quad \text{or} \quad 7\tfrac{1}{2}$$

Numbers like 16 or 81 or 5625 are called ***perfect squares,*** *i.e.*, we can find their square roots without any remainder. The great majority of numbers, however, are not perfect squares. When we take the square root of 731, for example, we find a remainder after getting the second figure. In cases of this sort we decide the degree of accuracy required and carry out the work to as many decimal places as we need. *Two decimal places are required in the square for each decimal place in the root.* Suppose that we wish to get the square root of 731 to 2 decimal places. We therefore need four figures after the decimal point. So we write 731.0000, and mark off the figures into pairs to the right and left of the decimal point: $\overline{7}\ \overline{31}.\overline{00}\ \overline{00}.$

Example

Find the square root of 731 to 2 decimal places.

SOLUTION.

$$\begin{array}{r} 2\ 7.0\ 3 \\ \overline{7}\ \overline{31.00}\ \overline{00} \\ 4 \\ 47\overline{)3\ 31} \\ 3\ 29 \\ 5403\overline{)\ 2\ 00\ 00} \\ 1\ 62\ 09 \\ \hline 37\ 91 \end{array}$$

EXPLANATION. The largest square less than 7 is 4, of which the square root is 2. Write the 2 over the 7 and the 4 under the 7. Subtracting 4 from 7 leaves a remainder of 3. Bring down the next pair, 31. Double the root already obtained; $2 \times 2 = 4$, the trial divisor. 4 goes into 33 eight times, but if we try 8, we will find it too large; so we try a smaller number, 7. Write the 7 over the 31 and after the trial divisor, making the complete divisor 47. Multiply the divisor 47 by the new root figure 7; $7 \times 47 = 329$. Subtract 329 from 331, leaving the remainder 2.

Bring down the next pair of figures, 00. Double the root already obtained, 27; $2 \times 27 = 54$, the new trial divisor. 54 into 20 goes 0 times. Write the 0 over the 00 and after the 54. Bring down the next pair of figures, 00, making the dividend, 20,000. Dividing the trial divisor, 540, into 2000 gives the figure 3 as the fourth figure of the root. Multiply the complete divisor 5403 by the new root figure 3; $3 \times 5403 = 16{,}209$. Subtracting 16,209 from 20,000 leaves a remainder of 3791. The fraction $\frac{3791}{5403}$, being more than $\frac{1}{2}$, we add 1 to the last figure of the root, making the answer, 27.04. The square root of 731 equals 27.04. We see that after the operation of obtaining the first figure of the root, all the other figures are obtained by exactly the same process.

The important things to bear in mind are:

(1) *Begin at the decimal point and divide the number into pairs of figures on each side of the point.*

(2) *Decide the number of decimal places the work requires.*

(3) *Arrange your work for that number of decimal places.*

(4) *Place the decimal point in the root directly over the decimal point in the square.*

The following examples show how to arrange the work for finding the square roots of various numbers:

(1) Find $\sqrt{3859}$ to 2 decimal places $\quad \overline{38}\ \overline{59.00}\ \overline{00}$

(2) Find $\sqrt{2}$ to 3 decimal places $\quad \overline{2.00}\ \overline{00}\ \overline{00}$

(3) Find $\sqrt{713.801}$	to 3 decimal places	$\overline{7}\ \overline{13.80}\ \overline{10}\ \overline{00}$
(4) Find $\sqrt{8.2733}$	to 3 decimal places	$\overline{8.27}\ \overline{33}\ \overline{00}$
(5) Find $\sqrt{2733816}$	to no decimal places	$\overline{2}\ \overline{73}\ \overline{38}\ \overline{16.}$
(6) Find $\sqrt{0.273}$	to 3 decimal places	$0.\overline{27}\ \overline{30}\ \overline{00}$
(7) Find $\sqrt{0.08601}$	to 3 decimal places	$0.\overline{08}\ \overline{60}\ \overline{10}$
(8) Find $\sqrt{3\frac{5}{8}}$	to 3 decimal places	$\overline{3.62}\ \overline{50}\ \overline{00}$
(9) Find $\sqrt{5\frac{2}{3}}$	to 2 decimal places	$\overline{5.66}\ \overline{67}$
(10) Find $\sqrt{6\frac{1}{3}}$	to 2 decimal places	$\overline{6.33}\ \overline{33}$

38. To Find the Square Root of a Fraction

The simplest method of finding the square root of a fraction is to convert the fraction into a decimal of twice as many places as are required in the root. For instance, to get the square root of $\frac{2}{3}$ to 3 decimal places, we convert the fraction $\frac{2}{3}$ to the decimal 0.666667 and find the square root of 0.666667; hence $\sqrt{\frac{2}{3}} = 0.816$.

To get the square root of $\frac{191}{3.14}$ to 2 decimal places, we divide 191 by 3.14 carrying the work to 4 decimal places; $\frac{191}{3.14} = 60.8280$. We then get the square root of 60.8280; hence $\sqrt{\frac{191}{3.14}} = 7.80$.

PROBLEMS

1. Find the square root of 4624.
2. Find the square root of 54,756.
3. Find the square root of 161.29.
4. Find the square root of 2316 to 2 decimal places.
5. Find the square root of 8.609 to 3 decimal places.
6. Find the square root of 75.3 to 3 decimal places.

7. Find the length of side of a square which contains 1000 sq. in.
8. Find the length of side of a square which contains 2 sq. in.
9. Find the length of side of a square which contains 5 sq. in.
10. Find the length of side of a square which contains 10 sq. ft.

In the following problems, perform the operations indicated.

11. $\sqrt{0.426}$ to 3 decimal places.
12. $\sqrt{0.4891}$ to 4 decimal places.
13. $\sqrt{0.0893}$ to 3 decimal places.
14. $\sqrt{0.00524}$ to 4 decimal places.
15. $\sqrt{0.000618}$ to 4 decimal places.
16. $\sqrt{\frac{1}{3}}$ to 3 decimal places.
17. $\sqrt{9\frac{3}{4}}$ to 3 decimal places.
18. $\sqrt{2\frac{7}{16}}$ to 3 decimal places.
19. $\sqrt{1.02}$ to 3 decimal places.
20. $\sqrt{\frac{2}{7}}$ to 3 decimal places.
21. $\sqrt{\frac{5}{12}}$ to 3 decimal places.
22. $\sqrt{\frac{23}{32}}$ to 3 decimal places.
23. $\sqrt{\frac{1.33}{3.14}}$ to 4 decimal places
24. $\sqrt{\frac{2763}{0.83}}$ to 1 decimal place
25. $\sqrt{\frac{11\frac{1}{8}}{96\frac{3}{4}}}$ to 4 decimal places.

39. Applications of Square Root

The commonest and most useful application of square root is in finding the third side of a right triangle, when two

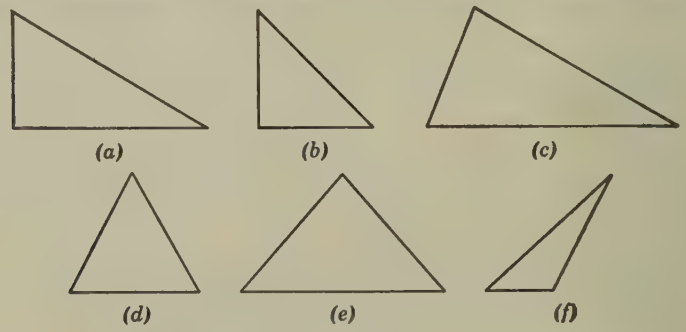


Fig. 42

of the sides are known. A ***right triangle*** is a triangle which has a right angle or square corner. Of the six triangles shown in Fig. 42 only (*a*) and (*b*) are right triangles, because they have one right angle.

In Fig. 43 are shown the names given to the three sides of a right triangle: ***base, altitude,*** and ***hypotenuse.***

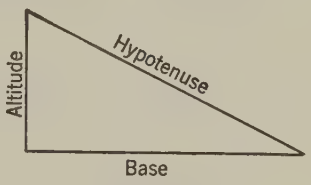


Fig. 43

To find the relation between the three sides, let us construct a right triangle with a 4″ base and a 3″ altitude, Fig. 44. Measuring the hypotenuse, we find a length of 5″. A square built on the hypotenuse is equal to the sum of the squares built on the base and the altitude.

$$\begin{aligned} \text{Base}^2 &= 4^2 = 16 \\ \text{Altitude}^2 &= 3^2 = 9 \\ \text{Sum of squares} &= 25 \end{aligned}$$

The square root of 25 = 5, the length of the hypotenuse. From this is derived the rule for the relation between the

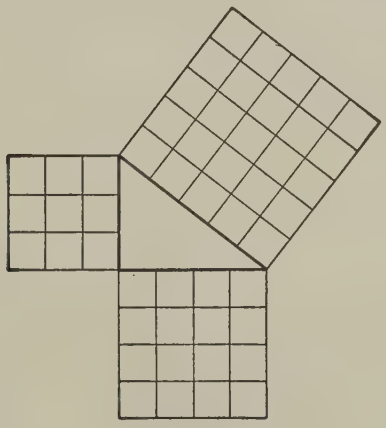


Fig. 44

three sides of a right triangle: ***the square of the hypotenuse is equal to the sum of the squares of the base and the altitude.***

If we call the base B, the altitude A, and the hypotenuse C, we can express the rule thus: $C^2 = A^2 + B^2$. To find C, the hypotenuse, we must square A, square B, add the squares and get the square root of the sum. This may be expressed by the formula

$$C = \sqrt{A^2 + B^2}$$

which says that the hypotenuse, C, is equal to the square root of the sum of A^2 and B^2.

Example 1

Find the hypotenuse of the right triangle in Fig. 45.

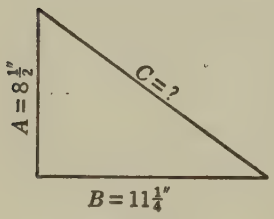


Fig. 45

SOLUTION.
$$C = \sqrt{A^2 + B^2}$$
$$= \sqrt{(8\tfrac{1}{2})^2 + (11\tfrac{1}{4})^2}$$
$$= \sqrt{72.25 + 126.5625}$$
$$= \sqrt{198.8125}$$
$$= 14.10$$

The length of the hypotenuse is 14.10″.

If the hypotenuse and one of the sides are given and we are required to find the other side, the work is the same except that we subtract the squares instead of adding them.

To find the altitude, we use the formula

$$A = \sqrt{C^2 - B^2}$$

To find the base, we use the formula

$$B = \sqrt{C^2 - A^2}$$

Example 2

Find the altitude of the right triangle in Fig. 46.

SOLUTION.
$$A = \sqrt{C^2 - B^2}$$
$$= \sqrt{20^2 - 16^2}$$
$$= \sqrt{400 - 256}$$
$$= \sqrt{144} = 12$$

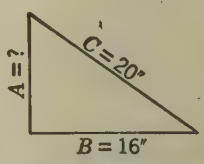


Fig. 46

The altitude is 12″.

Example 3

Find the base of the right triangle in Fig. 47.

SOLUTION. $B = \sqrt{C^2 - A^2}$

$= \sqrt{20^2 - 12^2}$

$= \sqrt{400 - 144}$

$= \sqrt{256} = 16$

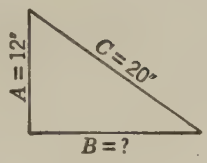


Fig. 47

The base is 16″.

Summary

To find the hypotenuse of a right triangle. Square both sides, add the squares, and get the square root of the sum.

To find the base or the altitude of a right triangle. Square the hypotenuse and the given side, subtract the square of the side from the square of the hypotenuse, and get the square root of the difference.

PROBLEMS

1 to 6. In the right triangles shown in Fig. 48, compute the length of the third side. Where dimensions are given in feet and inches, change them all to inches and, after obtaining the result, convert the answers into feet, inches, and a fraction of an inch.

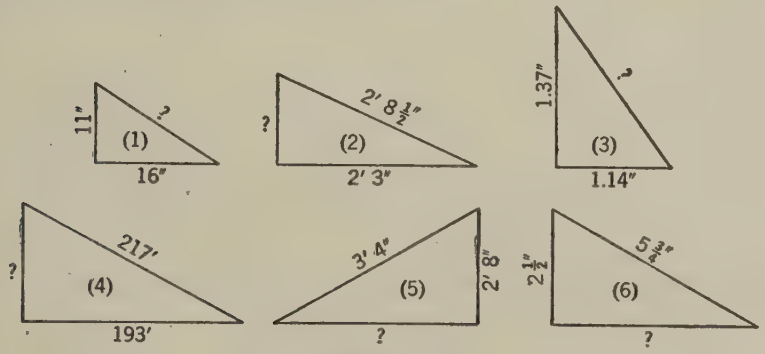


Fig. 48

7. Find the diagonal distance between the corners of a room 16′ 4″ long and 12′ 9″ wide. Give the answer in feet and inches.

8. Compute center-to-center distance between bolt holes in sketch, Fig. 49.

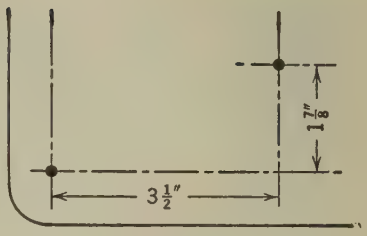


Fig. 49

9. In printing, good proportion calls for a page whose diagonal is twice the width. What should be the length of a page which is $3\frac{7}{8}''$ wide?

10. Compute the overall length along the center line of the piece of work shown in the sketch, Fig. 50.

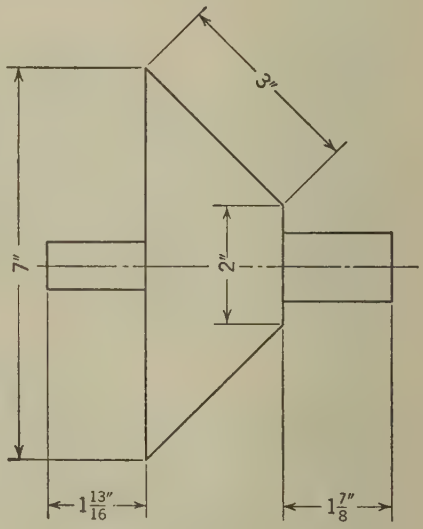


Fig. 50

11. Compute the length of the stair stringer shown in Fig. 51.

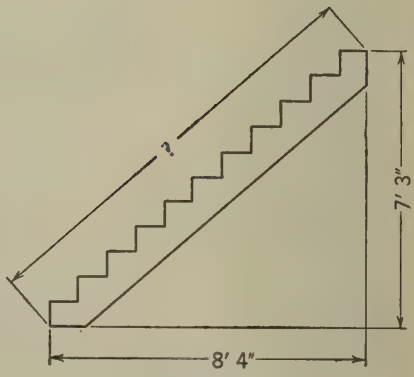


Fig. 51

12. Figure 52 shows a roof truss. Compute the length of a rafter. Add 10 in. for overhang.

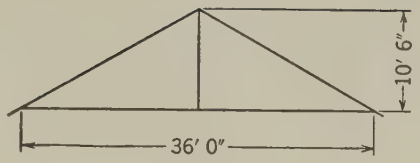


Fig. 52

13. In Fig. 53, pulley *A* is 18 ft. 4 in. above and 20 ft. 8 in. to the right of pulley *B*. Find the distance center to center of pulleys.

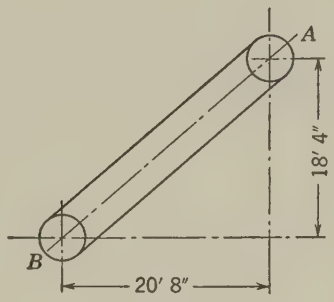


Fig. 53

14. In building a pier, the stringers were fastened to the piles with square-headed bolts 2″ on a side. If the heads are to be countersunk, what size holes should be bored in the stringers, allowing $\frac{1}{2}$ in. clearance around the bolt heads? See Fig. 54.

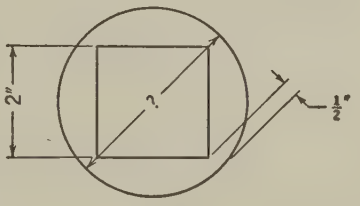


Fig. 54

40. Triangles

Triangles with which we shall deal are of the following kinds:

A ***right triangle*** is a triangle which has one right angle or square corner; Fig. 55.

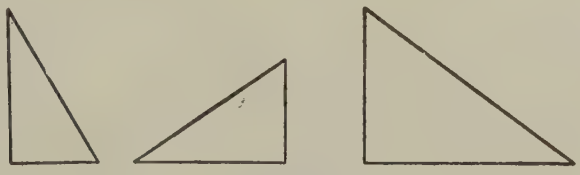


Fig. 55

An *isosceles triangle* is a triangle which has two equal sides and two equal angles; Fig. 56.

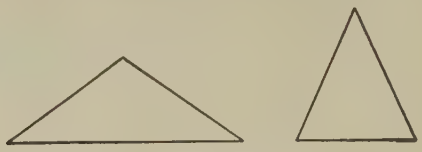


Fig. 56

An *equilateral triangle* is a triangle in which the three sides are equal and the three angles are equal; Fig. 57.

Fig. 57

A *scalene triangle* is a triangle which has no right angle and no two sides equal; Fig. 58. Scalene triangles are also called *oblique triangles.*

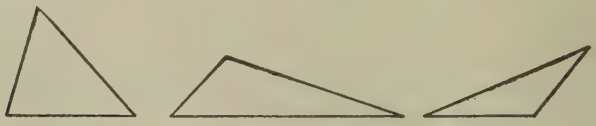


Fig. 58

41. Area of Triangles

In the rectangle, Fig. 59(*a*), the area is found by multiplying the base by the altitude. Area = 16 × 10 = 160 sq. in. Drawing the diagonal *CF* divides the rectangle into two equal right triangles, *CEF* and *CDF*.

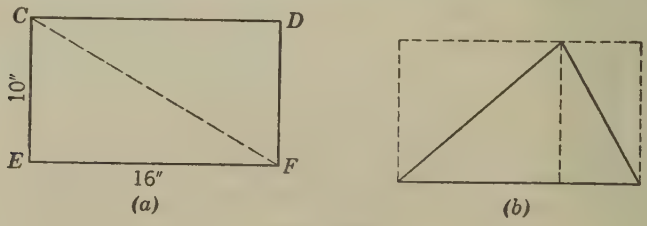


Fig. 59

The area of each right triangle is $\frac{1}{2}$ of 160 = 80 sq. in.; $\frac{1}{2} \times 16 \times 10 = 80$ sq. in. The area of the right triangle is therefore equal to $\frac{1}{2}$ the base times the altitude. Calling the altitude a and the base b, we find:

$$\text{Area of right triangle} = \tfrac{1}{2} \times b \times a$$

A study of Fig. 59(b) shows that the same rule applies to that triangle. In fact, the following is a general formula for finding the area of any triangle when the base and the altitude are known or can be computed:

$$\text{Area} = \tfrac{1}{2} \times b \times a$$

It is customary to omit the sign of multiplication in mathematical formulas such as the preceding. The formula is then written thus:

$$\text{Area} = \tfrac{1}{2}ba$$

The expression ba always means b *times* a. In writing formulas the sign of multiplication is omitted.

Example 1

Find the area of the right triangle shown in Fig. 60.

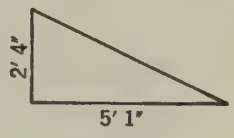


Fig. 60

SOLUTION. 2′ 4″ = 28″, the altitude

5′ 1″ = 61″, the base

$$\text{Area} = \tfrac{1}{2}ba$$

$$= \tfrac{1}{2} \times 61 \times 28$$

$$= 854 \text{ sq. in.}$$

$$= 5.93 \text{ sq. ft.}$$

Example 2

Find the area of the right triangle shown in Fig. 61.

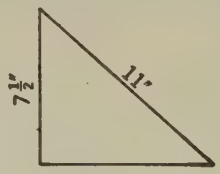


Fig. 61

SOLUTION. In this case, we first find the length of the base and then proceed as in Example 1.

$$b = \sqrt{11^2 - (7\tfrac{1}{2})^2} = \sqrt{121 - 56.25} = \sqrt{64.75} = 8.05 \text{ in.}$$

$$\text{Area} = \tfrac{1}{2}ba$$
$$= \tfrac{1}{2} \times 8.05 \times 7\tfrac{1}{2}$$
$$= 30.19 \text{ sq. in.}$$

42. Area of Isosceles Triangles

In isosceles and equilateral triangles, computations are easily made because a perpendicular dropped from the vertex, *i.e.*, the point opposite the base, divides the triangle into two equal right triangles. In Fig. 62, the perpendicular divides the isosceles triangle into two equal right triangles, the base of each being 16″ and the hypotenuse of each being 20″. The altitude is found by the usual method for finding the altitude of a right triangle:

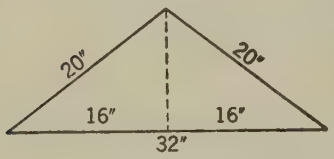


Fig. 62

$$\text{Altitude} = \sqrt{20^2 - 16^2} = \sqrt{400 - 256} = \sqrt{144} = 12''$$

The area of the isosceles triangle is found by the formula:

$$\text{Area} = \tfrac{1}{2}ba$$
$$= \tfrac{1}{2} \times 32 \times 12$$
$$= 192 \text{ sq. in.}$$

43. Area of Scalene Triangles

In scalene triangles the area is found by the same formula

$$\text{Area} = \tfrac{1}{2}ba$$

provided the base and the altitude are known. Sometimes, however, we know the three sides and do not know the altitude. In cases of this sort, a special method is used which does not require the use of the altitude for obtaining the area.

Example

Find the area of the scalene triangle in Fig. 63.

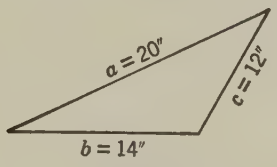


Fig. 63

SOLUTION. First add the three sides and take half the sum.

$$20 + 14 + 12 = 46; \quad \tfrac{1}{2} \text{ of } 46 = 23$$

Now take each side in turn from this half sum.

$$23 - 20 = 3$$

$$23 - 14 = 9$$

$$23 - 12 = 11$$

Then multiply together the half sum and the three quantities obtained by taking each side in turn from the half sum: $23 \times 3 \times 9 \times 11 = 6831$. The square root of this product is the area of the triangle; $\sqrt{6831} = 82.7$. The area is 82.7 sq. in.

The above method is conveniently expressed by the following formula:

$$\text{Area of a triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

We denote the three sides by the letters a, b, and c; s denotes *half* the sum of the sides. The formula reads as follows: Area = the square root of s times $(s - a)$ times $(s - b)$ times $(s - c)$. In Fig. 63, call the 20″ side a, the 14″ side b, and the 12″ side c.

$$s = \tfrac{1}{2} \text{ sum of sides} = \frac{20 + 14 + 12}{2} = \frac{46}{2} = 23$$

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

$$= \sqrt{23(23 - 20)(23 - 14)(23 - 12)}$$

$$= \sqrt{23 \times 3 \times 9 \times 11}$$

$$= \sqrt{6831}$$

$= 82.7$. The area is 82.7 sq. in.

PROBLEMS

1. Find the area of the triangles in Fig. 64. Give the answers to (*a*) and (*c*) in square feet, 1 decimal place; (*b*) and (*d*) in square inches, 1 decimal place.

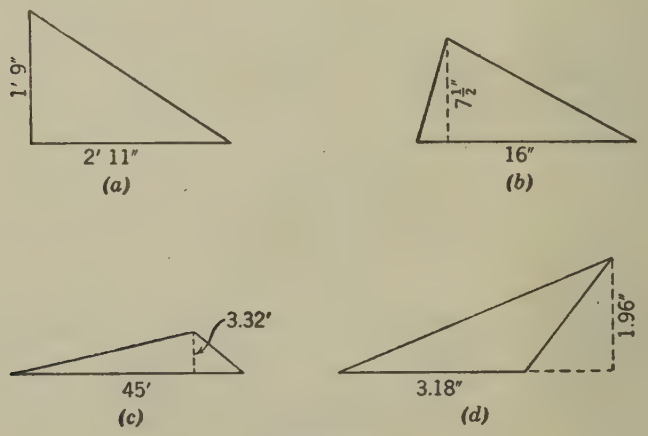


Fig. 64

2. Find the area of the right triangles in Fig. 65. Give all answers in square feet, 1 decimal place.

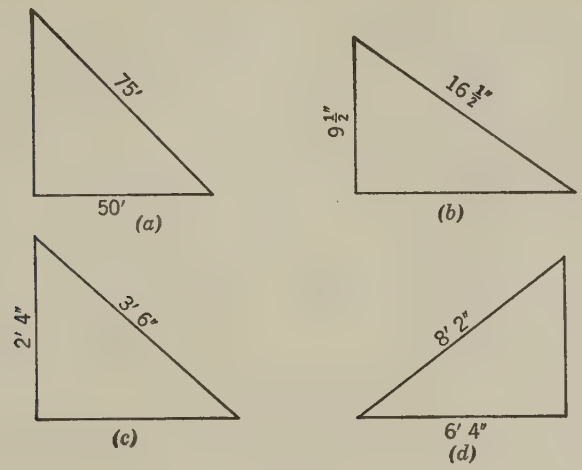


Fig. 65

3. Find the altitude and area of the isosceles triangles in Fig. 66.

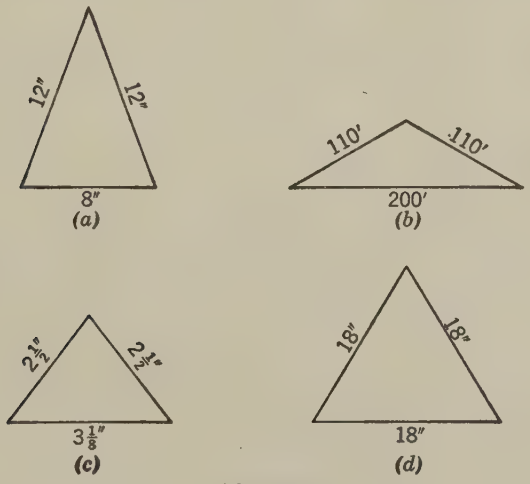


Fig. 66

4. Find the area and length of the equal sides of the isosceles triangles in Fig. 67.

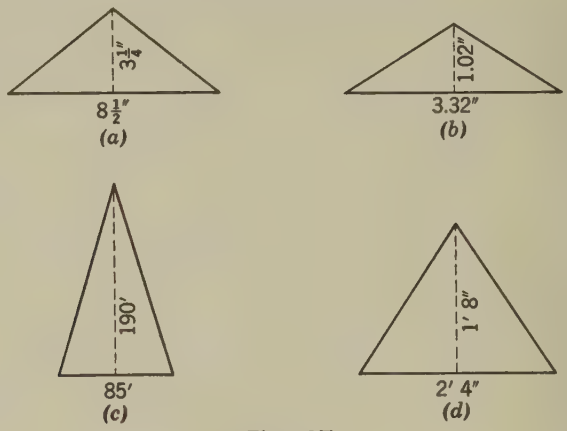


Fig. 67

5. Find the area of the scalene triangles with three sides given, Fig. 68.

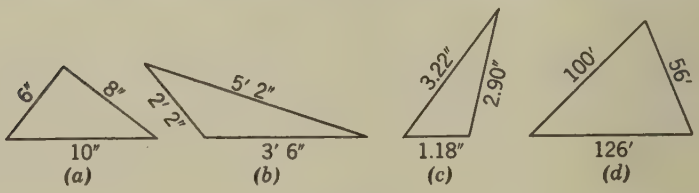


Fig. 68

44. Angles in Triangles

Angles are defined as follows:

A ***right angle*** is a 90-degree angle or a square corner.

An ***acute angle*** is an angle which is less than 90 degrees.

An ***obtuse angle*** is an angle which is more than 90 degrees.

A general law with respect to triangles is: ***The sum of the three angles in any triangle is equal to two right angles, or 2 × 90 degrees = 180 degrees.***

In a right triangle one angle is 90 degrees. Hence the sum of the two acute angles is 180 − 90 = 90 degrees.

In an isosceles triangle the angles opposite the equal sides are equal.

In an equilateral triangle, since all the sides are equal, the angles are all equal; each angle $= \frac{180}{3} = 60$ degrees.

Angles are measured in ***degrees, minutes,*** and ***seconds.***

$$1 \text{ degree} = 60 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

The symbols commonly used for indicating degrees, minutes, and seconds are: degrees (°), minutes (′), seconds (″). We write 27 degrees 18 minutes and 30 seconds, 27° 18′ 30″.

Example 1

In the right triangle, Fig. 69, find the value of angle B.

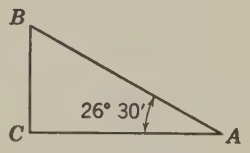


Fig. 69

SOLUTION.

$$\begin{array}{rr} A + B = 90^\circ = & 89^\circ\,60' \\ A = & 26^\circ\,30' \\ \hline \text{Angle } B = & 63^\circ\,30' \end{array}$$

EXPLANATION. Since angles A and B together are 90°, we subtract 26° 30′ from 90°, leaving 63° 30′ for angle B. In order to subtract, we change 90° to 89° 60′.

Example 2

In the isosceles triangle, Fig. 70, compute the base angles B and C.

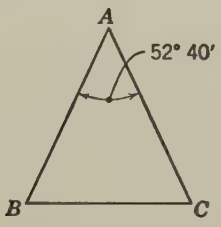


Fig. 70

SOLUTION.

$$\begin{array}{rr} A + B + C = 180^\circ = & 179^\circ\,60' \\ A = & 52^\circ\,40' \\ \hline B + C = & 127^\circ\,20' \end{array}$$

$$\text{Each base angle} = \frac{127^\circ\,20'}{2} = \frac{126^\circ\,80'}{2} = 63^\circ\,40'$$

EXPLANATION. Since the sum of the three angles is 180°, we subtract 52° 40′ from 180° (changed to 179° 60′), leaving 127° 20′ for the two base angles, B and C. Each base angle is one-half of 127° 20′. Changing 127° 20′ to 126° 80′ simplifies the division.

Example 3

In the scalene triangle, Fig. 71, angle $A = 52° 40′ 30″$ and angle $B = 27° 30′ 50″$. Find angle C.

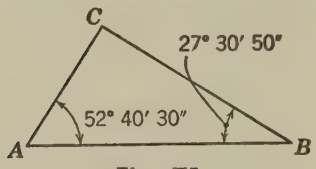


Fig. 71

SOLUTION.

$$\begin{aligned} A &= 52° 40′ 30″ \\ B &= 27° 30′ 50″ \\ \hline A + B &= 79° 70′ 80″ \\ &= 80° 11′ 20″ \end{aligned}$$

$$\begin{aligned} \text{Sum of three angles} = A + B + C &= 180° = 179° 59′ 60″ \\ A + B &= 80° 11′ 20″ \\ \hline \text{Angle } C &= 99° 48′ 40″ \end{aligned}$$

EXPLANATION. Since the sum of the three angles is 180°, we add the two given angles and subtract their sum from 180°.

Example 4

In the isosceles triangle, Fig. 72, the base angles are 47° 39′ 51″ each. Find the vertex angle, that is, the angle opposite the base.

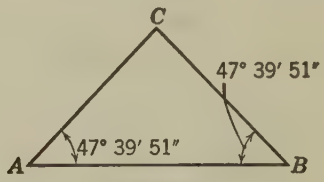


Fig. 72

SOLUTION.

$$\begin{aligned} \text{Base angle} &= 47° 39′ 51″ \\ \text{Base angle} &= 47° 39′ 51″ \\ \hline \text{Sum of base angles} &= 94° 78′ 102″ \\ &= 95° 19′ 42″ \end{aligned}$$

$$\begin{aligned} \text{Sum of three angles} = 180° &= 179° 59′ 60″ \\ \text{Sum of base angles} &= 95° 19′ 42″ \\ \hline \text{Vertex angle} &= 84° 40′ 18″ \end{aligned}$$

EXPLANATION. The base angles in an isosceles triangle are equal. To obtain the vertex angle, subtract the sum of the base angles from 180°.

PROBLEMS

Compute the missing angles in the triangles in Fig. 73.

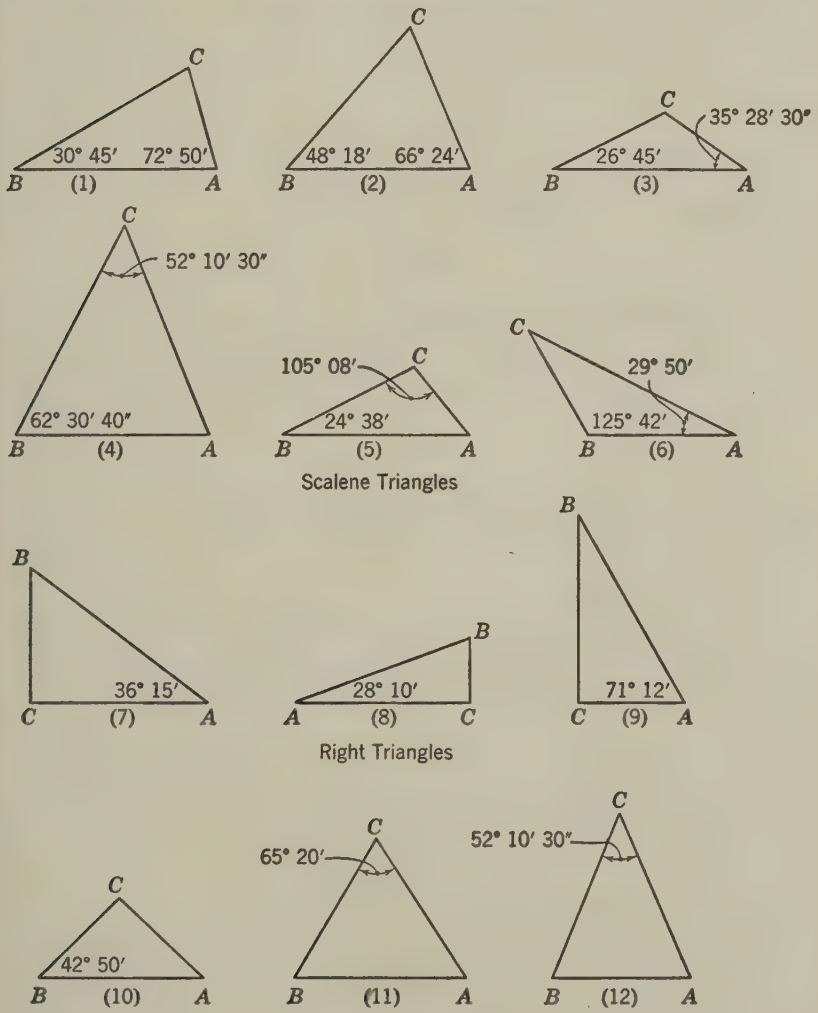
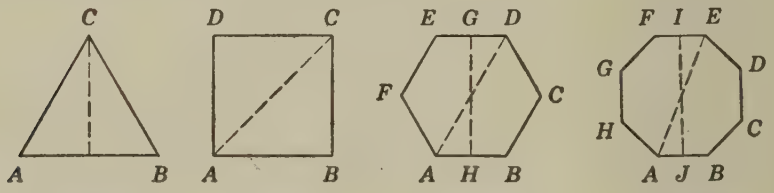


Fig. 73

45. Use of Constants

Figure 74 shows four regular figures which occur frequently in practical work. These figures are the equilateral triangle, the square, the hexagon or regular 6-sided figure, and the octagon or regular 8-sided figure.



a. Equilateral Triangle *b.* Square. *c.* Hexagon *d.* Octagon

Fig. 74

The Equilateral Triangle

The essential parts of the equilateral triangle are: the side, the altitude, the perimeter or distance around the figure, and the area.

The Square

The essential parts of the square are: the side, the diagonal [AC, Fig. 74(b)], the perimeter, and the area.

The Hexagon

The essential parts of the hexagon are: the side, the perimeter, the corners [AD in Fig. 74(c)], the flats, or perpendicular distance between parallel faces [GH in Fig. 74(c)], and the area.

The Octagon

The essential parts of the octagon are: the side, the perimeter, the corners [AE in Fig. 74(d)], the flats [IJ in Fig. 74(d)], and the area.

All these parts can be computed by the application of the laws for right triangles. Considerable labor is involved in this computation, but the work may be greatly shortened by means of certain ratios, called ***factors*** or ***constants,*** which have been derived for the purpose. A table of such constants for the equilateral triangle, the square, the hexagon, and the octagon is given on page 88. The following examples will illustrate the use of this table:

Example 1

Find the altitude of the equilateral triangle shown in Fig. 75.

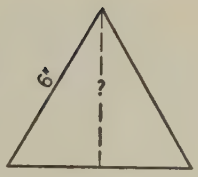


Fig. 75

SOLUTION. $A = .866S = .866 \times 6 = 5.196''$

EXPLANATION. From the table of constants for the equilateral triangle, we select the formula which gives the altitude. Substituting the value of S in this formula gives the altitude.

Example 2

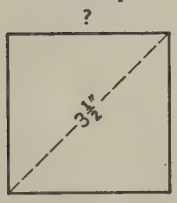


Fig. 76

Find the side of the square shown in Fig. 76.

SOLUTION. $S = .707D = .707 \times 3\frac{1}{2} = 2.475''$

EXPLANATION. From the table of constants for the square select the formula for finding the side. Substituting the value of D in this formula gives the side.

Example 3

Find the side of the octagon shown in Fig. 77.

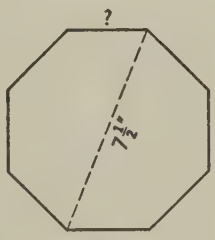


Fig. 77

SOLUTION.

$$S = .383C = .383 \times 7.5 = 2.873''$$

EXPLANATION. In the table of constants for the octagon, we see two formulas for finding the side. We select the formula which gives the side in terms of the corners. Substituting the value of C in this formula, we get the side.

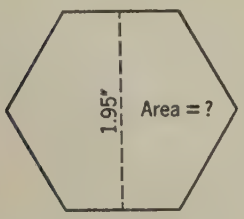


Fig. 78

Example 4

Find the area of the hexagon shown in Fig. 78.

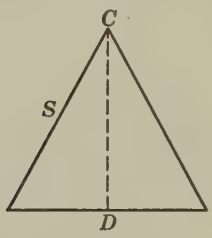
SOLUTION.

$$\text{Area} = .866F^2 = .866 \times 1.95^2$$
$$= .866 \times 3.8025 = 3.29 \text{ sq. in.}$$

EXPLANATION. In the table of constants for the hexagon we see three formulas for finding the area. We select the formula which gives the area in terms of the flats. Substituting the value of F in this formula gives the area.

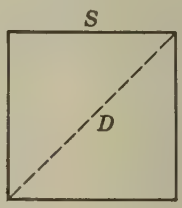
CONSTANTS

EQUILATERAL TRIANGLE



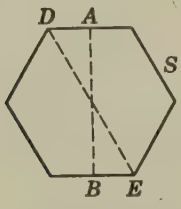
Let S = *Side;*
$CD = A$ = *Altitude.*
$A = 0.866S$;
$S = 1.155A$;
Area $= 0.433S^2$;
$= 0.577A^2$

SQUARE



Let S = *Side;*
D = *Diagonal.*
$D = 1.414S$;
$S = 0.707D$.
Area $= S^2$
$= 0.5D^2$

HEXAGON

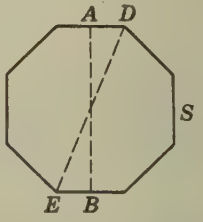


Let $AB = F$ = *Distance across flats;*
$DE = C$ = *Distance across corners;*
S = *Side.*

$F = 0.866C$; $F = 1.732S$;
$C = 2S$; $C = 1.155F$;
$S = 0.5C$; $S = 0.577F$;

Area $= 0.866F^2$;
$= 0.650C^2$;
$= 2.598S^2$.

OCTAGON



Let $AB = F$ = *Distance across flats;*
$DE = C$ = *Distance across corners;*
S = *Side.*

$F = 0.924C$; $F = 2.414S$;
$C = 1.083F$; $C = 2.613S$;
$S = 0.414F$; $S = 0.383C$;

Area $= 0.828F^2$;
$= 0.707C^2$;
$= 4.828S^2$.

PROBLEMS

1. Find the area of an equilateral triangle with $4\frac{1}{2}''$ sides.
2. Find the area of an equilateral triangle whose altitude is $4\frac{3}{4}''$.
3. Find the diagonal of a $1.75''$ square.
4. Find the side of a square whose diagonal is $3.45''$.
5. Find the area of a hexagon which is $10''$ across flats.
6. Find the area of a hexagon with $1\frac{3}{4}''$ sides.
7. Find the area of a hexagon which measures $\frac{7}{8}''$ across corners.
8. Find the area of an octagon which is $4.78''$ across flats.
9. Find the area of an octagon with $5\frac{1}{2}''$ sides.
10. Find the area of an octagon $9''$ across corners.
11. Find the side of an equilateral triangle whose altitude is $8\frac{1}{4}''$.
12. Find the flats of an octagon which has $6''$ sides.
13. Find the side of an octagon which measures $6.3''$ across flats.
14. Find the altitude of an equilateral triangle with $5\frac{1}{2}''$ sides.
15. Find the side of an octagon which measures $1\frac{7}{8}''$ across flats.
16. Find the area of a square whose diagonal is $7\frac{1}{2}''$.
17. Find the area of a hexagon which is $8\frac{1}{4}''$ across flats.
18. Find the corners of an octagon which is $6.03''$ across flats.
19. Find the area of an equilateral triangle with $3.16''$ sides.
20. Find the area of a hexagon with $1.09''$ sides.
21. A sharp V-thread, Fig. 79, has the cross-section of an equilateral triangle. If the pitch or distance AB is $\frac{1}{12}''$, find the depth CD.

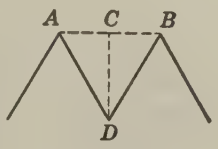


Fig. 79

22. What should be the diameter of a piece of round stock, Fig. 80, if we wish to mill a $1\frac{3}{4}''$ square on the end of it?

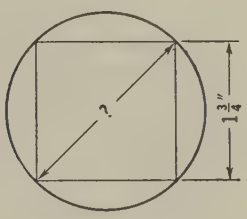


Fig. 80

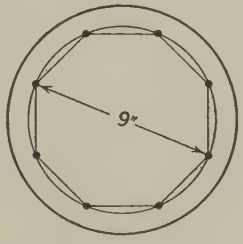


Fig. 81

23. Eight bolt holes are to be drilled evenly spaced on the circumference of a $9''$ circle, as in Fig. 81. What will be the distance center to center of holes?

24. We are required to make a wrench, Fig. 82, to fit a hexagonal nut which measures $2\frac{1}{4}''$ across corners. What must be the width of the opening between the jaws of the wrench?

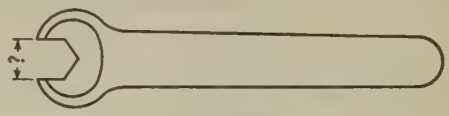


Fig. 82

25. How many feet of railing are required around an octagonal platform which measures 38′ 6″ across flats?

26. Find the corner measurement, Fig. 83, for laying out an octagon on a piece of $2\frac{1}{2}''$ square stock.

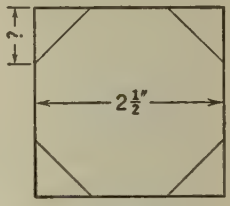


Fig. 83

27. A piece of $1\frac{3}{4}''$ round stock is to have a hexagonal end milled on it, as in Fig. 84. Find the distance across flats of the hexagon.

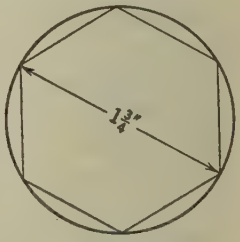


Fig. 84

28. Figure 85 shows a room with a bay window in the shape of a half hexagon. The distance across AB is 8′ 6″. Find the distance CD.

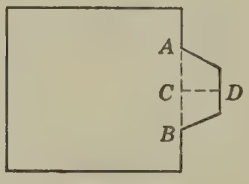


Fig. 85

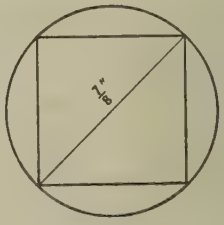


Fig. 86

29. What is the largest square that can be cut from a piece of $\frac{7}{8}''$ round stock? Fig. 86.

46. Four-sided Figures

Figures with four sides are called ***quadrilaterals.*** The various types of quadrilaterals are shown in Fig. 87.

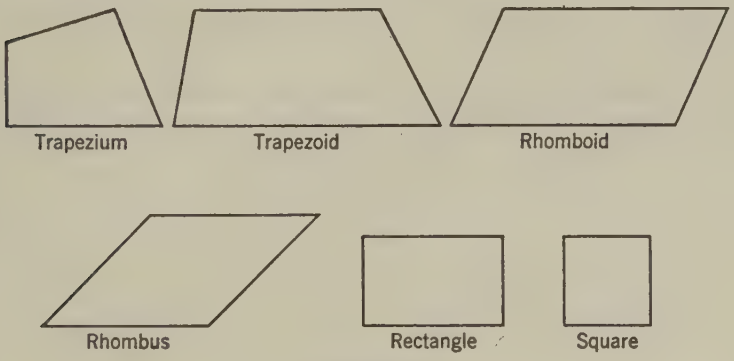


Fig. 87

A ***parallelogram*** is a quadrilateral which has two pairs of parallel sides. The types of parallelograms are:

The ***rectangle,*** in which all the angles are right angles.

The ***square,*** in which all the angles are right angles and the four sides are equal.

The ***rhomboid,*** which has two pairs of parallel sides but no right angles.

The ***rhombus,*** which has four equal sides but no right angles.

A ***trapezoid*** is a quadrilateral in which only two of the sides are parallel.

An ***isosceles trapezoid*** is a trapezoid in which the two non-parallel sides are equal.

A ***trapezium*** is a quadrilateral which has no two sides parallel; in other words, an irregular four-sided figure.

47. Area of Trapezoids

We can find the area of the trapezoid $ABCD$, Fig. 88, by the following method. Draw a center line parallel to and midway between the upper base AB and the lower base CD. Where this center line EF meets the sides, draw perpendicu-

lars to it, forming the rectangle *KLHG*. The rectangle *KLHG* is equal in area to the trapezoid *ABCD*, because the two triangles which are cut off the trapezoid in forming the rectangle are equal to the triangles which are gained. Triangle *KAE* is equal to triangle *EGD*, and triangle *BLF* is equal to triangle *FCH*.

The area of a rectangle is found by multiplying the base by the altitude. To find the area of the rectangle *KLHG*,

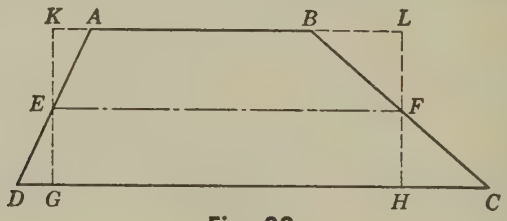


Fig. 88

we multiply the height *LH* by the width *EF*. But the area of the trapezoid *ABCD* is equal to the area of the rectangle *KLHG*. We can therefore find the area of the trapezoid by multiplying the height by the width along the center line *EF*. Since *EF* is half-way between the upper and the lower bases, it is equal to their average, or half their sum. We thus get the following formula for the area of a trapezoid:

$$\textit{Area of trapezoid} = \textit{Average width} \times \textit{Altitude}$$

Example 1

Find the area of the trapezoid in Fig. 89.

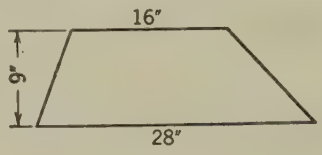


Fig. 89

SOLUTION. Area = Average width × Altitude

$$\text{Average width} = \frac{16 + 28}{2} = \frac{44}{2} = 22''$$

$$\text{Area} = 22 \times 9$$

$$= 198 \text{ sq. in.}$$

Example 2

Find the altitude of the isosceles trapezoid in Fig. 90.

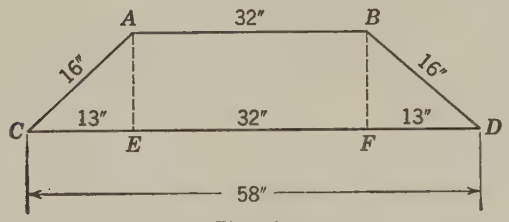


Fig. 90

SOLUTION. $CE + FD = 58 - 32 = 26''$

$$CE = \tfrac{26}{2} = 13''$$

$$AE = \sqrt{\text{Hypotenuse}^2 - \text{Base}^2}$$

$$= \sqrt{16^2 - 13^2} = \sqrt{256 - 169} = \sqrt{87} = 9.33''$$

EXPLANATION. The perpendiculars AE and BF cut off equal distances CE and FD on either side of EF. But, since $EF = AB = 32''$, that makes $CE = FD = \dfrac{58 - 32}{2} = \dfrac{26}{2} = 13''$. In triangle ACE, the base is $13''$ and the hypotenuse is $16''$. Hence, the altitude $= \sqrt{16^2 - 13^2} = 9.33''$.

PROBLEMS

1. Find the area of the trapezoids in Fig. 91.

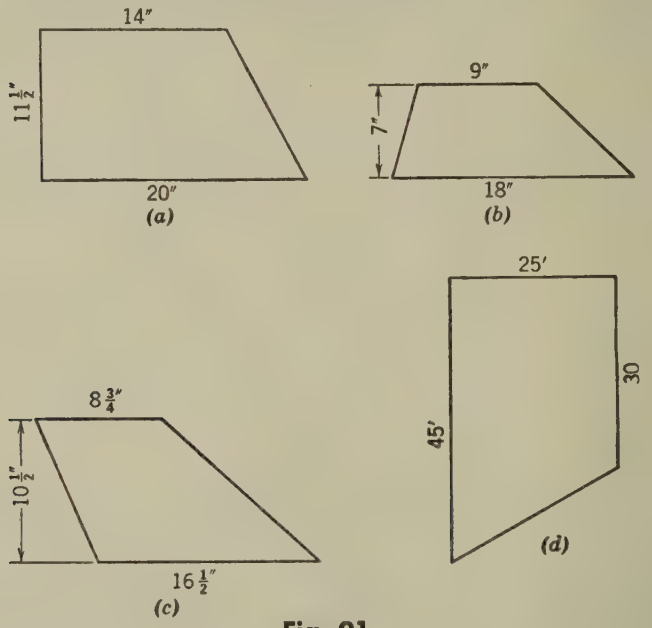


Fig. 91

2. Find the area and length of equal sides of the isosceles trapezoids in Fig. 92.

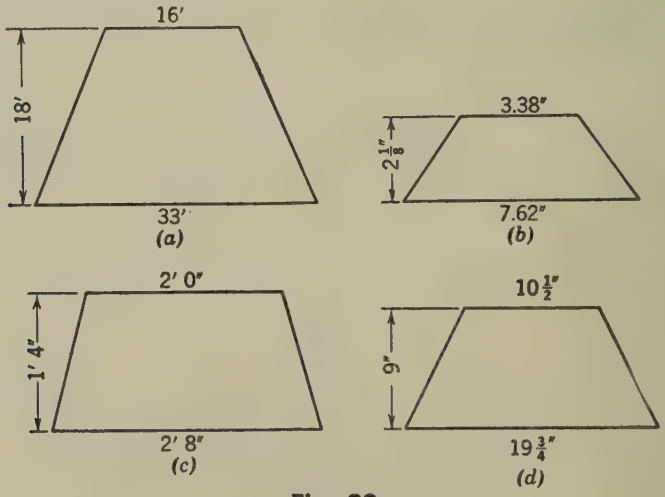


Fig. 92

3. Find the altitude and area of the isosceles trapezoids in Fig. 93.

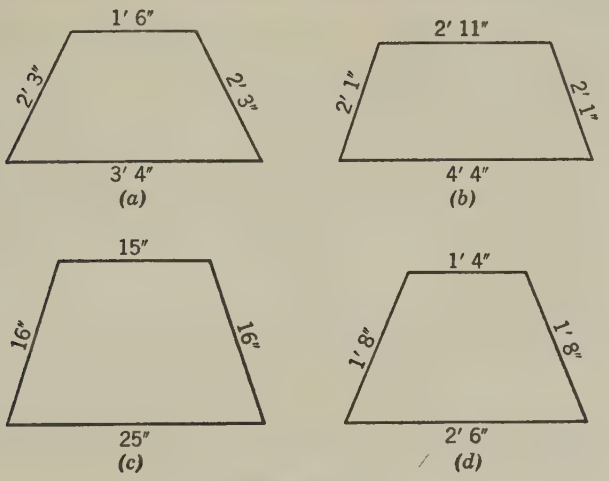


Fig. 93

48. Area of Composite Figures

Very often we are required to obtain the area of figures which do not belong to any of the classes we have studied. Such figures can sometimes be divided into smaller parts, the area of which we can get by means of rules already learned.

Example 1

Find the area of the surface in Fig. 94.

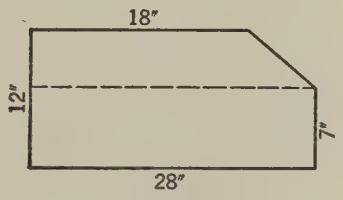


Fig. 94

SOLUTION. This figure can be divided as shown by the dotted construction line into a trapezoid and a rectangle.

The altitude of the trapezoid $= 12 - 7 = 5''$

The average width of the trapezoid $= \dfrac{18 + 28}{2} = \dfrac{46}{2} = 23''$

Area of trapezoid $= 23 \times 5 = 115$ sq. in.
Area of rectangle $= 28 \times 7 = 196$ sq. in.
Total area $= 311$ sq. in.

Example 2

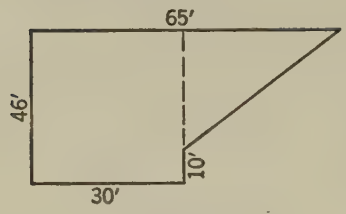


Fig. 95

Find the area of the surface in Fig. 95.

Solution. The dotted construction line divides the figure into a rectangle and a triangle.

Altitude of triangle = 46 − 10 = 36′
Base of triangle = 65 − 30 = 35′

Area of triangle = $\frac{1}{2} \times 35 \times 36$ = 630 sq. ft.
Area of rectangle = 30 × 46 = 1380 sq. ft.
Total area = 2010 sq. ft.

PROBLEMS

Find the area of the composite figures shown in Fig. 96. Give the answers in square feet, 1 decimal place.

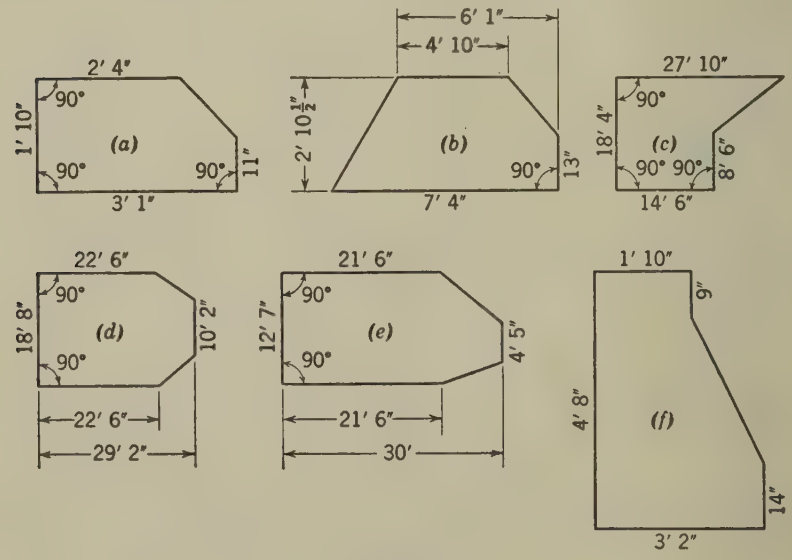


Fig. 96

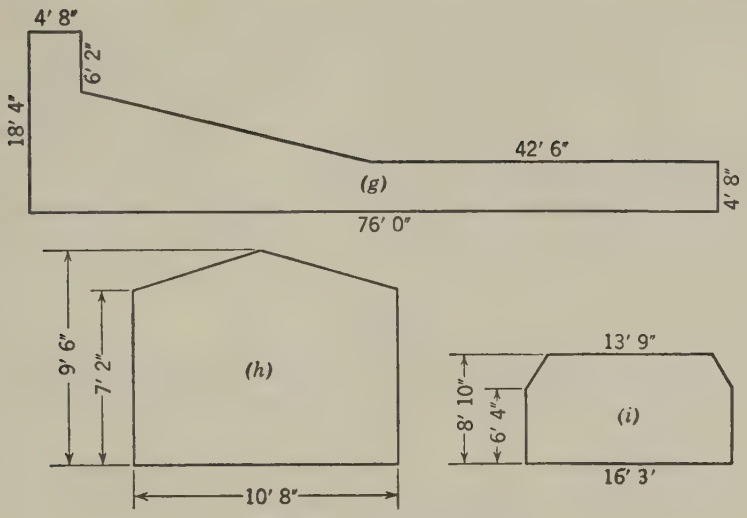


Fig. 96 (Continued)

49. Scale

Diagrams of objects are generally drawn smaller than the objects they represent. For example, the top of an end table 3′ long and 1′ 3″ wide may be represented by a diagram 3″ long and $1\frac{1}{4}$″ wide, Fig. 97. Every inch in the diagram

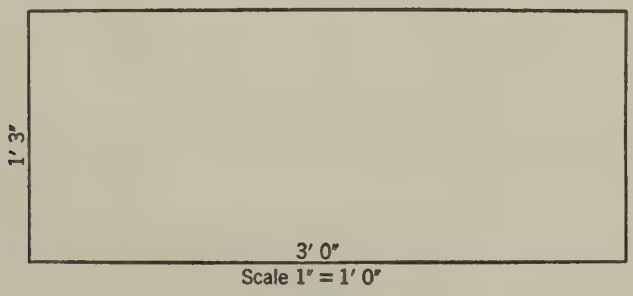


Fig. 97

would therefore represent a foot length in the table. And, since the ratio of 1″ to 1′ is 1 to 12, we say that the diagram is drawn to a scale of $\frac{1}{12}$, 1:12, or 1″ = 1′. In general, scale means the ratio of any dimension on the drawing to the corresponding dimension on the object represented.

50. To Find the Drawing Measure

If a drawing is to be made to a scale of $\frac{1}{4}$, it means that every distance on the object is to be represented by a line one-fourth as long. An object $8'\ 4\frac{1}{2}''$ long drawn to a $\frac{1}{4}$ scale would be represented by a diagram $2'\ 1\frac{1}{8}''$ in length, or $\frac{1}{4}$ of $8'\ 4\frac{1}{2}''$. Hence the rule: *To find the length of any line on the diagram, multiply the dimension on the object by the scale ratio.*

PROBLEMS

1. A box $5'\ 4''$ long, $4'\ 8''$ wide, and $3'\ 0''$ high is drawn to a scale of $\frac{1}{8}$. Compute the length of the lines in the diagram that will represent the length, width, and height of the box.

2. The length of an object is $2'\ 4''$. Find the length of the corresponding line in a diagram drawn to: (*a*) $\frac{1}{2}$ size; (*b*) $\frac{1}{4}$ size; (*c*) scale: $\frac{3}{8}$; (*d*) scale: $\frac{1}{8}'' = 1'$; (*e*) scale: $\frac{1}{8}$.

3. A drawing of an object is made to scale: $\frac{3}{4}$. Find the length of the lines in the drawing that will represent the following measurements on the object: (*a*) $1'\ 6''$; (*b*) $6\frac{1}{2}''$; (*c*) $3'\ 5\frac{1}{2}''$; (*d*) $11\frac{3}{4}''$; (*e*) $14''$.

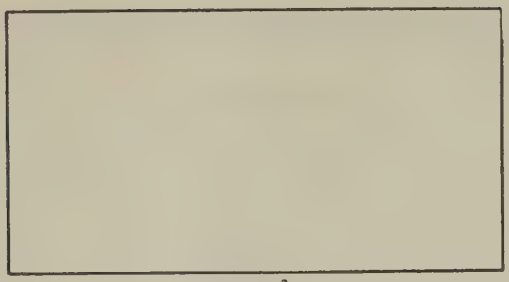
4. A drawing is to be made to a scale of $1'' = 1'$. Compute the dimensions in the drawing that will correspond to the following dimensions on the object: (*a*) $3'\ 9''$; (*b*) $2'\ 3''$; (*c*) $2'\ 6''$; (*d*) $18''$; (*e*) $4'\ 0''$.

51. To Find the Actual Dimension on the Object from the Measured Dimension on the Diagram

If the scale of a diagram is $\frac{1}{8}$, it means that the length of a line on the object is 8 times as long as the corresponding line in the diagram. Thus, if a line in a diagram measures $1\frac{3}{4}''$, the corresponding dimension on the object will be $8 \times 1\frac{3}{4} = 14''$; or $1\frac{3}{4}'' \div \text{scale ratio} = 1\frac{3}{4} \div \frac{1}{8} = 1\frac{3}{4} \times \frac{8}{1} = 14''$. Hence the rule: *To find the measure on an object corresponding to the length of a line on the diagram, divide the diagram measure by the scale ratio.*

PROBLEMS

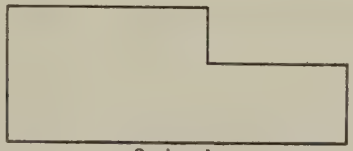
1. The diagram in Fig. 98 is drawn to a scale of $\frac{3}{4}$. The length of the diagram measures $2\frac{1}{2}''$ and the width measures $1\frac{3}{8}''$. Find the corresponding dimensions of the object.



Scale = $\frac{3}{4}$

Fig. 98

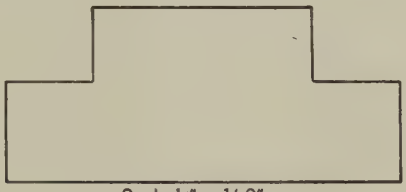
2. Measure the lines in the diagram in Fig. 99 and compute the actual dimensions of the object.



Scale = $\frac{1}{8}$

Fig. 99

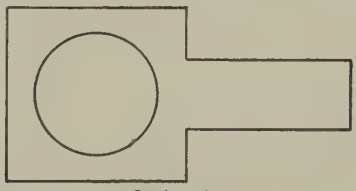
3. Measure the lines in the diagram in Fig. 100 and compute the actual dimensions of the object.



Scale $\frac{1}{4}$" = 1' 0"

Fig. 100

4. Measure the lines in the diagram in Fig. 101 and compute the actual dimensions of the object.



Scale = $\frac{1}{4}$

Fig. 101

5. Measure the long and short diameters of the ellipse in Fig. 102 and compute the actual diameters of the object represented.

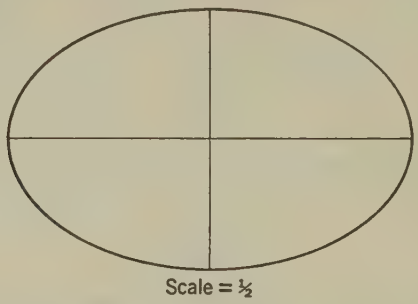


Fig. 102

52. To Find the Area of a Figure Drawn to a Certain Scale

Measure each line in the diagram and compute the actual length of the corresponding lines in the object. Then proceed to find the area represented according to the rules given for the various figures.

Example

Find the area of the surface represented by the rectangle in Fig. 103.

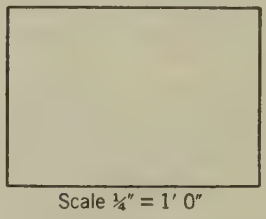


Fig. 103

SOLUTION. Measuring the length and width of the rectangle in the figure, we find that the dimensions are $1\frac{1}{4}''$ and $\frac{7}{8}''$ respectively. Since the scale is $\frac{1}{4}'' = 1'$, the scale ratio is $\frac{\frac{1}{4}''}{12''} = \frac{1}{4} \times \frac{1}{12} = \frac{1}{48}$. In other words the length of a line on the object is 48 times the corresponding length on the diagram. Hence the actual length of the object $= 48 \times 1\frac{1}{4}'' = 48 \times \frac{5}{4}'' = 60''$, and the actual width of the object is $48 \times \frac{7}{8}'' = 42''$.

The actual area of the surface represented = $60 \times 42 = 2520$ sq. in. = 17.5 sq. ft.

PROBLEMS

1. Find the area represented by the rectangle in Fig. 104 if $AB = 1\frac{3}{8}''$ and $BC = \frac{3}{4}''$.

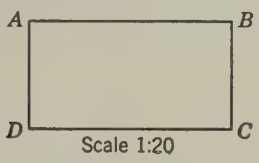


Fig. 104

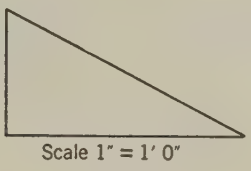


Fig. 105

2. Find the actual area of a disc represented by a circle whose diameter is $1\frac{1}{4}''$ when drawn to a scale of $\frac{1}{8}$.

3. The base of the right triangle in Fig. 105 is $1\frac{1}{8}''$ and the altitude is $\frac{5}{8}''$. Find the actual area represented by the diagram.

4. Measure on the diagram in Fig. 106 the base and altitude and compute the actual area represented by the triangle.

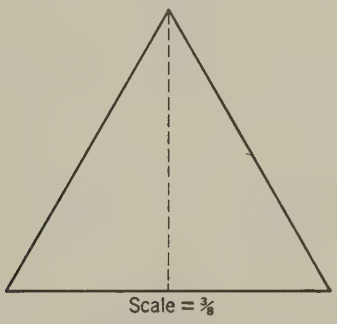


Fig. 106

5. Measure the altitude and bases of the trapezoid in Fig. 107 and compute the area represented by the diagram.

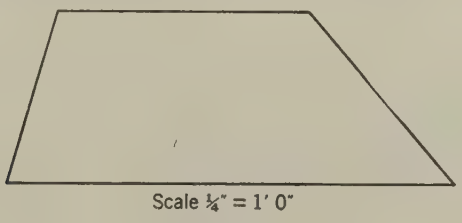


Fig. 107

53. Circles

The perimeter of the circle is called the ***circumference.*** The distance from the center to the circumference of the circle is called the ***radius.***

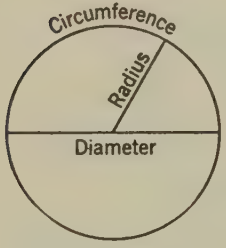


Fig. 108

The distance across the circle through the center is called the ***diameter.*** The diameter is equal to two radii. See Fig. 108.

When we speak of a 12″ pulley, we mean a pulley 12″ in diameter. Similarly, a piece of $\frac{1}{2}$″ round stock means a piece of stock $\frac{1}{2}$″ in diameter. A 10″ circle is a circle whose diameter is 10″.

54. Circumference

It has been found by measurement and computation that the circumference of a circle is approximately 3.14 times the diameter. The number 3.14 is represented by the Greek letter π (pi). The relation between the diameter and the circumference of a circle is expressed by the formula

$$\text{Circumference} = \pi D$$

where D stands for the diameter of the circle.

Example

Find the circumference of a 16″ circle.

SOLUTION. $\text{Circumference} = \pi D$

$= 3.14 \times 16$

$= 50.24$ in.

NOTE. The value 3.14 for π is only approximate. The exact value of π cannot be computed, but for ordinary use, 3.14 or $3\frac{1}{7}$ is close enough. For more accurate work we use the value 3.1416 for π, which is close enough for any computation that may arise in the course of trade work. The following tabulation shows a comparison of the three values of π mentioned above:

$$3.14$$

$$3.1416$$

$$3\tfrac{1}{7} = 3.1429$$

The first three figures of each of these values being the same, the approximations 3.14 and $3\frac{1}{7}$ are accurate enough for ordinary work. In the succeeding pages, use $\pi = 3.14$ or $\pi = 3\frac{1}{7}$ unless more accurate work is called for. The number $3\frac{1}{7}$ is more conveniently expressed as $\frac{22}{7}$ where cancellation may be employed in the computation.

PROBLEMS

1. Find the circumference of a 14″ circle.
2. Find the circumference of a 5″ circle.
3. Find the circumference of a 2″ circle.
4. Find the circumference of a 1″ circle.
5. Find the circumference of a $2\frac{1}{8}$″ circle.
6. Find the circumference of a $1\frac{7}{8}$″ circle.
7. Find the circumference of a $3\frac{1}{4}$″ circle.
8. Find the circumference of a circle 4′ 9″ in diameter.
9. Find the circumference of a circle 135′ in diameter.
10. What is the circumference of a 32″ pulley?
11. What must be the length of a label to fit around a can 3″ in diameter, allowing $\frac{1}{4}$″ for pasting?
12. What must be the length of a bar of steel to make a rim for a carriage wheel 4′ 2″ in diameter, allowing 1″ for weld?
13. An emery wheel is 8″ in diameter. How many inches will a point on the circumference travel while the wheel makes one revolution?
14. What distance does an automobile travel with each complete turn of the wheels if the wheels are $29\frac{1}{2}$″ in diameter?
15. A circular running track in a gymnasium has an average diameter of 62′. Find the circumference.
16. A water tank is 10′ 6″ in diameter. Find the length of the tie rods around it, allowing 6″ for fastenings.
17. What must be the length of a belt connecting two 16″ pulleys which are 3′ 8″ center to center?

55. To Find the Diameter of a Circle

When the circumference is given, the diameter is found by dividing the circumference by π. The formula is:

$$D = \frac{\text{Circumference}}{\pi}$$

Example

Find the diameter of a circle whose circumference is 42.5″.

SOLUTION. $$D = \frac{\text{Circumference}}{\pi}$$

$$= \frac{42.5}{3.14} = 13.54 \text{ in.}$$

PROBLEMS

1. Find the diameter of a circle whose circumference is 12″.
2. Find the diameter of a circle whose circumference is 280′.
3. Find the diameter of a circle whose circumference is 3.73″.
4. Find the diameter of a circle whose circumference is 3′ 9″.
5. Find the diameter of a circle whose circumference is 11″.
6. What is the diameter of a shaft if the distance around the shaft is $23\frac{1}{2}$″?
7. What is the pitch diameter of a gear if the circumference of the pitch circle is 20.42″?
8. The circumference of a pulley is 75.4″. Find its diameter.
9. What is the diameter of a piece of round stock whose circumference is 8.53″?
10. What is the diameter of a cylindrical tank if the distance around the tank is 62.5′?
11. One lap around a circular running track is 220 yds. What is the diameter of the track?
12. What must be the diameter of an emery wheel if a point on the circumference travels 20.41″ for each revolution?
13. The circumference of the trunk of a tree measures 7′ 4″. What is its diameter?

56. To Find the Area of a Circle

METHOD 1. To find the area of a circle, we multiply the square of the radius by π. The formula is:

$$\text{Area} = \pi R^2$$

Example

Find the area of an 11″ circle.

SOLUTION. $\text{Area} = \pi R^2$; $R = \frac{11}{2} = 5.5$″

$$\text{Area} = 3.14 \times 5.5^2$$

$$= 3.14 \times 30.25 = 94.99 \text{ sq. in.}$$

PROBLEMS

1. Find the area of a 2″ circle.
2. Find the area of a 14″ circle.
3. Find the area of a 65′ circle.
4. Find the area of a $4\frac{1}{2}$″ circle.
5. Find the area of a $3\frac{1}{8}$″ circle.
6. Find the area of a $1\frac{1}{4}$″ circle.
7. Find the area of a circle 1.85″ in diameter.
8. Find the area of a circle 3.6″ in diameter.
9. Find the area of a circle 5′ 9″ in diameter.
10. How many square feet of pavement in a circle 108′ in diameter?
11. A sheet of steel $\frac{1}{8}$″ thick and 1′ square weighs 5.08 lb. What is its weight after an 8″ hole is cut in it?
12. The top of a gas tank 72′ 8″ in diameter is to be painted at a cost of $0.45 per square yard. Find the cost.
13. Find the area of a piston head $3\frac{3}{4}$″ in diameter.
14. Find the total pressure on the bottom of a cylindrical tank 8′ 4″ in diameter if the pressure is 920 lb. per square foot.
15. A splice plate 18″ × 6″, Fig. 109, has 10 rivet holes each $\frac{7}{8}$″ in diameter punched in it. What is the net area of the plate?

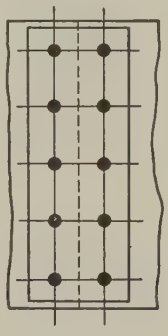


Fig. 109

METHOD 2. To find the area of a circle, we multiply the square of the diameter by $\frac{\pi}{4}$. The formula is:

$$\text{Area} = \frac{\pi D^2}{4}$$

Since $\frac{\pi}{4} = \frac{3.14}{4} = 0.785$, we may write the formula thus:

$$\text{Area} = 0.785D^2$$

Example

Find the area of a 15″ circle.

SOLUTION. $\text{Area} = 0.785D^2$

$= 0.785 \times 15^2$

$= 0.785 \times 225 = 176.6$ sq. in.

PROBLEMS

NOTE. *A simple way to check the area of a circle approximately is to multiply the square of the diameter by 0.8.*

1. Find the area of a 5″ circle.
2. Find the area of a 16″ circle.
3. Find the area of a 72′ circle.
4. Find the area of a $5\frac{1}{2}$″ circle.
5. Find the area of a 9″ circle.
6. Find the area of a $2\frac{1}{2}$″ circle.
7. Find the area of a circle 2.64″ in diameter.
8. Find the area of a circle 5.8″ in diameter.
9. Find the area of a circle 6′ 3″ in diameter.
10. Find the net area of a manhole cover 26″ in diameter with 13 vent holes each 1″ in diameter.
11. Find the area of the doorway with the semicircular top shown in Fig. 110.

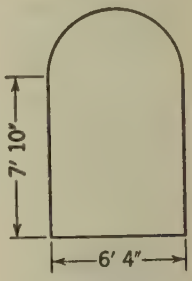


Fig. 110

12. Find the area of the bottom of the clothes boiler with semicircular ends shown in Fig. 111.

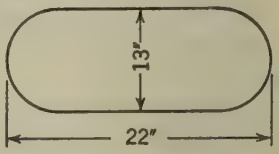


Fig. 111

57. To Find the Diameter When the Area of the Circle Is Given

METHOD 1. To find the diameter of a circle when we know the area, we use the formula

$$D = 2\sqrt{\frac{\text{Area}}{\pi}}$$

which reads as follows:

Diameter = twice the square root of the area divided by π

This formula indicates the following operations:

1. Divide the area by 3.14.
2. Find the square root of the quotient.
3. Multiply the square root of the quotient by 2.

Example

Find the diameter of a circle whose area is 36.8 sq. in.

SOLUTION. $D = 2\sqrt{\frac{\text{Area}}{\pi}}$

$$= 2\sqrt{\frac{36.8}{3.14}}$$

$$= 2\sqrt{11.7197}$$

$$= 2 \times 3.42 = 6.84''$$

PROBLEMS

1. Find the diameter of a circle whose area is 100 sq. in.
2. Find the diameter of a circle whose area is 5000 sq. ft.
3. Find the diameter of a circle whose area is 5.78 sq. in.
4. Find the diameter of a circle whose area is 1.19 sq. in.
5. Find the diameter of a circle whose area is 30.5 sq. in.
6. What is the diameter of a circular disc whose area is $5\frac{1}{4}$ sq. in.?
7. A round steel rod can safely bear a pull of 16,000 lb. per square inch. What must be the diameter of a rod to carry 125,000 lb.?
8. What must be the diameter of a rivet to have a cross-section area of 0.6 of a square inch?
9. A copper wire is required to have a cross-section area of $\frac{1}{2}$ sq. in. What must be the diameter?
10. What must be the diameter of a pipe to have an area of 4.78 sq. in.?
11. What must be the diameter of a cylinder if the base area is required to be 125 sq. in.?

METHOD 2. We can also find the diameter of a circle by means of the formula

$$D = \sqrt{\frac{\text{Area}}{0.785}}$$

which reads as follows:

Diameter = the square root of the area divided by 0.785

This formula indicates the following operations:

1. Divide the area by 0.785.
2. Find the square root of the quotient.

Example

Find the diameter of a circle whose area is 25.5 sq. in.

SOLUTION. $D = \sqrt{\frac{\text{Area}}{0.785}} = \sqrt{\frac{25.5}{0.785}}$

$= \sqrt{32.47} = 5.70$ in.

PROBLEMS

1. Find the diameter of a circle whose area is 80 sq. in.
2. Find the diameter of a circle whose area is 1000 sq. ft.
3. Find the diameter of a circle whose area is 14.62 sq. in.
4. Find the diameter of a circle whose area is 3.35 sq. in.
5. Find the diameter of a circle whose area is 41.8 sq. in.
6. Find the diameter of a circle whose area is 7.5 sq. in.
7. What must be the diameter of a steel wire to have a cross-section area of $\frac{1}{8}$ sq. in.?
8. A circular wooden post is required to have a cross-section area of 20 sq. in. What must the diameter be?
9. What must be the diameter of a steel rod whose cross-section area is $2\frac{1}{2}$ sq. in.?
10. The base area of a cylindrical tank is 250 sq. ft. What is its diameter?

58. To Find the Diameter of a Circle Equal in Area to the Combined Areas of Two or More Circles

This involves the following processes:

1. Find the area of each circle.
2. Find the sum of the areas.
3. Find the diameter of the circle whose area is equal to the sum of the areas of the individual circles.

Example 1

Find the diameter of a circle whose area is equal to the combined areas of a 3″ circle and a 4″ circle.

SOLUTION.

$$\text{Area of } 3'' \text{ circle} = 3.14 \times 1.5 \times 1.5 = 7.07 \text{ sq. in.}$$
$$\text{Area of } 4'' \text{ circle} = 3.14 \times 2 \times 2 = 12.56 \text{ sq. in.}$$
$$\text{Combined areas} = 19.63 \text{ sq. in.}$$

$$D = 2\sqrt{\frac{\text{Area}}{\pi}} = 2\sqrt{\frac{19.63}{3.14}} = 2\sqrt{6.2516} = 2 \times 2.50 = 5$$

The diameter is 5 in.

Example 2

Find the diameter of a circle whose area is equal to the combined areas of three 5″ circles and two 4″ circles.

SOLUTION.

$$\text{Area of } 5'' \text{ circle} = 3.14 \times 2.5 \times 2.5 = 19.63 \text{ sq. in.}$$

$$\text{Area of three } 5'' \text{ circles} = 3 \times 19.63 = 58.89 \text{ sq. in.}$$

$$\text{Area of } 4'' \text{ circle} = 3.14 \times 2 \times 2 = 12.56 \text{ sq. in.}$$

$$\text{Area of two } 4'' \text{ circles} = 2 \times 12.56 = 25.12 \text{ sq. in.}$$
$$\text{Combined areas} = 84.01 \text{ sq. in.}$$

$$D = 2\sqrt{\frac{\text{Area}}{\pi}} = 2\sqrt{\frac{84.01}{3.14}} = 2\sqrt{26.7548} = 2 \times 5.17 = 10.34$$

The diameter of the required circle is 10.34 in.

If we are required to find the diameter of a circle whose area is equal to the *difference* between the areas of two other circles, we proceed as before except that we subtract the areas instead of adding them.

PROBLEMS

1. Find the diameter of a circle equal in area to a 6″ circle and a 4″ circle.

2. Find the diameter of a circle equal in area to the combined areas of a 2″ circle, a 3″ circle, and a 5″ circle.

3. Find the diameter of a circle whose area is equal to the area of six circles each $1\frac{1}{2}''$ in diameter.

4. What must be the diameter of a circle whose area is equal to the sum of the areas of three $1''$ circles and three $\frac{1}{2}''$ circles?

5. What must be the diameter of a circle whose area is equal to the sum of the areas of ten $2\frac{1}{2}''$ circles?

6. Two $3''$ pipes run together into one pipe. What should be the diameter of the large pipe to carry off the flow from the other two pipes?

7. A $2''$ pipe and another pipe are used to carry off the flow from a $3''$ pipe. Find the size of the other pipe.

8. How many $1''$ pipes are required to carry as much water as a $2''$ pipe?

9. How many air ducts $10''$ in diameter are required to have the same cross-section area as one air duct of rectangular cross-section, $24'' \times 40''$?

10. How many copper wires of No. 20 B. & S. gage ($0.032''$ diameter) are required to have the same cross-section area as a No. 0000 B. & S. gage wire whose diameter is $0.46''$?

59. Short Method of Comparing Areas of Circles

It is frequently convenient to make use of a shorter method of comparing the areas of circles. We know that the area of a $5''$ square is equal to the combined areas of a $4''$ square and a $3''$ square. See Fig. 112.

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

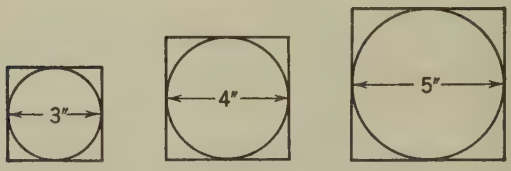


Fig. 112

To find the size of a square equal in area to the combined areas of two or more squares, we add those areas and find the square root of the sum.

$\sqrt{3^2 + 4^2} = \sqrt{25} = 5''$, the size of square required

This same rule holds for circles, i.e., ***the areas of circles bear the same ratio to each other as the squares of their***

diameters. The area of a 5″ circle is equal to the combined areas of a 4″ circle and a 3″ circle. Instead of actually computing the areas of the circles, we square their diameters and get the square root of the sum of the squares.

Example

Find by the short method the diameter of a circle whose area is equal to the combined areas of a 6″ circle, an 8″ circle, and a 9″ circle.

SOLUTION. Diameter $= \sqrt{6^2 + 8^2 + 9^2}$

$= \sqrt{36 + 64 + 81}$

$= \sqrt{181}$

$= 13.45''$

A circle 13.45″ in diameter has an area which is equal to the sum of the areas of a 6″ circle, an 8″ circle, and a 9″ circle.

PROBLEMS

Work all the problems in the preceding section by the short method and compare the results with those obtained by the long method.

60. Area of Ring Sections

To find the area of a ring section, such as Fig. 113, subtract the area of the inside circle from the area of the outside circle.

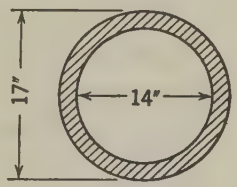


Fig. 113

Example

Find the area of cross-section of a pipe whose inside diameter is 14″ and outside diameter, 17″.

SOLUTION.

Area of 17″ circle $= 3.14 \times 8\frac{1}{2} \times 8\frac{1}{2} = 226.98$ sq. in.

Area of 14″ circle $= 3.14 \times 7 \times 7 \quad = 153.94$ sq. in.

Net area $= 73.04$ sq. in.

The area of the ring section is 73.04 sq. in.

PROBLEMS

1. Find the area of a flat ring whose outside diameter is 11″ and inside diameter 7″.

2. Find the area of cross-section of a brass pipe $2\frac{1}{2}$″ inside diameter, if the metal is $\frac{1}{4}$″ thick.

3. How many square feet of pavement are there in a path around a fountain 32′ in diameter if the path is 6′ 6″ wide?

4. What is the area of cross-section of a hollow cast iron column if the outside diameter is 14″ and the metal is $1\frac{1}{4}$″ thick?

5. At $4.85 per square yard, find the cost of paving a circular walk whose inside diameter is 125′. The walk is 8′ 6″ wide.

6. Find the area of a washer whose inside diameter is $\frac{1}{2}$″ and whose outside diameter is $\frac{5}{8}$″.

7. How many square feet of sod are required for a circular lawn 150′ in diameter if the center of the lawn is occupied by a fountain 35′ in diameter?

8. What is the cross-section area of a pipe whose inside and outside diameters are respectively $3\frac{1}{2}$″ and $3\frac{3}{4}$″?

9. A flat steel ring $12\frac{1}{2}$″ in diameter has a $2\frac{1}{2}$″ hole in the center. What is its area?

10. What is the cross-section area of a steel collar, $\frac{3}{16}$″ thick, whose inside diameter is $3\frac{3}{8}$″?

61. Short Method of Finding the Area of Ring Sections

There is a short method of considerable practical value for finding the area of a ring section. The rule is:

Multiply the average circumference of the ring by the thickness of the ring.

In Fig. 114 the inside diameter is 18″ and the outside diameter is 26″. The thickness of the ring is, therefore, $\frac{26 - 18}{2} = \frac{8}{2} = 4$″. The average diameter is half the sum of the inside diameter and the outside diameter; $\frac{26 + 18}{2} = \frac{44}{2} = 22$″. The circumference of a 22″ circle is $3.14 \times 22 = 69.08$″.

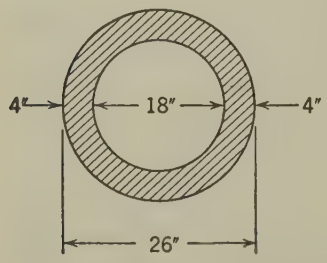


Fig. 114

Area of ring section = Average circumference $\times$ Thickness

$= 69.08 \times 4$

$= 276.32$ sq. in.

Example

Find the area of a flat ring 26″ inside diameter and 33″ outside diameter.

SOLUTION. Area = Average circumference $\times$ Thickness

$$\text{Av. diam.} = \frac{26 + 33}{2} = \frac{59}{2} = 29\tfrac{1}{2}''$$

$$\text{Thickness} = \frac{33 - 26}{2} = \frac{7}{2} = 3\tfrac{1}{2}''$$

$$\text{Area} = 3.14 \times 29\tfrac{1}{2} \times 3\tfrac{1}{2}$$

$= 324.6$ sq. in.

PROBLEMS

Work all the problems in the preceding section by the short method and compare the results with those obtained by the long method.

62. Arcs and Sectors of Circles

A part of a circle like AOB in Fig. 115 is called a ***sector*** of the circle. A part of the circumference like AB is called an ***arc***. The sector is bounded by the two radii OA and OB, and the arc AB. The angle AOB at the center is called the ***central angle*** of the sector. A central angle is measured by the same number of degrees as its intercepted arc. When we speak of a 40° sector, we mean a sector whose central angle is 40°. Since one degree of arc means $\frac{1}{360}$ of the circumference of the circle, the length of a 40° arc is evidently $\frac{40}{360}$ of the circumference of the circle.

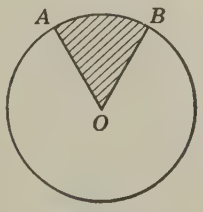


Fig. 115

Example 1

Find the length of a 35° arc of a 16″ circle.

SOLUTION.

Circumference of 16″ circle $= 3.14 \times 16 = 50.24''$

Length of 35° arc $= \frac{35}{360} \times 50.24 = 4.885''$

Example 2

Find the area of a 40° sector of a 16″ circle.

SOLUTION.

Area of 16″ circle $= 3.14 \times 8 \times 8 = 201.06$ sq. in.

Area of 40° sector $= \frac{40}{360} \times 201.06 = 22.34$ sq. in.

EXPLANATION. Since a 1° sector contains $\frac{1}{360}$ of the area of the circle, a 40° sector contains $\frac{40}{360}$ of the area of the circle.

PROBLEMS

Find (*a*) the length of arc and (*b*) the area of sector in each of the following cases:

1. 60° sector of a 9″ circle
2. 27° sector of a $3\frac{1}{2}$″ circle
3. 94° sector of a 45′ circle
4. 118° 30′ sector of a $4\frac{1}{2}$″ circle
5. 36° 30′ sector of a circle 2′ $8\frac{1}{2}$″ in diameter
6. 90° sector of a 20″ circle
7. 45° sector of a 24″ circle
8. 30° sector of a 3′ circle
9. 15° sector of a circle 18″ in diameter
10. 40° 45′ sector of a circle 2′ 4″ in diameter

63. Circles and Other Figures

It is often important to compute the relation between the circle and such common figures as the square, the equilateral triangle, the hexagon, and the octagon. A circle is said to be ***inscribed*** in another figure when it touches all the sides of that figure. In Fig. 116 the circles are inscribed in a triangle, square, hexagon, and octagon.

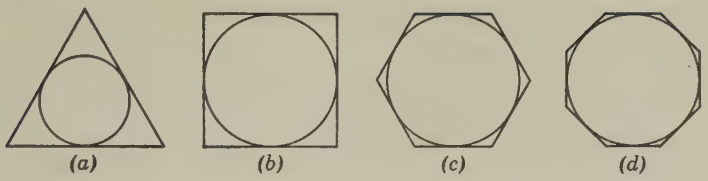


Fig. 116

A figure is said to be ***inscribed*** in a circle when all the corners of the figure touch the circumference of the circle. In Fig. 117 the triangle, square, etc., are inscribed in the circles. The circles are said to be ***circumscribed*** about the figures.

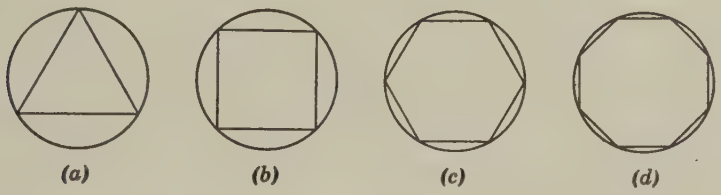


Fig. 117

Most of the relations between the circle and the inscribed and circumscribed figures are evident from an inspection of the figure. In Fig. 116(*b*) the diameter of the inscribed circle equals the side of the square. In Fig. 116(*c*) and Fig. 116(*d*) the diameters of the inscribed circles are equal to the flats of the hexagon and the octagon respectively. In Fig. 117(*b*) the diameter of the circumscribed circle is equal to the diagonal of the square; in Figs. 117(*c*) and 117(*d*) the diameters of the circles are equal to the corners of the hexagon and the octagon respectively.

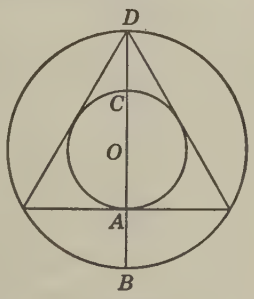


Fig. 118

Figure 118 illustrates the relation between an equilateral triangle and its inscribed and circumscribed circles. The diameter DB of the outer circle is divided by the construction into four equal parts; $DC = CO = OA = AB$.

Hence AC, the diameter of the inscribed circle, equals $\frac{2}{3}$ of the altitude of the triangle.

Also AD, the altitude of the triangle, equals $\frac{3}{4}$ of the diameter of the circumscribed circle.

The expression 2:3:4 states the relation between these figures.

Example 1

Find the area of a square inscribed in a 9″ circle. See Fig. 119.

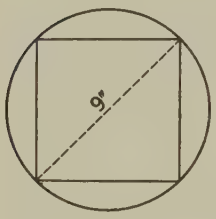


Fig. 119

SOLUTION. The diagonal of the square is equal to the diameter of the circle, 9″.

Side of square

$$= .707 \times D = .707 \times 9 = 6.363''$$

Area of square

$$= 6.363 \times 6.363 = 40.49 \text{ sq. in.}$$

Example 2

Find the area of a circle inscribed in an octagon with $3\frac{1}{2}''$ sides. See Fig. 120.

SOLUTION. The diameter of the circle is equal to the flats of the octagon.

$$\text{Flats of octagon} = 2.414 \times S = 2.414 \times 3\tfrac{1}{2} = 8.45''$$

$$\text{Area of circle} = 3.14 \times 4.23 \times 4.23 = 56.2 \text{ sq. in.}$$

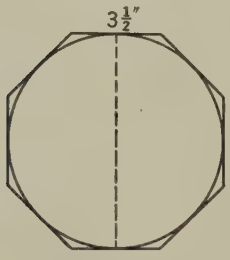


Fig. 120

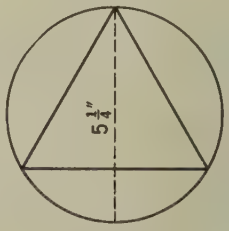


Fig. 121

Example 3

Find the area of an equilateral triangle inscribed in a $5\frac{1}{4}''$ circle. See Fig. 121.

SOLUTION. The altitude of the triangle is equal to $\frac{3}{4}$ of the diameter of the circle.

$$\text{Altitude of triangle} = \tfrac{3}{4} \times 5\tfrac{1}{4} = 3.94''$$

$$\text{Area of triangle} = 0.577 \times A^2$$

$$= 0.577 \times 3.94 \times 3.94 = 8.88 \text{ sq. in.}$$

PROBLEMS

NOTE. *Draw a neat diagram for each problem. This will show the relation between the figures.*

1. Find the area of a square inscribed in a $6\frac{1}{2}''$ circle.

2. Find the area of an equilateral triangle inscribed in a $35'$ circle.

3. Find the area of a hexagon inscribed in a $1\frac{1}{4}''$ circle.

4. Find the area of an octagon inscribed in a $4''$ circle.

5. Find the area of a circle inscribed in a $3\frac{1}{2}''$ square.

6. Find the area of a circle inscribed in an equilateral triangle whose sides are $4.72''$.

7. Find the area of a circle inscribed in an octagon with $1\frac{1}{2}''$ sides.

8. Find the area of a circle inscribed in a hexagon with $5\frac{3}{4}''$ sides.

9. Find the flats of a hexagon inscribed in a $\frac{3}{4}''$ circle.

10. Find the flats of an octagon inscribed in a $12''$ circle.

11. A circle is inscribed in a hexagon with $2\frac{1}{2}''$ sides. What is the diameter of the circle?

12. Eight holes equally spaced are drilled on the circumference of a $12\frac{1}{2}''$ circle. What is the straight-line distance between centers of holes?

13. What is the distance between holes (center-to-center) if 3 holes, equally spaced, are drilled on the circumference of a $3\frac{1}{4}''$ circle?

14. Find the depth of cut required to mill a square end on a $1\frac{3}{4}''$ round shaft.

15. Find the depth of cut required to mill a triangular end on a round shaft $\frac{7}{8}''$ in diameter.

64. Segments of Circles

A portion of the area of a circle like the shaded part in Fig. 122 is called a ***segment*** of a circle. It is bounded by an arc and a straight line which joins the ends of the arc. A straight line like AB, which joins the ends of an arc, is called a ***chord***. The angle BOA formed

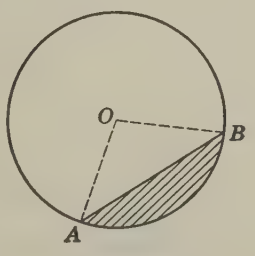


Fig. 122

at the center of the circle by the radii to the ends of the arc is called the ***central angle.*** When we speak of a 65-degree segment, we mean a segment whose central angle is 65 degrees.

By means of the relations between circles and inscribed figures studied in the last section, we can find the area of certain segments of a circle.

Example

Find the area of a 90° segment of a 24″ circle, Fig. 123.

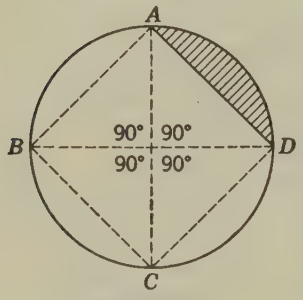


Fig. 123

SOLUTION. There are four 90° angles in a circle. Drawing these four angles and drawing chords between the ends of the arcs, we construct the square *ABCD*. Subtracting the area of this square from the area of the circle will leave four segments like the one whose area we are trying to find. Dividing the result by 4 gives the area of one segment.

The diagonal of the square is equal to the diameter of the circle, 24″.

$$\text{Side of square} = 0.707 \times D = 0.707 \times 24'' = 16.97''$$

$$\text{Area of square} = 16.97 \times 16.97 = 288.0 \text{ sq. in.}$$

$$\text{Area of 24″ circle} = 3.14 \times 12 \times 12 = 452.4 \text{ sq. in.}$$

Subtract area of square from area of circle and divide by 4.

$$\text{Area of circle} = 452.4 \text{ sq. in.}$$

$$\text{Area of square} = \underline{288.0} \text{ sq. in.}$$

$$\text{Area of 4 segments} = 164.4 \text{ sq. in.}$$

$$\text{Area of segment} = \frac{164.4}{4} = 41.10 \text{ sq. in.}$$

PROBLEMS

1. Find the area of a segment bounded by a chord and a 120° arc of an 8″ circle.

2. Find the area of a segment bounded by a chord and a 90° arc of a 16″ circle.

3. Find the area of a segment bounded by a chord and a 60° arc of a 25″ circle.

4. Find the area of a segment bounded by a chord and a 45° arc of a 36″ circle.

5. In milling a square end on a $1\frac{1}{2}$″ round shaft, how many square inches of cross-section area are removed?

65. The Ellipse

The most important things about the ellipse are the ***major axis,*** *AB* (also called the long diameter), the ***minor axis,*** *CD* (also called the short diameter), the circumference or perimeter, and the area. See Fig. 124.

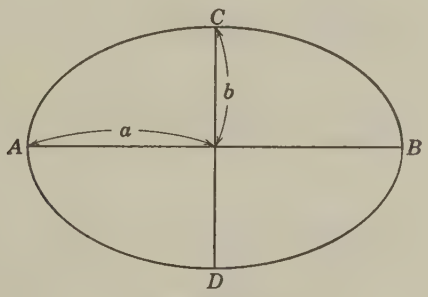


Fig. 124

The area of the ellipse is found by the formula

$$\text{Area} = \pi ab$$

where $a = \frac{1}{2}$ the long diameter; $b = \frac{1}{2}$ the short diameter; and $\pi = 3.14$ as in the circle formulas.

Example

Find the area of an ellipse 16″ long and 12″ wide.

SOLUTION. $\text{Area} = \pi ab$

$$a = \tfrac{16}{2} = 8; \; b = \tfrac{12}{2} = 6$$

$$\text{Area} = 3.14 \times 8 \times 6 = 150.7 \text{ sq. in.}$$

PROBLEMS

1. Find the area of an ellipse with an 8″ long diameter and a 5″ short diameter.

2. Find the area of an ellipse whose major axis is $6\frac{1}{2}''$ and minor axis $4\frac{1}{4}''$.

3. Find the area of an ellipse whose long diameter is 4.56″ and short diameter 2.48″.

4. Find the area of an ellipse with a long diameter of 65′ and a short diameter of 43′.

5. What is the area of an ellipse which is 5′ 2″ long and 3′ 4″ wide?

66. To Find a Diameter of the Ellipse When the Area and the Other Diameter Are Given

For this computation we transpose the formula, Area = πab, to read as follows:

$$\text{For finding the long diameter, } a = \frac{\text{Area}}{\pi b}$$

$$\text{For finding the short diameter, } b = \frac{\text{Area}}{\pi a}$$

Example 1

Find the long diameter of an ellipse whose area is 45 sq. in. and short diameter $6\frac{1}{2}''$.

$$\text{Solution. } a = \frac{\text{Area}}{\pi b} = \frac{45}{3.14 \times 3\frac{1}{4}} = \frac{45}{10.2} = 4.42''$$

$$\text{Long diameter} = 2 \times a = 2 \times 4.42 = 8.84''$$

Example 2

Find the short diameter of an ellipse whose area is 120 sq. in. and long diameter $14\frac{1}{2}''$.

$$\text{Solution. } b = \frac{\text{Area}}{\pi a} = \frac{120}{3.14 \times 7\frac{1}{4}} = \frac{120}{22.8} = 5.27''$$

$$\text{Short diameter} = 2 \times b = 2 \times 5.27 = 10.54''$$

PROBLEMS

1. An ellipse has an area of 275 sq. in. and a short diameter of 13″. Find its long diameter.

2. Find the short diameter of an ellipse whose long diameter is 4″ and area 8.18 sq. in.

3. An ellipse whose long diameter is 42.6′ is required to have an area of 1000 sq. ft. Find the short diameter.

4. Find the minor axis of an ellipse with a major axis of 13′ and an area of 105 sq. ft.

5. An ellipse with a minor axis of $5\frac{1}{2}''$ is required to have an area of 40 sq. in. Find the major axis.

67. Perimeter of an Ellipse

To find the perimeter of an ellipse, we use the following approximate formula:

$$\text{Perimeter} = \pi\sqrt{2(a^2 + b^2)}$$

which says that the perimeter is equal to π times the square root of twice the sum of a^2 and b^2. The several steps in the process are:

1. Substitute in the formula the values of π, a, and b.
2. Square a and b.
3. Add these squares.
4. Multiply the sum of the squares by 2.
5. Find the square root of the product.
6. Multiply the square root of the product by 3.14.

Example

Find the perimeter of an ellipse with a long diameter of 18″ and a short diameter of 14″.

SOLUTION.
$$\begin{aligned}\text{Perimeter} &= \pi\sqrt{2(a^2 + b^2)}\\ &= 3.14\sqrt{2(9^2 + 7^2)}\\ &= 3.14\sqrt{2(81 + 49)}\\ &= 3.14\sqrt{2 \times 130}\\ &= 3.14\sqrt{260}\\ &= 3.14 \times 16.12\\ &= 50.62''\end{aligned}$$

PROBLEMS

Find the perimeters of the ellipses in Problems **1** to **5**, Section 65.

68. Cube Root

In working with solid figures, we sometimes find it necessary to get the cubes and cube roots of certain numbers. A number used as a factor *three times* is said to be *cubed* or *raised to the third power*. Thus $12 \times 12 \times 12 = 12$ cubed $= 1728$. This is shown by placing the exponent 3 above and to the right of the number, thus: $12^3 = 1728$.

In expressing the cube root of a number, we write it as follows: $\sqrt[3]{1728} = 12$. This means that the cube root of 1728 is 12; or, 12 is the number which must be used as a factor three times to obtain 1728.

Example

Find the value of the expression 4^3.

SOLUTION. $4^3 = 4 \times 4 \times 4 = 64$.

PROBLEMS

1. Find the cube of 2.
2. Find the value of 11^3.
3. Find the cube of 31.
4. Find the cube of 2.16.
5. How many cubic inches in an 18″ cube? How many cubic feet?
6. How many cubic inches in a cube 1.09″ on a side?
7. Find the volume of a 75′ cube.
8. Find the cube of 6.7.
9. How many cubic inches in a cubic foot?
10. How many cubic feet in a cubic yard?

The following list of the cubes and the cube roots of numbers should be memorized by the student.

CUBES	CUBE ROOTS
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$

Cubes	Cube Roots
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$

69. To Find the Cube Root of a Whole Number

Example

Find the cube root of 103,823.

Solution.

$$\begin{array}{rr|rr} & & 4 & 7 \\ \hline & & 103 & 823 \\ & & 64 & \\ \hline 3 \times 40^2 = & 4800 & 39 & 823 \\ 3 \times 40 \times 7 = & 840 & & \\ 7^2 = & 49 & & \\ \hline & 5689 & 39 & 823 \\ \hline & & 0 & \end{array}$$

Explanation. Divide the number into sets of three figures from right to left. The first set is 103. The largest cube less than 103 is 64, of which the cube root is 4. Write the root figure 4 over the set 103 and the cube 64 under it. Subtracting 64 from 103 leaves 39. Bring down the next set 823, making the new dividend 39,823. We must now get a *trial divisor* in order to obtain the next figure in the root. This is found by squaring 10 times the root already obtained and multiplying by 3. Thus 10 times the root already obtained $= 10 \times 4 = 40$; $40^2 = 1600$; and $3 \times 1600 = 4800$, the trial divisor. The trial divisor, 4800, goes 7 times into 39,823, making 7 the second figure of the root. Write this above the second set of figures. To complete the divisor, we must add two things: first, $3 \times 40 \times 7$, that is, 10 times the root already obtained multiplied by 3 and by the new figure 7 ($3 \times 40 \times 7 = 840$); second, the square of the new figure in the root, 7 ($7^2 = 49$). The sum of these three partial divisors, $4800 + 840 + 49$, equals 5689, the complete divisor. Multiply this by the new figure, 7; $7 \times 5689 = 39{,}823$, which leaves no remainder. The cube root of 103,823 is 47.

Proof. $47 \times 47 \times 47 = 103{,}823$

If the number had been 103.823, the work would have been exactly the same except that the decimal point would appear between the 4 and the 7, making the answer 4.7.

70. To Find the Cube Root of Decimals

Example

Find the cube root of 6.37 to 2 decimal places.

SOLUTION.

		8	
		1. 9 5	
		6.370 000	
		1	
3×10^2	= 300	5 370	
$3 \times 10 \times 9$	= 270		
9^2	= 81		
	651	5 859	
$3 \times 10 \times 8$	= 240		
8^2	= 64		
	604	4 832	
3×180^2	= 97200	538 000	
$3 \times 180 \times 5$	= 2700		
5^2	= 25		
	99925	499 625	
		38 375	

EXPLANATION. Since each decimal place in the root requires 3 places in the power, we must have 6 figures after the point; so we add 4 ciphers. Separate the number into sets of 3 figures to the right and left of the decimal point. The largest cube less than 6 is 1, of which the cube root is 1. Write the root 1 above the first set of figures and the power 1 below it. Subtracting 1 from 6 leaves 5. Bring down the next set. $10 \times 1 = 10$; $3 \times 10^2 = 3 \times 100 = 300$, the trial divisor. The trial divisor 300 goes 17 times into 5370; but 9 is the largest single figure, so we try 9. To complete the divisor, we add $3 \times 10 \times 9 = 270$; and $9^2 = 81$. The total divisor is $300 + 270 + 81 = 651$; $9 \times 651 = 5859$, which shows that the number 9 is too large.

We will try 8. To the trial divisor we must add $3 \times 10 \times 8 = 240$ and 8^2 or 64. The total divisor is $300 + 240 + 64 = 604$; $8 \times 604 = 4832$. Subtraction leaves 538. Bring down the next set, making the new dividend 538,000.

To obtain the next figure, we get a trial divisor in the same way: $10 \times 18 = 180$; $180^2 = 32{,}400$; $3 \times 180^2 = 3 \times 32{,}400 = 97{,}200$, the trial divisor. Dividing 97,200 into 538,000 gives the new figure 5. To complete the divisor, add $3 \times 180 \times 5 = 2700$, and

$5^2 = 25$, making the total divisor 97,200 + 2700 + 25 = 99,925. $5 \times 99{,}925 = 499{,}625$. Subtracting this from 538,000 leaves 38,375. This being less than half of the last divisor, 99,925, we drop it. The cube root of 6.37 is 1.85.

PROBLEMS

1. Find the cube root of 13,824.
2. Find the cube root of 14,348,907.
3. Find the cube root of 2 to 2 decimal places.
4. Find the cube root of 5 to 3 decimal places.
5. Find the cube root of 0.77 to 3 decimal places.
6. Find the cube root of 2560 to 1 decimal place.
7. What must be the size of a cube to contain 100 cu. in.?
8. Find the cube root of 3.6355 to 3 decimal places.
9. Find the cube root of 87.3 to 2 decimal places.
10. Find the cube root of 1865.97 to 2 decimal places.

71. Prisms and Cylinders

Prisms are solid figures with parallel edges and uniform cross-section. If the edges are perpendicular to the bases, the prisms are called ***right prisms*** (Fig. 125).

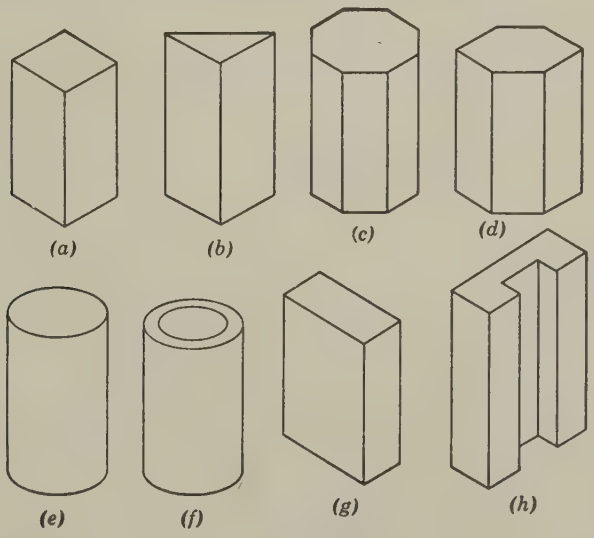


Fig. 125

When the base is a circle, the figure is called a ***cylinder.***

72. Volume of Prisms

The cubic content or capacity of a solid figure is called the *volume.*

The volume of any prism or cylinder is found by multiplying the area of the base by the height or altitude. Expressed as a formula,

$$\text{Volume} = \text{Area of base} \times \text{Altitude}$$

Example 1

Find the volume of a square prism 16″ high whose base is a $9\frac{1}{2}$″ square, Fig. 126.

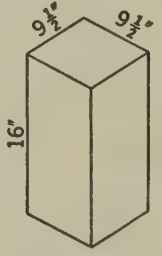


Fig. 126

SOLUTION. Volume = Area of base × Altitude

Area of base = $9\frac{1}{2} \times 9\frac{1}{2}$ = 90.25 sq. in.

Volume = 90.25 × 16

= 1444 cu. in.

Example 2

Find the volume of a cylinder $4\frac{1}{2}$″ in diameter and 6′ 4″ long.

SOLUTION. Volume = Area of base × Altitude

Area of base = $3.14 \times 2\frac{1}{4} \times 2\frac{1}{4}$ = 15.90 sq. in.

Altitude = 6′ 4″ = 76″

Volume = 15.90 × 76

= 1208.4 cu. in.

Example 3

Find the volume of the solid shell of a hollow cylinder, Fig. 127, $16\frac{1}{2}''$ high; the inside diameter is $8\frac{1}{2}''$ and the outside diameter $10''$.

SOLUTION.

$$\text{Volume} = \text{Area of base} \times \text{Altitude}$$

$$\text{Area of base (flat ring)} = \text{Average circumference} \times \text{Thickness}$$

$$= 3.14 \times 9\tfrac{1}{4} \times \tfrac{3}{4}$$

$$= 21.8 \text{ sq. in.}$$

$$\text{Volume} = 21.8 \times 16\tfrac{1}{2}$$

$$= 359 \text{ cu. in.}$$

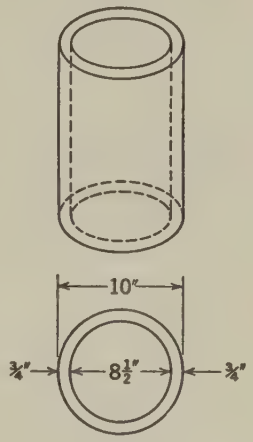


Fig. 127

PROBLEMS

1. Find the volume of a rectangular prism whose base is $12'' \times 4''$ and altitude $36''$.

2. Find the volume of a prism $8\frac{1}{2}''$ high if the base is a hexagon $5\frac{1}{2}''$ across flats.

3. Find the volume of a prism whose base is an equilateral triangle with $4\frac{3}{4}''$ sides and whose altitude is $20''$.

4. A prism whose base is an octagon with $15''$ sides is $4\frac{1}{2}''$ high. Find its volume.

5. How many cubic feet are there in a cylinder $16''$ in diameter and $42'\ 8''$ long? (1 cu. ft. = 1728 cu. in.)

6. Find the capacity in gallons of an elliptical tank $31''$ long if the base is an ellipse $21'' \times 17''$. (1 gallon = 231 cu. in.)

7. How many cubic inches of metal are there in a length of pipe $9'\ 6''$ long if the inside diameter is $6''$ and the metal is $\frac{7}{16}''$ thick?

8. Find the capacity in gallons of a cylindrical tank $14'$ in diameter and $11'\ 6''$ high.

9. Find the capacity in gallons of an elliptical tank $12'\ 6''$ long if the long diameter of the ellipse is $5'$ and the short diameter is $3'\ 4''$.

10. Find the weight of a bar of brass $5''$ in diameter and $6'\ 2''$ long. Brass weighs 512 lb. per cubic foot.

73. To Find the Height of a Prism or Cylinder

When we know the volume of a prism or cylinder and the area of the base, we can find the altitude by the following formula:

$$\text{Altitude} = \frac{\text{Volume}}{\text{Area of base}}$$

Example

Find the height of a cylinder 26″ in diameter to contain 6500 cu. in.

SOLUTION.

$$\text{Area of base} = \pi R^2 = 3.14 \times 13 \times 13 = 530.66 \text{ sq. in.}$$

$$\text{Altitude} = \frac{\text{Volume}}{\text{Area of base}} = \frac{6500}{530.66} = 12.25''$$

PROBLEMS

1. How high must a cylindrical tank 8′ 3″ in diameter be in order to have a capacity of 2000 gallons?

2. A gasoline tank whose cross-section is an ellipse 13″ × 10″ is required to contain 16 gallons. What must be its length?

3. A cylindrical container 7″ in diameter is required to have a capacity of 1 gallon. How high must it be?

4. Find the height of a 5-gallon cylindrical container if the diameter is 12″.

5. Find the depth of a tank 4′ 10″ square to have a capacity of 1000 gallons.

6. A milk container $3\frac{1}{2}$″ square is required to hold 1 quart. Find the height.

7. How high must a cylindrical can $4\frac{1}{2}$″ in diameter be in order to contain 2 quarts?

8. What should be the depth of the water in an octagonal tank 10′ 6″ across flats in order to provide for 10,000 gallons?

9. Find the depth of a rectangular vat 2′ 6″ × 5′ 10″ to contain 500 gallons.

10. The volume of a steel shaft 6″ in diameter is 2560 cu. in. Find its length.

74. To Find the Area of the Base of a Prism or Cylinder

When the volume and the altitude are known, divide the volume by the altitude. The formula is:

$$\text{Area of base} = \frac{\text{Volume}}{\text{Altitude}}$$

Example

A cylindrical tank 11′ high has a volume of 1050 cu. ft. Find the area of the base.

SOLUTION. $\text{Area of base} = \dfrac{\text{Volume}}{\text{Altitude}}$

$$= \frac{1050}{11} = 95.5 \text{ sq. ft.}$$

If the diameter of the cylinder is required, we use the formula, $D = 2\sqrt{\dfrac{\text{Area}}{\pi}}$. In this example,

$$D = 2\sqrt{\frac{95.5}{3.14}} = 2\sqrt{30.4140} = 2 \times 5.51 = 11.02'$$

PROBLEMS

1. Find the area of the base of a 25-gallon tank 22″ high.
2. If the tank in Problem 1 is a cylinder, find the diameter of the base.
3. If the tank in Problem 1 has a square base, find the length of the sides.
4. A cylindrical tank 6′ 8″ high is required to contain 200 gallons. What should be its diameter?
5. What should be the diameter of a cylindrical container 10″ high to have a capacity of 1 gallon?
6. Find the diameter of a 1-quart cylindrical container 8″ high.
7. A square water tank 12′ 8″ high has a capacity of 5000 gallons. Find the length of the sides.
8. Find the diameter of an oil drum $31\frac{1}{2}$″ high to contain 63 gallons.
9. A cylindrical 10-gallon container is 20″ high. Find the diameter.
10. A circular vat 18″ deep is required to have a capacity of 350 gallons. Find its diameter.

75. Lateral Surface of Prisms and Cylinders

By *lateral surface* is meant the area of the surface in the *sides* of the prism. A sheet of paper that can be wrapped around the prism or cylinder to cover the outside will be a

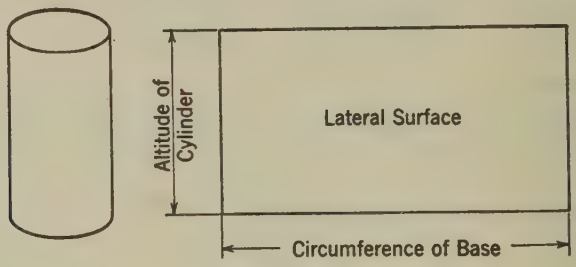


Fig. 128

rectangle as high as the prism or cylinder and as long as its perimeter. See Fig. 128. We thus obtain the formula for the lateral surface of a prism.

$$\text{Lateral surface} = \text{Perimeter of base} \times \text{Altitude}$$

Example 1

Find the lateral surface of a cylinder 6″ in diameter and 12″ high.

SOLUTION.

$$\text{Lateral surface} = \text{Perimeter of base} \times \text{Altitude}$$

$$\text{Perimeter of base} = 3.14 \times 6 = 18.84''$$

$$\text{Lateral surface} = 18.84 \times 12$$

$$= 226.08 \text{ sq. in.}$$

Example 2

Find the lateral surface of a prism whose altitude is $5\frac{1}{2}''$ and whose base is a hexagon $11\frac{1}{2}''$ across flats.

SOLUTION.

$$\text{Lateral surface} = \text{Perimeter of base} \times \text{Altitude}$$

$$\text{Side of hexagon} = 0.577 \times F = 0.577 \times 11\tfrac{1}{2} = 6.64''$$

$$\text{Perimeter of base} = 6 \times 6.64 = 39.84''$$

$$\text{Lateral surface} = 39.84 \times 5\tfrac{1}{2}$$

$$= 219.3 \text{ sq. in.}$$

PROBLEMS

1. Find the lateral surface of a cylinder 18″ in diameter and 12″ high.

2. Find the lateral surface of a prism with a 16″ square base and 8′ 4″ high.

3. Find the lateral surface of a triangular prism with 14″ sides and 32″ high.

4. Find the lateral surface of a prism 6′ 10″ high. The base is a hexagon with 2′ 6″ sides.

5. Find the lateral surface of an octagonal prism 2′ 4″ high. The base is an octagon 2′ 4″ across flats.

6. At 55¢ per square yard find the cost of painting the sides of a cylindrical gas tank 62′ high and 85′ in diameter.

7. A sheet metal duct is 28″ in diameter and 65′ 6″ long. How many square feet of metal are required for this duct, making no allowances for joints?

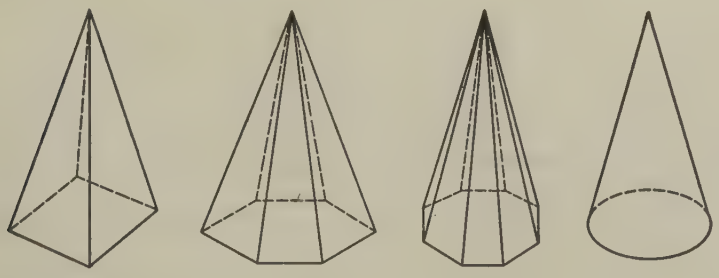
8. The sides and bottom of a vat 4′ 2″ square and 6′ 8″ high are lined with sheet lead. How many square feet of lead are required?

9. A rectangular tank 4′ 10″ by 6′ 8″ and 5′ 6″ high is lined (sides and bottom) with sheet copper. How many square feet of copper are required?

10. A sheet metal duct of rectangular cross-section 22″ by 40″ is 38′ 6″ long. Find the number of square feet of metal required.

76. Pyramids and Cones

Figures like those shown in Fig. 129(*a*), (*b*), and (*c*) are called ***pyramids.*** When the base is a circle, the figure is called a ***cone*** (Fig. 129(*d*)).



(a) *(b)* *(c)* *(d)*

Fig. 129

77. Volume of Pyramids and Cones

Figure 130 shows a comparison between a cylinder and a cone of the same base and altitude. *The volume of the cone is one-third the volume of the cylinder.* The volume of the cylinder being equal to the product of the area of the base and the altitude, the volume of the cone $= \frac{1}{3} \times$ area of base $\times$ altitude. The same rule applies to pyramids.

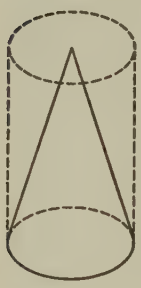


Fig. 130

Volume of pyramid or cone $= \frac{1}{3} \times$ Area of base $\times$ Altitude

Example 1

Find the volume of a cone 8″ high, the base being 6″ in diameter.

SOLUTION. Volume $= \frac{1}{3} \times$ Area of base $\times$ Altitude

Area of base $= 3.14 \times 3 \times 3 = 28.26$ sq. in.

Volume $= \frac{1}{3} \times 28.26 \times 8$

$= 75.4$ cu. in.

Example 2

Find the volume of a pyramid $27\frac{1}{2}$″ high, the base being a rectangle 19″ $\times$ 16″.

SOLUTION. Volume $= \frac{1}{3} \times$ Area of base $\times$ Altitude

Area of base $= 19 \times 16 = 304$ sq. in.

Volume $= \frac{1}{3} \times 304 \times 27\frac{1}{2}$

$= 2786.7$ cu. in.

PROBLEMS

1. Find the volume of a cone 36″ high if the base is 15″ in diameter.

2. Find the volume of a hexagonal pyramid $9\frac{1}{2}$″ high, the base being a hexagon with 5″ sides.

3. Find the volume of an elliptical cone $18\frac{1}{2}$″ high, the base being an ellipse 39″ $\times$ 56″.

4. Find the volume of a pyramid whose base is a 28″ square and whose altitude is 12″.

5. Find the volume of a pyramid whose base is an equilateral triangle with 30″ sides and whose altitude is 56″.

6. Find the volume of a pyramid whose base is an octagon 6″ across flats and whose altitude is $8\frac{1}{2}$″.

7. Find the volume of a pyramid whose base is a rectangle 16″ × $12\frac{1}{2}$″ and whose altitude is 5′ 2″.

8. A quantity of sand dumped upon the ground assumed a conical shape with a base 32′ in diameter and a height of 11′. How many cubic yards of sand are there in the pile?

9. An oil container in the shape of an inverted cone is $9\frac{1}{2}$″ high and has a base whose diameter is 11″. Find the capacity.

10. If cast iron weighs 0.26 lb. per cubic inch, what is the weight of a casting in the shape of a square pyramid with a $6\frac{1}{2}$″ square base and $3\frac{1}{4}$″ high?

78. Lateral Surface of Pyramids and Cones

In the square pyramid shown in Fig. 131 the lateral surface is seen to be made up of four equal isosceles triangles

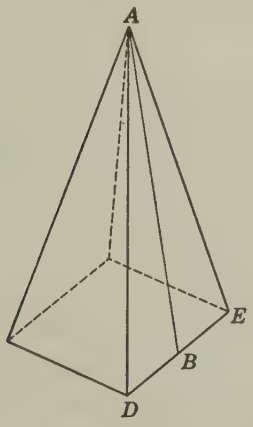


Fig. 131

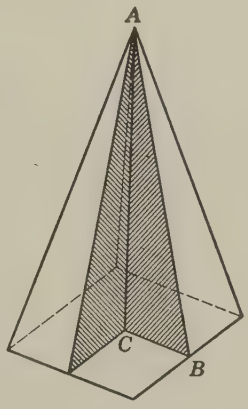


Fig. 132

like ADE. The base of each triangle is a side of the base of the pyramid. We can get the area of one of these triangles and multiply it by 4 to get the lateral surface of the pyramid.

$$\text{Area of triangle} = \tfrac{1}{2} \times \text{Base} \times \text{Altitude}$$

If the pyramid in Fig. 131 is 12″ high and the base is a 10″ square, then the base of triangle ADE is 10″, but the

altitude of the triangle is the line AB, called the ***slant height of the pyramid.***

Figure 132 shows the pyramid with one-quarter of it removed to show the actual height AC and the slant height AB. The triangle ABC (Fig. 133) is a right triangle with the slant height AB as the hypotenuse. The altitude of this right triangle, AC, is 12″, the altitude of the pyramid. The base BC of the triangle is half the distance across the square; $\frac{1}{2}$ of $10'' = 5'' = BC$. The hypotenuse $AB = \sqrt{AC^2 + BC^2} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13''$, the slant height.

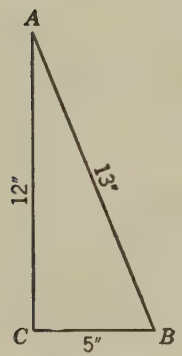


Fig. 133

This slant height, AB, is the altitude of the triangle ADE, which is one of the sides of the pyramid.

Area of triangle $ADE = \frac{1}{2}ba$

$= \frac{1}{2} \times 10 \times 13 = 65$ sq. in.

The lateral surface of the pyramid is 4 times the area of one of the sides: $4 \times 65 = 4 \times \frac{1}{2} \times 10 \times 13 = 260$ sq. in.

But $4 \times 10 = 40''$, the perimeter of the base. The lateral surface, therefore, equals $\frac{1}{2} \times$ the perimeter of the base $\times$ the slant height $= \frac{1}{2} \times 40 \times 13 = 260$ sq. in.

The rule for finding the lateral surface of a pyramid is: *Lateral surface* $= \frac{1}{2} \times$ *Perimeter of base* $\times$ *Slant height.*

In each case the slant height of the pyramid is the hypotenuse of a right triangle whose altitude is the altitude of the pyramid. The base of this right triangle is seen by a study of Fig. 132. In the cone, the base of the triangle is one-half the diameter of the base of the cone. In the hexagonal pyramid and the octagonal pyramid, the base of the triangle is one-half the flats of the base of the pyramid. In the triangular pyramid the base of the triangle is one-third the altitude of the triangular base.

Example 1

Find the lateral surface of a pyramid whose base is a hexagon $9\frac{1}{2}''$ across flats and whose altitude is $16''$.

SOLUTION.

$$\text{Lateral surface} = \tfrac{1}{2} \times \text{Perimeter of base} \times \text{Slant height}$$

$$\text{Side of hexagon} = 0.577 \times F = 0.577 \times 9\tfrac{1}{2} = 5.46''$$

$$\text{Perimeter} = 6 \times 5.46 = 32.76''$$

$$\text{Slant height} = \sqrt{4.75^2 + 16^2} = \sqrt{22.56 + 256}$$

$$= \sqrt{278.56} = 16.71''$$

$$\text{Lateral surface} = \tfrac{1}{2} \times 32.76 \times 16.71 = 273.5 \text{ sq. in.}$$

Example 2

Find the lateral surface of a cone $1'\ 9''$ high, the base being $6'\ 8''$ in diameter.

SOLUTION.

$$\text{Lateral surface} = \tfrac{1}{2} \times \text{Perimeter of base} \times \text{Slant height}$$

$$\text{Perimeter of base} = 3.14 \times 80 = 251.2''$$

$$\text{Slant height} = \sqrt{40^2 + 21^2} = \sqrt{1600 + 441}$$

$$= \sqrt{2041} = 45.2''$$

$$\text{Lateral surface} = \tfrac{1}{2} \times 251.2 \times 45.2$$

$$= 5677 \text{ sq. in.}$$

PROBLEMS

1. Find the lateral surface of a square pyramid with a $4\frac{1}{2}''$ square base and an altitude of $10\frac{1}{2}''$.

2. Find the lateral surface of a pyramid whose base is an octagon with $4''$ sides and whose altitude is $6''$.

3. Find the lateral surface of a cone whose base is $8\frac{1}{2}''$ in diameter and whose altitude is $18''$.

4. Find the lateral surface of a pyramid whose base is an equilateral triangle with $9''$ sides and whose altitude is $12''$.

5. Find the lateral surface of a pyramid whose base is a rectangle 14″ × 20″ and whose altitude is 12″.

6. What is the altitude of a cone whose base is 36′ in diameter and whose slant height is 24′?

7. Find the lateral surface of a pyramid whose base is a hexagon with 10″ sides and whose altitude is 30″.

8. The top of a copper boiler is conical in shape with a base 12′ 3″ in diameter and an altitude of 3′ 6″. How many square feet of copper were required to make it, allowing 3% for seams?

9. How many square feet of tin are required to cover a roof in the shape of a square pyramid 9′ 4″ high, the base being a 28′ square?

79. Frustums of Pyramids and Cones

The frustum of a pyramid or cone is much more common in practical work than the pyramid or cone. When a pyramid or cone is cut at any point below the apex by a plane parallel to the base, the portion below the cutting plane is called a ***frustum*** of a pyramid or a ***frustum*** of a cone. Figure 134 shows various frustums.

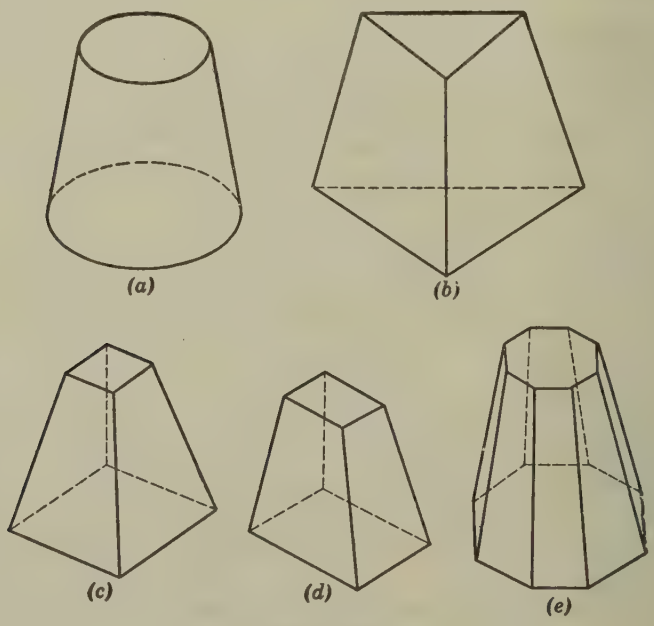


Fig. 134

The volume of the frustum is obtained by the following formula:

$$\text{Volume} = \tfrac{1}{3}h(B + b + \sqrt{B \times b}\,)$$

where h is the height, B is the area of the large base, and b is the area of the small base. The formula says that the volume = $\frac{1}{3}$ × the height × the sum of three things: the area of the large base, the area of the small base, and the square root of their product. The process consists of the following steps:

1. Compute the area of B, the large base.
2. Compute the area of b, the small base.
3. Multiply B by b and get the square root of the product.
4. Add the three quantities in the parentheses.
5. Multiply $\frac{1}{3}$ of the height by the sum of the three quantities in the parentheses.

Example 1

Find the volume of the frustum of a cone 15″ high, the upper base being 16″ in diameter and the lower base 9″ in diameter.

Solution.

$$\text{Volume} = \tfrac{1}{3} \times h \times (B + b + \sqrt{B \times b}\,)$$

B = Area of 16″ circle = $3.14 \times 8 \times 8 = 201.0$ sq. in.

b = Area of 9″ circle = $3.14 \times 4\tfrac{1}{2} \times 4\tfrac{1}{2} = 63.6$ sq. in.

$$\text{Volume} = \tfrac{1}{3} \times 15 \times (201.0 + 63.6 + \sqrt{201.0 \times 63.6}\,)$$

$$= \tfrac{1}{3} \times 15 \times (201.0 + 63.6 + \sqrt{12783.6}\,)$$

$$= \tfrac{1}{3} \times 15 \times (201.0 + 63.6 + 113.1)$$

$$= \tfrac{1}{3} \times 15 \times 377.7$$

$$= 1888.5 \text{ cu. in.} = 1.09 \text{ cu. ft.}$$

This formula is the general formula and applies to all frustums of cones and pyramids regardless of the shape of the base. For the frustum of the cone, however, the formula can be simplified

and much time saved in the computation. The following is the simplified formula for finding the volume of the frustum of a cone:

$$\text{Volume} = .262h(D^2 + d^2 + Dd)$$

where D = diameter of large base
d = diameter of small base
h = altitude

Example 2

Using the simplified formula, find the volume of the frustum of a cone with bases 6″ and 10″ in diameter and 12″ high.

SOLUTION.
$$\begin{aligned}\text{Volume} &= .262h(D^2 + d^2 + Dd)\\ &= .262 \times 12(10^2 + 6^2 + 10 \times 6)\\ &= .262 \times 12(100 + 36 + 60)\\ &= .262 \times 12 \times 196\\ &= 616.2 \text{ cu. in.}\end{aligned}$$

PROBLEMS

1. Find the volume of the frustum of a square pyramid 15″ high, the upper base being a 42″ square and the lower base a 26″ square.

2. Find the volume of the frustum of a hexagonal pyramid 7″ high, the upper base being a hexagon with 3″ sides and the lower base a hexagon with 8″ sides.

3. Find the volume of the frustum of a cone 10″ high, the lower base being 3″ in diameter and the upper base 6″ in diameter.

4. Find the volume of the frustum of an elliptical pyramid 8″ high, the upper base being an ellipse 7″ × 9″ and the lower base an ellipse 14″ × 18″.

5. Find the capacity in gallons of a pail 11″ deep with a top diameter of 10″ and a bottom diameter of 8″.

6. Find the volume of a frustum of a rectangular pyramid $4\frac{1}{2}$″ high, the upper base being a rectangle 12″ × 16″ and the lower base a rectangle 9″ × 12″.

7. What is the capacity in cubic feet of a square coal hopper in the shape of a frustum of a square pyramid 13′ 0″ high, the upper base being a 22′ square and the lower base a $5\frac{1}{2}$′ square?

8. An oil measure in the shape of a frustum of a cone is $11\frac{1}{2}$″ deep, with a top diameter of $5\frac{1}{4}$″ and a bottom diameter of $7\frac{1}{2}$″. How many quarts of oil can it contain?

9. A concrete base for a heavy machine has the form of a frustum of a rectangular pyramid. The top is 16′ 4″ × 4′ 8″ and the bottom is 32′ 8″ × 9′ 4″. The depth is 3′ 8″. How many cubic yards of concrete were used in building this foundation?

10. A basin in the form of a frustum of an elliptical pyramid 8″ deep is 18″ × 15″ at the top and 12″ × 10″ at the bottom. What is its capacity in gallons?

80. To Find the Altitude of the Frustum of a Pyramid or a Cone

In designing vessels, we are frequently required to find the height for a given capacity. The volume formula is modified to read as follows:

$$h = \frac{3 \times \text{Volume}}{B + b + \sqrt{B \times b}}$$

that is, the height is equal to 3 times the volume divided by $B + b + \sqrt{B \times b}$.

Example 1

Find the height of a frustum of a cone to contain 2000 cu. in. The upper base is 16″ in diameter and the lower base 12″ in diameter.

SOLUTION. $h = \dfrac{3 \times \text{Volume}}{B + b + \sqrt{B \times b}}$

B = Area of 16″ circle = $3.14 \times 8 \times 8 = 201.0$ sq. in.

b = Area of 12″ circle = $3.14 \times 6 \times 6 = 113.0$ sq. in.

$$\sqrt{B \times b} = \sqrt{201.0 \times 113.0} = 150.7$$

$$h = \frac{3 \times 2000}{201.0 + 113.0 + 150.7}$$

$$= \frac{6000}{464.7}$$

$$= 12.9\text{″, the altitude}$$

The simplified formula for finding the altitude of the frustum of a cone is:

$$h = \frac{\text{Volume}}{.262(D^2 + d^2 + Dd)}$$

Example 2

Find the answer to Example 1 by means of the simplified formula.

SOLUTION. $$h = \frac{\text{Volume}}{.262(D^2 + d^2 + Dd)}$$

$$= \frac{2000}{.262(16^2 + 12^2 + 16 \times 12)}$$

$$= \frac{2000}{.262(256 + 144 + 192)}$$

$$= \frac{2000}{.262 \times 592}$$

$$= \frac{2000}{155.1} = 12.9''$$

PROBLEMS

1. Find the altitude of the frustum of a square pyramid to contain 400 cu. in., the upper base to be an 8″ square and the lower base a 10″ square.

2. Find the altitude of the frustum of a cone to contain 2772 cu. in., the top to be 18″ in diameter and the bottom $13\frac{1}{2}$″ in diameter.

3. What must be the height of a frustum of a hexagonal pyramid to contain 200 cu. in., the upper base being a hexagon with 2″ sides and the lower base a hexagon with 5″ sides?

4. Find the altitude of the frustum of a rectangular pyramid which has a volume of 6000 cu. ft., an upper base 30′ × 15′ and a lower base 40′ × 20′.

5. What must be the height of a gallon measure with a top diameter of $4\frac{1}{4}$″ and a bottom diameter of $7\frac{1}{4}$″?

6. A bin in the form of the frustum of a square pyramid is required to have a capacity of 8000 bushels. What must be its height if the top is a 50′ square and the bottom a 35′ square? (1 bu. = 2150.4 cu. in.)

7. Find the depth of a 10-gallon pail with a top diameter of 14″ and a bottom diameter of 10″.

8. A hopper whose top is 10′ square and whose bottom is 2′ square is required to have a capacity of 500 cu. ft. Find the height.

81. To Find the Lateral Surface of the Frustum of a Cone or a Pyramid

Figure 135 shows the lateral surface of the frustum of a square pyramid to be made up of four trapezoids like *EFHG*. Suppose the upper base to be a 9″ square, the lower base a 14″ square, and the altitude 5″. Then each side is a trapezoid with bases of 9″ and 14″ and an altitude *AB* which is

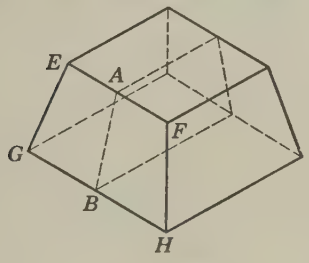


Fig. 135

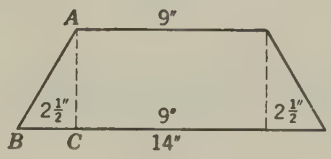


Fig. 136

the slant height of the frustum. Figure 136 shows a cross-section through the figure as shown by the dotted lines in Fig. 135. The slant height *AB* is the hypotenuse of the right triangle *ABC*, in which the altitude is the height of the frustum, 5″, and the base is half the difference between the upper and lower bases of the trapezoid.

$$\text{Base } BC = \frac{14 - 9}{2} = \frac{5}{2} = 2\tfrac{1}{2}''$$

The slant height,

$$AB = \sqrt{5^2 + 2\tfrac{1}{2}^2} = \sqrt{25 + 6.25} = \sqrt{31.25} = 5.59''$$

$$\text{Average width of } EFHG = \frac{9 + 14}{2} = 11.5''$$

Area of trapezoid *EFGH* (one side of frustum)

$$= \text{Average width} \times \text{Slant height}$$

$$= 11.5 \times 5.59 = 64.285 \text{ sq. in.}$$

$$\text{Lateral surface} = 4 \times \text{Area of one side}$$

$$= 4 \times 64.285 = 257.14 \text{ sq. in.}$$

A more convenient method of finding the lateral surface is by means of the following formula:

Lateral surface = Average perimeter of base × Slant height

$$\begin{aligned} \text{Perimeter of upper base} &= 4 \times 9 = 36'' \\ \text{Perimeter of lower base} &= 4 \times 14 = 56'' \\ \text{Sum of perimeters} &= 92'' \end{aligned}$$

Average perimeter $= \frac{92}{2} = 46''$, which is the same as 4 times the average width of one side.

$$\text{Lateral surface} = 46 \times 5.59 = 257.14 \text{ sq. in.}$$

Example

Find the lateral surface of the frustum of a cone 6″ high with upper and lower bases respectively 11″ and 7″ in diameter (Fig. 137).

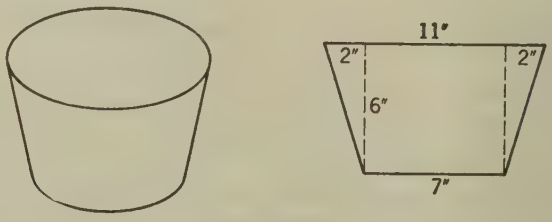


Fig. 137

SOLUTION.

Lateral surface = Average perimeter of bases × Slant height

$$\begin{aligned} \text{Perimeter of upper base} &= 3.14 \times 11 = 34.54'' \\ \text{Perimeter of lower base} &= 3.14 \times 7 = 21.98'' \\ & \qquad\qquad 56.52'' \end{aligned}$$

$$\text{Average perimeter} = \frac{56.52}{2} = 28.26''$$

Altitude of right triangle = 6″

$$\text{Base of right triangle} = \frac{11 - 7}{2} = \frac{4}{2} = 2''$$

$$\text{Slant height} = \sqrt{6^2 + 2^2} = \sqrt{36 + 4} = \sqrt{40} = 6.32''$$

$$\begin{aligned} \text{Lateral surface} &= 28.26 \times 6.32 \\ &= 178.60 \text{ sq. in.} \end{aligned}$$

PROBLEMS

1. Find the lateral surface of the frustum of a square pyramid 16′ high, the upper base being a 28′ square and the lower base an 18′ square.

2. Find the lateral surface of the frustum of a hexagonal pyramid $9\frac{1}{2}''$ high, the upper base of which is a hexagon with $3\frac{1}{2}''$ sides and the lower base a hexagon with 9″ sides.

3. Find the lateral surface of the frustum of an octagonal pyramid 20″ high, the upper base being an octagon with 14″ sides and the lower base an octagon with 22″ sides.

4. Find the lateral surface of the frustum of a cone 6′ 10″ high, the upper base being 7′ 2″ in diameter and the lower base 9′ 4″ in diameter.

5. Find the lateral surface of the frustum of a triangular pyramid 12″ high whose upper base is an equilateral triangle with 10″ sides and whose lower base is an equilateral triangle with 20″ sides.

6. Each set of figures, (*a*) to (*e*), represents a frustum of a cone with dimensions as given. Find the lateral surface in each case. Give all answers in square feet, 1 decimal place.

	Large Diameter	Small Diameter	Height
(*a*)	1′ 10″	1′ 4″	1′ 6″
(*b*)	32″	18″	6″
(*c*)	14″	6″	12″
(*d*)	8′ 4″	2′ 8″	3′ 6″
(*e*)	12″	9″	11″

82. Spheres

The volume of a sphere or ball may be found by the formula

$$\text{Volume} = \tfrac{4}{3}\pi R^3$$

in which R is the radius.

Example

Find the volume of a 6″ sphere.

Solution. $\text{Volume} = \tfrac{4}{3}\pi R^3$

$$= \tfrac{4}{3} \times \tfrac{22}{7} \times 3^3$$

$$= \tfrac{4}{3} \times \tfrac{22}{7} \times 27 = 113.1 \text{ cu. in.}$$

PROBLEMS

1. Find the volume of a 32″ sphere.
2. Find the volume of a 1″ sphere.
3. Find the volume of a 2″ sphere.
4. Find the volume of a 3″ sphere.
5. Find the volume of a 16″ sphere.
6. Find the volume of a 35′ sphere.
7. Find the volume of a sphere $\frac{3}{4}$″ in diameter.
8. Find the volume of a sphere $1\frac{1}{2}$″ in diameter.
9. Find the volume of a hemisphere 11″ in diameter.
10. Find the volume of a hemisphere 3′ 4″ in diameter.

83. To Find the Diameter of a Sphere When the Volume Is Known

In this computation the formula for volume is modified to read as follows:

$$D = \sqrt[3]{\frac{6 \text{ Volume}}{\pi}}$$

To find the diameter, we multiply the volume by 6, divide the product by π, and get the cube root of the result.

Example

Find the diameter of a sphere whose volume is 200 cu. in.

SOLUTION.

$$D = \sqrt[3]{\frac{6 \text{ Volume}}{\pi}} = \sqrt[3]{\frac{6 \times 200}{3.14}} = \sqrt[3]{382.1656} = 7.26''$$

PROBLEMS

1. Find the diameter of a sphere which contains 1 cu. in.
2. Find the diameter of a sphere which contains 250 cu. in.
3. Find the diameter of a sphere which contains 8.1813 cu. in.
4. Find the diameter of a hemisphere which contains 122.72 cu. in.
5. Find the diameter of a hemisphere which contains 6.283 cu. in.

84. To Find the Surface of a Sphere

The surface of a sphere is equal to four times the area of a circle with the same diameter.

$$\text{Surface of sphere} = 4\pi R^2$$

Example

Find the surface area of a 6″ sphere.

SOLUTION. $\text{Surface} = 4 \times \pi \times R^2$

$= 4 \times 3.14 \times 3^2$

$= 4 \times 3.14 \times 9 = 113.04$ sq. in.

PROBLEMS

1. Find the area of the surface of a 22″ sphere.
2. Find the area of the surface of a $1\frac{1}{2}$″ sphere.
3. Find the area of the surface of a sphere 100′ in diameter.
4. Find the area of the surface of a sphere 1″ in diameter.
5. Find the area of the surface of a sphere 1.4″ in diameter.

85. Volume of a Ring

The volume of a ring of circular cross-section as in Fig. 138 is found by multiplying the circumference of the center line circle by the area of cross-section.

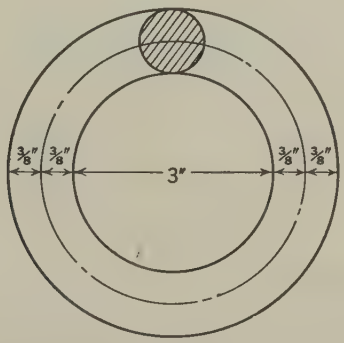


Fig. 138

Example

The ring in Fig. 138 has an inside diameter of 3″, and the cross-section of the ring is a $\frac{3}{4}$″ circle. Find its volume.

SOLUTION.

Volume = Circumference of center line circle × Area of cross-section

Diameter of center line circle = $3 + \frac{3}{8} + \frac{3}{8} = 3\frac{3}{4}$″

Circumference of center line circle = $3.14 \times 3\frac{3}{4} = 11.78$″

Area of cross-section = $3.14 \times \frac{3}{8} \times \frac{3}{8} = 0.442$ sq. in.

Volume = 11.78×0.442

$= 5.21$ cu. in.

PROBLEMS

1. Find the volume of the rim of a handwheel of circular cross-section if the outside diameter is 10″ and the inside diameter 9″.

2. Find the volume of a ring of circular cross-section if the inside diameter is 5″ and the ring is $\frac{7}{8}$″ thick.

3. Find the volume of a ring of circular cross-section if the outside diameter is $6\frac{1}{2}$″ and the ring is $\frac{3}{4}$″ thick.

4. A ring having a circular cross-section has an inside diameter of 7″ and a thickness of $\frac{1}{2}$″. Find the volume.

5. The rim of a handwheel with an outside diameter of 8″ is $\frac{3}{4}$″ thick. What is the volume?

86. Volume of Composite Solid Figures

Many objects are made up of parts, the volumes of which can be found by the rules studied in the preceding articles. In cases of this sort we get the volume of each part and add the results to get the total volume of the object.

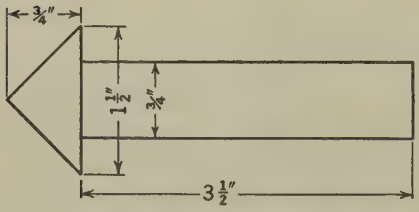


Fig. 139

Example 1

Find the volume of the steeple-head rivet in Fig. 139.

SOLUTION.

Volume of head (cone) $= \frac{1}{3} \times \frac{3}{4} \times \frac{22}{7} \times \frac{3}{4} \times \frac{3}{4} = 0.442$ cu. in.

Volume of body (cylinder) $= \frac{7}{2} \times \frac{22}{7} \times \frac{3}{8} \times \frac{3}{8} = 1.547$ cu. in.

Total $= 1.989$ cu. in.

Example 2

Find the volume of the taper bushing shown in Fig. 140.

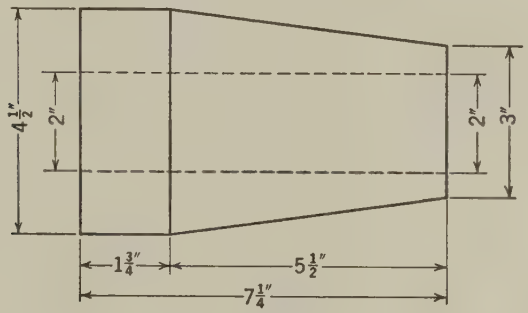


Fig. 140

SOLUTION.

Vol. of cyl. part

$= 3.14 \times 2\frac{1}{4} \times 2\frac{1}{4} \times 1\frac{3}{4} = 27.8$ cu. in.

Vol. of tapered part

$= .262h(D^2 + d^2 + Dd)$

$= .262 \times 5\frac{1}{2}[(4\frac{1}{2})^2 + 3^2 + 4\frac{1}{2} \times 3]$

$= .262 \times 5\frac{1}{2}(20.25 + 9 + 13.5)$

$= .262 \times 5.5 \times 42.75 = 61.6$ cu. in.

Gross volume $= 89.4$ cu. in.

Volume of cylindrical hole through the bushing

$= 3.14 \times 1 \times 1 \times 7\frac{1}{4} = 22.8$ cu. in.

Net volume or volume of the object $= 66.6$ cu. in.

EXPLANATION. First consider the entire object as a solid, then subtract the volume of the cylindrical hole.

PROBLEMS

1. Find the volume of the cone-head rivet shown in Fig. 141.

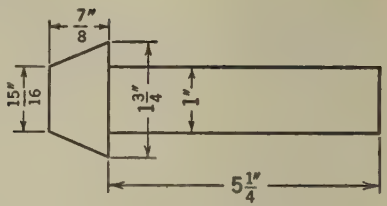


Fig. 141

2. Find the volume of the countersunk rivet shown in Fig. 142.

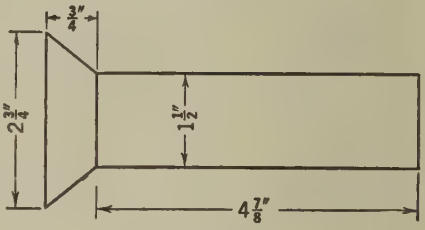


Fig. 142

3. Find the volume of the hollow cast iron column shown in Fig. 143.

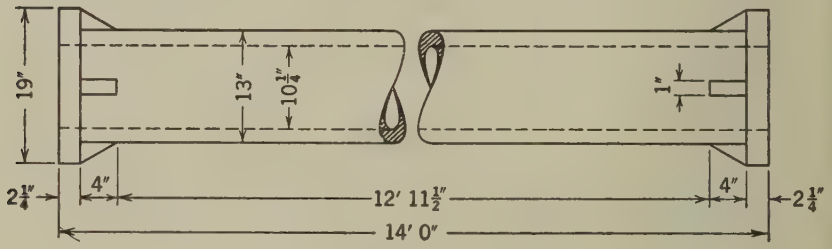


Fig. 143

4. Find the capacity of the oil can shown in Fig. 144.

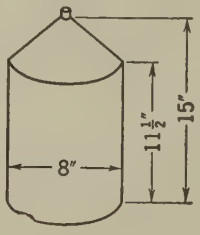


Fig. 144

5. Find the capacity of a cylindrical pail with a hemispherical bottom if the diameter is 10″ and the overall depth is 14″.

87. Summary of Formulas for Mensuration

Right triangle, sides. Hyp. $= \sqrt{\text{Alt.}^2 + \text{Base}^2}$

Alt. $= \sqrt{\text{Hyp.}^2 - \text{Base}^2}$

Base $= \sqrt{\text{Hyp.}^2 - \text{Alt.}^2}$

Triangle. Area $= \frac{1}{2}ba$, when the base and altitude are given

Area $= \sqrt{s(s-a)(s-b)(s-c)}$, when the three sides are given

Rectangle. Area = Base × Altitude

Trapezoid. Area = Average width × Altitude

Circle. Circumference $= \pi D$

Area $= \pi R^2$ or $\frac{\pi D^2}{4}$

Diameter $= \frac{\text{Circumference}}{\pi}$

Diameter $= 2\sqrt{\frac{\text{Area}}{\pi}}$ or $D = \sqrt{\frac{4 \text{ Area}}{\pi}}$

Diameter of circle equal in area to two or more circles = the square root of the sum of the squares of their diameters

Length of arc =

$$\frac{\text{No. of degrees in arc}}{360} \times \text{Circum. of circle}$$

Area of sector =

$$\frac{\text{No. of degrees in sector}}{360} \times \text{Area of circle}$$

Flat ring. Area = Average circumference × Thickness

Ellipse. Area $= \pi ab$ (a and b are half the long and the short diameters)

$$b = \frac{\text{Area}}{\pi a}; \quad a = \frac{\text{Area}}{\pi b}$$

$$\text{Perimeter} = \pi\sqrt{2(a^2 + b^2)}$$

Prism and cylinder.

$$\text{Volume} = \text{Area of base} \times \text{Altitude}$$

$$\text{Area of base} = \frac{\text{Volume}}{\text{Altitude}}$$

$$\text{Altitude} = \frac{\text{Volume}}{\text{Area of base}}$$

$$\text{Lateral surface} = \text{Perimeter of base} \times \text{Altitude}$$

Pyramid and cone.

$$\text{Volume} = \tfrac{1}{3} \times \text{Area of base} \times \text{Altitude}$$

$$\text{Area of base} = \frac{3 \times \text{Volume}}{\text{Altitude}}$$

$$\text{Altitude} = \frac{3 \times \text{Volume}}{\text{Area of base}}$$

$$\text{Lateral surface} = \tfrac{1}{2} \times \text{Perimeter of base} \times \text{Slant height}$$

Frustums of pyramids and cones.

$$\text{Volume} = \tfrac{1}{3}h(B + b + \sqrt{B \times b})$$

$$h = \frac{3 \times \text{Volume}}{B + b + \sqrt{B \times b}}$$

Lateral surface = Average perimeter of bases $\times$ Slant height

Sphere.

$$\text{Volume} = \tfrac{4}{3}\pi R^3$$

$$\text{Diameter} = \sqrt[3]{\frac{6 \text{ Volume}}{\pi}}$$

$$\text{Surface} = 4\pi R^2$$

Solid ring. Volume = Circumference of the center line circle × Area of cross-section

Equilateral triangle, square, hexagon, and octagon. For constants in reference to these figures, see page 88.

88. Weights of Materials

Table V at the back of this text gives the weight per cubic inch and per cubic foot of various materials. The weight of any object may be found by computing its volume in cubic inches or cubic feet and multiplying this volume by the unit weight, *i.e.*, the weight per cubic inch or cubic foot of that material.

Example 1

Find the weight of an iron casting whose volume is 7.35 cu. ft.

SOLUTION. $7.35 \times 450 = 3307.5$ lb.

EXPLANATION. We find in Table V, page 564, that the weight of cast iron is 450 lb. per cu. ft. Therefore the weight of the casting is 7.35×450 or 3307.5 lb.

Example 2

Find the weight of a sheet of steel 6′ 8″ long, 2′ 4″ wide, and $\frac{1}{4}$″ thick. See Fig. 145.

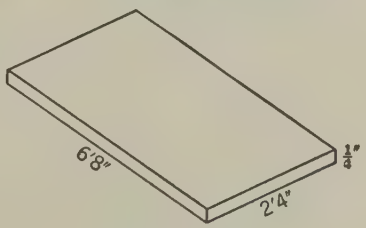


Fig. 145

SOLUTION.

$80 \times 28 \times \frac{1}{4} \times .283 = 158.5$ lb.

EXPLANATION. The product $80 \times 28 \times \frac{1}{4}$ gives the volume of the sheet in cubic inches. Multiplying by 0.283, the weight per cubic inch of steel, gives the weight of the sheet of steel.

Example 3

Find the weight of the rim of a cast iron flywheel of the following dimensions: inside diameter 48″, outside diameter $56\frac{1}{2}$″, face $6\frac{1}{2}$″ wide. See Fig. 146.

SOLUTION. $3.14 \times 52\frac{1}{4} \times 4\frac{1}{4} \times 6\frac{1}{2} \times \frac{450}{1728} = 1181$ lb.

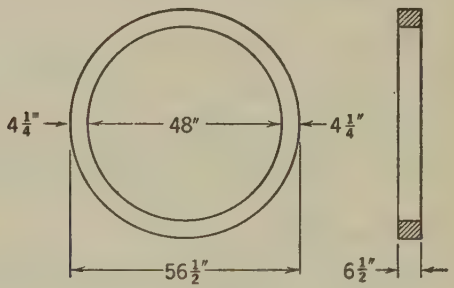


Fig. 146

EXPLANATION. Volume of a ring-section = Average circumference × Area of cross-section

$$\text{Diameter of center line circle} = \frac{48 + 56\frac{1}{2}}{2} = \frac{104\frac{1}{2}}{2} = 52\frac{1}{4}''$$

$$\text{Average circumference} = 3.14 \times 52\frac{1}{4}$$

$$\text{Area of cross-section} = 4\frac{1}{4} \times 6\frac{1}{2}$$

The product of $3.14 \times 52\frac{1}{4} \times 4\frac{1}{4} \times 6\frac{1}{2}$ gives the volume in cubic inches. Dividing by 1728 changes this to cubic feet. Since each cubic foot of cast iron weighs 450 lb., we multiply by 450, giving the result 1181 lb.

PROBLEMS

1. Find the weight of a rectangular steel plate 3′ 3″ × 2′ 1″ and $\frac{3}{4}$″ thick.

2. Find the weight of a 20′ length of wrought iron pipe whose inside diameter is 3.548″ and outside diameter 4″.

3. Find the weight of 1000 round U. S. Standard wrought iron washers whose outside diameter is $3\frac{3}{4}$″, with a $1\frac{3}{4}$″ hole and a thickness of $\frac{11}{64}$″.

4. Find the weight of 500 wrought iron washers 5″ square and $\frac{3}{8}$″ thick. The hole is $1\frac{1}{2}$″ in diameter.

5. Find the weight of a cast iron manhole cover with semicircular ends, if the overall length is 4′ 2″, the width 2′ 1″, and the thickness $1\frac{1}{2}$″.

89. Weight of Castings from Patterns

It is frequently possible to estimate very closely the weight of a casting from the weight of the pattern.

Example

Find the weight of a cast iron casting made from a white pine pattern weighing $9\frac{1}{4}$ lb.

SOLUTION. Volume of pattern $= \frac{9\frac{1}{4}}{25} = 0.37$ cu. ft.

Weight of casting $= 0.37 \times 450 = 166.5$ lb.

EXPLANATION. Since a cubic foot of white pine weighs approximately 25 lb., the volume of the pattern will be $\frac{9\frac{1}{4}}{25}$ or 0.37 of a cubic foot. A cubic foot of cast iron weighs 450 lb. Hence 0.37 of a cubic foot will weigh 0.37×450 or 166.5 lb.

PROBLEMS

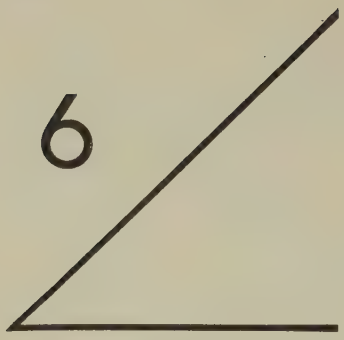
1. Find the weight of a brass casting made from a cherry pattern which weighs $7\frac{3}{4}$ lb.

2. A pattern made of white oak weighs 5 lb. $7\frac{1}{2}$ oz. What will a copper casting made from it weigh?

3. The ratio of the weight of white pine to cast iron is 1 to 16. If the pattern weighs 4 lb. 14 oz., what will an iron casting weigh?

4. Bell metal weighs 14.2 times as much as mahogany. If a pattern of mahogany weighs 8 lb. 6 oz., what will the bell metal casting weigh?

5. From the table of weights of materials on page 564 compute the following weight ratios: (*a*) pine to brass; (*b*) maple to cast iron; (*c*) mahogany to copper; (*d*) pine to bell metal; (*e*) oak to aluminum.



6 Practical Computation

90. Methods

The skilled mechanic can always be recognized by the methods he employs in doing a job. He lays out his work systematically; he makes use of the shortest and most convenient methods. He invariably selects the proper tool for the job. He knows how to check the accuracy of his work within the limits of accuracy required by the type of work he is doing. All of which may be summarized by saying that he applies common sense to his work.

Practical computation is similar to any other type of practical work. The most effective work is done when the right methods are employed. The work can be so arranged as to require the minimum number of operations and to make each operation stand out as a distinct step. Various aids, such as tables, constants, slide rules, and handbooks, may be used to facilitate the work and make the computation quick, clear, and accurate. In the following pages are given some of the methods of computation which a practical man would employ.

91. Arrangement of Work

Of foremost importance is the arrangement of the work on the sheet. The work may be either a hopeless jumble of figures or it may be arranged as a clear and definite statement of the problem, with each step in the process plainly indi-

cated. The following example will serve to illustrate both methods.

Example

Find the weight of a steel plate $\frac{3}{4}''$ thick, with shape and size as shown in Fig. 147.

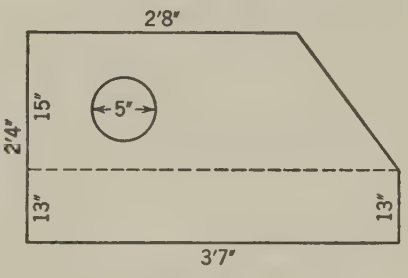


Fig. 147

SOLUTION.

2	2	3
12	12	12
24	24	36
8	4	7
32	28	43
	13	
	15	

43	32	2.5	1121.5
13	43	2.5	19.625
129	2)75	125	1101.875
43	37.5	50	.75
559	15	6.25	5509375
	1875	3.14	7713125
	375	2500	826.406250
	562.5	625	.284
	559	1875	3305624
	1121.5	19.6250	6611248
			1652812
			234.699304

The answer is 235 lb.

The above is a typical example selected at random from thousands which have come under the authors' observation. The result is correct, but it is almost impossible to check it. There is nothing to show whether some important step has not been omitted; nothing to indicate the various processes incidental to obtaining the result. Compare the following solution:

SOLUTION. Change dimensions to inches. $2'\ 8'' = 32''$

$2'\ 4'' = 28''$

$3'\ 7'' = 43''$

Divide the figure into a rectangle and a trapezoid.

Area of rectangle = 43×13 = 559 sq. in.

Area of trapezoid = 37.5×15 = 562.5 sq. in.
(average width of trapezoid

$$= \frac{43 + 32}{2} = 37.5'')$$

Total area (gross) = 1121.5 sq. in.

Area of 5″ circle
$= 3.14 \times 2.5 \times 2.5$ = 19.63 sq. in.

Net area = 1101.87 sq. in.

Volume = $1101.87 \times 0.75 = 826.41$ cu. in.

Weight = $826.41 \times 0.284 = 234.71$ lb.

The answer is 235 lb.

The result is the same in both cases, but in the second solution it is always possible to go back to the problem and see whether all the factors have been taken into account. Every operation is labeled. The steps are indicated only; the actual work of division, multiplication, etc., being done on some other part of the sheet, where it will not interfere with the thought processes as carried out in the solution. It is often essential for the computer or some other person to review the calculation some time after it was made, and in the second instance it is very easy for him to see how the result was obtained. In the foregoing pages typical solutions of many examples have been given. The student should study the arrangement of the steps in these typical solutions and use a similar arrangement in his work.

92. Accuracy

All computation should be correct within the limits of accuracy determined by the nature of the problem. This does not mean that every example should be worked out to 3 or 4 decimal places. There are some types of work, such as tool and gage making, where an error of $\frac{1}{1000}$ of an inch is sufficient to cause rejection of the work. In other calcu-

lations such accuracy is entirely superfluous and involves a lot of unnecessary work in the computation. In most types of practical work the result is close enough if the first three or four significant figures are correct. This is the degree of accuracy that is obtained by the use of the common 10″ slide rule.

Several points should be considered in deciding the degree of accuracy desirable in any given problem.

1. *The accuracy of the data on which the problem is based.* Care must be taken that the data on which the problem is based are correct.

2. *The degree of accuracy of the data.* If measurements, formulas, factors, and unit values are only approximate, nothing is gained by carrying the answer to a fine point.

3. *The nature of the material.* The length of a leather belt or a wooden stringer should be given to the nearest $\frac{1}{8}$″ or $\frac{1}{4}$″. The diameter of a steel rod or the length of a brass bushing should be much more closely figured.

4. *The purpose for which the measurement or computation is made.* If we are computing the weight of a heavy casting or the capacity of a large tank, the nearest whole pound or gallon is close enough. If we are finding the depth of a gear tooth or a check measurement on a small tap, the answer should be given to the nearest .001″ or .0001″.

5. *The instruments to be used in making the measurements.* If we are going to measure the length of a piece of stock with a two-foot rule, a measurement to the nearest sixteenth of an inch is close enough. If a micrometer or a vernier caliper is to be used for finding the diameter of a wire, the measurement should be given to the nearest .001″ or .0001″.

6. *Reading the answer to a problem.* For example, if the computation shows a measurement of 1.108732″, and the measurement is to be made to the nearest 0.001″, the answer should be read 1.109″. If 4 decimal places are required, the answer should be read 1.1087″. If the result is close enough when 2 decimal places are used, the answer is 1.11″. In each case, when the remainder after the last figure

is more than $\frac{1}{2}$, 1 is added to the last figure; if less than $\frac{1}{2}$, the remainder is dropped. If the remainder is exactly $\frac{1}{2}$, leave the last figure unchanged if it is even and add 1 if it is odd.

To sum up, the student should bear in mind that the thing to be sought is common-sense accuracy, that is, a degree of accuracy sufficient for the type of work in hand.

93. Use of Diagrams

Diagrams are an important aid in the solution of practical problems. The student should acquire the habit of making a neat sketch or diagram to illustrate his work. On this sketch all dimensions and all construction lines should be clearly indicated. The drawing may be made free-hand and need not be to scale. However, the general proportions should be maintained. A line 9″ long should not be drawn shorter than a line 7″ long; a 10″ circle should be drawn approximately twice as large as a 5″ circle; a 30° angle should be drawn approximately 30°.

The student will find that a careful sketch will often simplify a problem which at first appears very difficult. A sketch is really a graphic statement of the problem and, as such, usually easier to understand than the written statement. A little practice will enable the student to make such sketches very rapidly, and he will find that the time so used is well spent, since it results in a distinct improvement in speed and accuracy.

94. Use of Tables

To simplify computation and save time, many tables have been prepared covering every type of work. A computer who knows how to use his tables intelligently can work much more quickly and accurately than one who is not able to avail himself of such aids. The most generally useful tables are those commonly called "Tables of Properties of Numbers." These tables give lists of numbers, usually from 1 to 1000, with their squares, cubes, square roots, and cube roots.

They also give the circumferences and areas of circles of which the numbers are the diameters. Some of these tables are very elaborate and rather bulky; but a table from 1 to 1000, contained in a few pages, is convenient to handle and can be of great value. Table I in the Appendix is such a table and, within its limits, should be consulted constantly by the student so that he can familiarize himself with its use. The following examples will serve to illustrate the use of this table.

Example 1

Find the square root of 236.

SOLUTION. Find 236 in the right- or left-hand column and read across until you come to the column marked "Square Root." We find that the square root of 236 is 15.3623. If 2 decimal places are sufficient for our work, we call the answer 15.36, the remainder being less than $\frac{1}{2}$. If only 1 decimal place is required, the answer is 15.4. If 3 places are required, the answer is 15.362.

Example 2

Find the circumference of a circle whose diameter is 19″. Answer to 2 decimal places.

SOLUTION. Under the column "Circum.," Table I, on the line with number 19, we find 59.690. The answer is 59.69″.

Example 3

Find the cube root of 741 to 2 decimal places.

SOLUTION. Under the column "Cube Root," on the line with number 741, we find 9.0491. The answer to 2 decimal places is 9.05.

Example 4

Find the value of 79^3.

SOLUTION. Under the column "Cube," on the line with number 79, we find the answer, 493,039.

Example 5

Find the area of a 13″ circle to 1 decimal place.

SOLUTION. Under the column, "Area," on the line with number 13, we find 132.732. This answer to 1 decimal place is 132.7 sq. in.

Example 6

Find the square of 27.3.

SOLUTION. Under the column "Square," on line 273, we find 74,529. We move the decimal point in the square 2 places to the left for each place in the number, making the answer 745.29.

Example 7

Find the area of a circle 3.78″ in diameter, giving the answer to 2 decimal places.

SOLUTION. In the column "Area," on line 378, we find 112,221. We move the decimal point to the left 4 places, that is, 2 places for each place in the number, making the answer 11.2221. Since we require only 2 places, the answer is 11.22 sq. in.

Example 8

Find the cube of 8.93 to 3 decimal places.

SOLUTION. In the column "Cube," on line 893, we find 712,121,957. We move the decimal point to the left 3 places in the cube for each place in the number, making the result 712.121957. Since only 3 places are required, the answer is 712.122.

Example 9

Find the area of a $3\frac{1}{4}$″ circle to 3 decimal places.

SOLUTION. Since $3\frac{1}{4}$ can be written 3.25, we look in the column "Area," on line 325 and find 82,957.7. Moving the decimal point to the left 4 places, that is, 2 places for each place in the number, we get 8.29577. Since only 3 places are required, the answer is 8.296 sq. in.

Example 10

Find the area of a $\frac{7}{8}''$ circle to 3 decimal places.

SOLUTION. $\frac{7}{8}'' = 0.875''$. The area of a circle 875″ in diameter is 601,320 sq. in. Moving the decimal point to the left 6 places (2 places for each place in the diameter), we get 0.601320. To 3 decimal places, the answer is 0.601 sq. in.

The table whose use is illustrated by the above examples is only one of the many useful tables contained in standard handbooks. The student should learn the use of the various tables in his handbooks. He will find that an intelligent use of them will, to a great extent, eliminate the drudgery of computation.

95. Mental Approximation of Results

Many of the errors which occur in computation are due to wrong location of the decimal point, or the omission of one or more of the factors involved in the problem. Such errors are usually very large and readily discovered, if a mental approximation of the result is made before the computation is actually performed. Before solving a problem, the student should look over the figures and decide in advance the approximate answer. *He should know about how large the answer should be;* and if the answer obtained by computation differs very much from this figure, he can be quite sure that an error has been made. Let us take, for example, a problem in which we are required to find the capacity of a certain tank and we make a mental approximation of 75 gallons. If we get an answer of 828 gallons as the result of computation, we know that it is too large and proceed to look for the error. Most likely the decimal point has been misplaced, and the answer should be 82.8 gallons. This error is easily found by looking over the work. No matter how rough the approximation may be, it will still be close enough to indicate whether or not the answer is a reasonable one.

96. Short Methods

Very few of the so-called "short methods" are of practical value. As a rule it takes more time to acquire facility in

their use than their limited application warrants. Some of them, however, are easily learned and of sufficient value to justify the time spent in mastering them. In the different chapters various short methods are given, among which the following are noteworthy.

1. Cancellation.
2. Division and multiplication by 10 and powers of 10.
3. Use of tables.
4. Use of logarithms.
5. Use of slide rule.

97. Checking Results

It is essential that the student form the habit of checking all computations. He should never be satisfied with the answer to a problem until he is quite sure that it is correct. Large errors are easily avoided by the application of the method of approximate answers. Other errors may be discovered by having two persons solve the problem independently. If the results agree, there is every probability that the answer is correct. It is hardly likely that two people working independently will make the same mistake.

In checking problems, we should avoid retracing the steps of the first solution. We should try, as far as possible, to obtain the result by different methods.

1. *Addition.* If a column is added *up* the first time, add it *down* the second time.

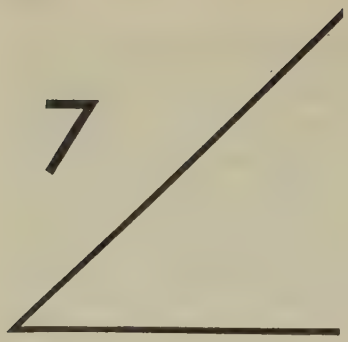
2. *Subtraction.* In subtraction, addition of the subtrahend and the difference should give the minuend; the student should perform such an addition automatically every time he makes a subtraction.

3. *Multiplication.* In checking multiplication, it is convenient to change the multiplier for the multiplicand, or we may divide the product by one of the factors.

4. *Division.* In division we can check by multiplying the divisor by the quotient and adding the remainder, if any.

The *slide rule* is of great value in checking multiplication, division, roots, and powers, particularly when an approximate check is sufficient for the purpose. It is especially valuable where a number of factors are involved, making ordinary methods of checking laborious and uncertain.

Special methods of checking can be developed for particular cases in which a certain typical operation or a series of operations is frequently repeated.



7 Logarithms

98. Definitions

The *logarithm* of a number is the exponent of the power to which a constant number must be raised to produce the given number. The constant number is called the ***base.*** In the common or Briggs system of logarithms the base is 10. The common logarithm of a number is therefore the power to which 10 must be raised to produce the given number. Thus, since $10^2 = 100$, that is, since 10 must be raised to the second power to equal 100, we say the logarithm of 100 is 2, written in abbreviated form, $\log 100 = 2$. Likewise $\log 10{,}000 = 4$, since 10 must be raised to the fourth power to produce 10,000.

Since,

$10^4 = 10000$, $\log 10000 = 4$	$10^{-1} = .1$, $\log .1 = -1$
$10^3 = 1000$, $\log 1000 = 3$	$10^{-2} = .01$, $\log .01 = -2$
$10^2 = 100$, $\log 100 = 2$	$10^{-3} = .001$, $\log .001 = -3$
$10^1 = 10$, $\log 10 = 1$	$10^{-4} = .0001$, $\log .0001 = -4$
$10^0 = 1$, $\log 1 = 0$	

It is evident from the above table that the logarithm of a number which lies between 1000 and 10,000 lies between 3 and 4; that is, it is 3 plus a fraction. The logarithm of a number which lies between 10 and 100 lies between 1 and 2; that is, it is 1 plus a fraction. The logarithm of a number which lies between .1 and 1 lies between -1 and 0; that is,

it is -1 plus a fraction. The logarithm of a number which lies between .001 and .01 lies between -3 and -2; that is, it is -3 plus a fraction. Thus log .008 is $\bar{3}.90309$. The minus sign is placed above the 3 instead of in front of it, to show that the 3 only is negative, while the decimal part of the logarithm is plus, or positive.

99. Characteristic and Mantissa

The integral part of a logarithm is called the ***characteristic;*** the decimal part is the ***mantissa.*** The mantissa is always found in a table of logarithms. The characteristic is determined mentally by the following rules:

Rule 1

To find the characteristic of the logarithm of a number greater than 1:

The characteristic of the logarithm of a number greater than 1 is 1 less than the number of figures before the decimal point in the given number.

Thus, log 352.6 = 2.5473; that is, the characteristic of the logarithm of 352.6 is 2, because the integral part of the number contains three figures, namely 3, 5, and 2.

PROBLEMS

What is the characteristic of the logarithm of each of the following numbers?

1. 64	**2.** 300	**3.** 5600	**4.** 24.3	**5.** 8
6. 7.02	**7.** 5006	**8.** 400000	**9.** 32.78	**10.** 400.6
11. 1.609	**12.** 236.84	**13.** 300.9	**14.** 64200	**15.** 27.624
16. 5.18	**17.** 518.6	**18.** 518.64	**19.** 51.86	**20.** 5.186
21. 9	**22.** 1	**23.** 1001	**24.** 12.01	**25.** 120.1

Since the characteristic is 1 less than the number of figures before the decimal point, it follows conversely that the number of figures before the decimal point in a number corresponding to a given logarithm will be 1 more than the characteristic. Thus, if the characteristic is 1, there must be two figures before the decimal point in the number whose

LOGARITHMS OF NUMBERS

	0	1	2	3	4	5	6	7	8	9
1	0000	0414	0792	1139	1461	1761	2041	2304	2553	2788
2	3010	3222	3424	3617	3802	3979	4150	4314	4472	4624
3	4771	4914	5051	5185	5315	5441	5563	5682	5798	5911
4	6021	6128	6232	6335	6435	6532	6628	6721	6812	6902
5	6990	7076	7160	7243	7324	7404	7482	7559	7634	7709
6	7782	7853	7924	7993	8062	8129	8195	8261	8325	8388
7	8451	8513	8573	8633	8692	8751	8808	8865	8921	8976
8	9031	9085	9138	9191	9243	9294	9345	9395	9445	9494
9	9542	9590	9638	9685	9731	9777	9823	9868	9912	9956
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981

LOGARITHMS OF NUMBERS

	0	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

logarithm has the characteristic 1. If the characteristic is 2, there must be three figures before the decimal point in the number, and so on.

PROBLEMS

How many figures must there be before the decimal point in the numbers if the characteristics of their logarithms are as follows?

1. 0	**2.** 4	**3.** 6	**4.** 5	**5.** 2
6. 1	**7.** 3	**8.** 0	**9.** 2	**10.** 3

Rule 2

To find the characteristic of the logarithm of a number less than 1:

The characteristic of the logarithm of a decimal fraction is a negative number and is 1 more than the number of zeros between the decimal point and the first significant figure.

Thus, the logarithm of .00642 is $\bar{3}.8075$, because there are two zeros between the decimal point and the first significant figure, 6. Note that the minus sign is placed *above* the characteristic to show that it only is negative, while the *mantissa is always positive.*

PROBLEMS

What is the characteristic of the logarithm of each of the following numbers?

1. .4	**2.** .04	**3.** .004	**4.** .0004
5. .0135	**6.** .06002	**7.** .548	**8.** .0063
9. .25	**10.** .025	**11.** .0306	**12.** .81
13. .8	**14.** .008	**15.** .245	**16.** .00085
17. .00009	**18.** .604	**19.** .825	**20.** .3264

Rule 3

To find the mantissa of the logarithm of a number:

The mantissa is found in the table and is the same for all numbers made up of the same significant figures, no matter where the decimal point is located.

Hence, to find the logarithm of a number, determine the characteristic according to the rules given above and annex

to it the mantissa for the significant figures as found in the table. Thus we have:

$\log 32500 = 4.5119$	$\log .325 = \bar{1}.5119$
$\log 3250 = 3.5119$	$\log .0325 = \bar{2}.5119$
$\log 325 = 2.5119$	$\log .00325 = \bar{3}.5119$
$\log 32.5 = 1.5119$	$\log .000325 = \bar{4}.5119$
$\log 3.25 = 0.5119$	$\log .0000325 = \bar{5}.5119$

Note that in each case the mantissa is that given in the table for the significant figures 325.

100. Use of Tables

To find the logarithm of a number:

Example 1

Find the logarithm of 5.

SOLUTION. $\text{Log } 5 = 0.6990$

EXPLANATION. Since the number consists of one integer, namely 5, the characteristic is 1 less than 1, namely, 0. (See Rule 1.) To find the mantissa, we look for the number 5 in the first column of the table of logarithms on page 166 and since there is no decimal after the integer 5, we take the mantissa 6990 which we find in the 0 column. Placing the decimal point before the mantissa, we get $\log 5 = 0.6990$.

Example 2

Find the logarithm of 728.

SOLUTION. $\text{Log } 728 = 2.8621$

EXPLANATION. Since there are 3 integers in the number, the characteristic is 2. (Rule 1.) To find the mantissa, we look for 72 in the first column on page 167, and reading across the page we find the number 8621 under 8 at the top. That is, 8621 is the mantissa for 728. Placing the decimal point between the characteristic and the mantissa, we get $\log 728 = 2.8621$.

Example 3

Find the logarithm of 57.4.

SOLUTION. Log 57.4 = 1.7589

EXPLANATION. Since there are two figures in the integral part of the number, the characteristic is one less than 2, namely 1. To find the mantissa, we look in the first column of the table for the number 57, and reading across the page, we find the number 7589 under 4 at the top, that is, 7589 is the mantissa of 574. We place the decimal point between the characteristic and the mantissa, giving the log 57.4 = 1.7589.

Example 4

Find the logarithm of 2648.

SOLUTION. Log 2648 = 3.4229

EXPLANATION. We set down the characteristic 3 because the number contains 4 figures before the decimal point. To find the mantissa, imagine the decimal point after the third figure. This would make the number 264.8. The mantissa for 2648 is the same as for 264.8. To obtain the mantissa for 264.8, we proceed as follows:

Mantissa of 264	=	4216
Mantissa of 265	=	4232
Difference	=	16

Since 264.8 lies $\frac{8}{10}$ of the way from 264 to 265, the mantissa of 264.8 will lie $\frac{8}{10}$ of the way from 4216 to 4232; that is, the mantissa of 264.8 is 4216 + $\frac{8}{10}$ of 16 = 4216 + 13 = 4229. Hence the logarithm of 2648 = 3.4229.

The process of computing an intermediate number between two numbers in a table is known as ***interpolation.***

To find the logarithm of a decimal fraction:

Example 5

Find the logarithm of .05834.

SOLUTION. Log .05834 = $\bar{2}$.7660

EXPLANATION. The characteristic is $\bar{2}$ because there is one zero between the decimal point and the first significant figure (Rule 2). To find the mantissa, we proceed as if 5834 were an integral number, as follows:

Mantissa of 583 = 7657
Mantissa of 584 = 7664
Difference = 7

$$\text{Mantissa of } 583.4 = 7657 + \tfrac{4}{10} \text{ of } 7 = 7657 + 3 = 7660$$

Therefore, $\log .05834 = \bar{2}.7660.$

PROBLEMS

Find the logarithms of the following numbers.

1. 9	**2.** 28	**3.** 70	**4.** 120	**5.** 400
6. 245	**7.** 248.6	**8.** 2.486	**9.** .0524	**10.** .345
11. 2600	**12.** 6735	**13.** .000382	**14.** .004965	**15.** 234.56
16. .4008	**17.** 12.3	**18.** 4.207	**19.** .8935	**20.** .000089
21. 149200	**22.** 80.06	**23.** .538	**24.** .0538	**25.** 53.8

To find the number corresponding to a given logarithm:

Example 1

Find the number whose logarithm is 1.5051.

SOLUTION. The number whose logarithm is 1.5051 is 32.

EXPLANATION. In the table of logs we find the mantissa 5051 corresponds to the number 32. The characteristic 1 tells us that there must be two figures before the decimal point. Hence, the number whose log is 1.5051 is 32.

Example 2

Find the number whose logarithm is 2.8609.

SOLUTION. The number whose logarithm is 2.8609 is 726.

EXPLANATION. In the table, we find the mantissa 8609 on line 72 in the column under 6. Therefore the number corresponding to the mantissa 8609 is 726. Since the characteristic is 2,

there must be three figures before the decimal point. Hence, the number whose log is 2.8609 is 726.

Example 3

Find the number whose logarithm is 3.3927.

SOLUTION. The number whose log is 3.3927 is 2470.

EXPLANATION. In the table of logs we find that the mantissa 3927 corresponds to the number 247. The characteristic 3 tells us that there must be 4 integral figures; hence we add a cipher to 247 and get the number 2470.

Likewise, the number whose log is 2.3927 is 247
the number whose log is 1.3927 is 24.7
the number whose log is 0.3927 is 2.47
the number whose log is $\bar{1}$.3927 is .247
the number whose log is $\bar{2}$.3927 is .0247

Example 4

Find the number whose log is $\bar{3}$.5595.

The mantissa 5595 cannot be found in the table. The nearest mantissa *below* 5595 is 5587, which is the log of 362. Since the given mantissa lies between 5587 and 5599, the number will be between 362 and 363. The difference between the mantissas for 362 and 363 is $5599 - 5587 = 12$, while the difference between the given mantissa and that of 362 is $5595 - 5587 = 8$. Hence the number corresponding to mantissa 5595 is $362 + \frac{8}{12} = 362.7$. But the characteristic $\bar{3}$ tells us there must be 2 zeros between the decimal point in the number and the first significant figure; hence the number corresponding to the logarithm $\bar{3}$.5595 is .003627.

PROBLEMS

Find the numbers whose logarithms are:

1. 2.9212	**2.** 1.9212	**3.** 0.9212	**4.** $\bar{1}$.9212	**5.** 4.9212
6. 2.6390	**7.** 3.3862	**8.** 0.8761	**9.** 0.9559	**10.** $\bar{1}$.8348
11. 3.7057	**12.** 2.7378	**13.** 2.0152	**14.** 1.0099	**15.** $\bar{2}$.8951
16. $\bar{2}$.9225	**17.** $\bar{3}$.9530	**18.** $\bar{1}$.8239	**19.** 4.9488	**20.** $\bar{4}$.9488
21. 2.8879	**22.** 1.7870	**23.** 3.5385	**24.** $\bar{1}$.4594	**25.** 0.6284

101. Multiplication by Use of Logarithms

To find the product of two or more numbers, add their logarithms; the sum is the log of the product.

Example 1

Multiply 32.4 by 71.6.

SOLUTION. Log 32.4 = 1.5105
Log 71.6 = 1.8549

Log of product = 3.3654 ∴ The product is 2319.4.

If we multiply 32.4 by 71.6, we get 2319.8. The discrepancy between the exact product and the one obtained by logarithms is due to the fact that only 4-place log tables are used. Greater accuracy can be obtained by the use of a 6-place table.

Example 2

Find the value of $6.45 \times 25.92 \times .088$.

SOLUTION. Log 6.45 = 0.8096
Log 25.92 = 1.4136
Log .088 = $\bar{2}$.9445

Log of product = 1.1677 ∴ The product is 14.71.

PROBLEMS

Find the approximate value of the following by means of logs.

1. $2 \times 7 \times 5$
2. $.6 \times 8.5 \times 40$
3. 124×532
4. 788×436
5. 42.5×684
6. 232×456
7. 14.8×12.7
8. 46.87×4.24
9. $.444 \times 3.25$
10. $3.1416 \times .063$
11. 5280×16400
12. $.085 \times .006254$
13. 6053×286.7
14. 2.005×82.695
15. $788.1 \times .00342$
16. $72 \times 320 \times .654$
17. $34.8 \times 28.6 \times 14.2$
18. $1765 \times 5.5 \times .075 \times 23.8$
19. $144 \times 6.25 \times .875$
20. $24.5 \times 24.5 \times .7854$
21. $5.4 \times 8 \times 48.56 \times .085$
22. $9876 \times .00325 \times 2.59$
23. $320.8 \times 2.31 \times 4.42$
24. $2004 \times .00375 \times 543$
25. $12.22 \times 5.26 \times .125$

102. Division by Use of Logarithms

To find the quotient of two numbers, subtract the log of the divisor from the log of the dividend; the difference is the log of the quotient.

Example 1

Divide 864 by 16.

SOLUTION. Log 864 = 2.9365
Log 16 = 1.2041
Log of quotient = 1.7324 ∴ The quotient is 54.

Example 2

Divide 1.82 by 1.3.

SOLUTION. Log 1.82 = 0.2601
Log 1.3 = 0.1139
Log of quotient = 0.1462 ∴ The quotient is 1.403.

Example 3

Divide 872 by 93.

SOLUTION. Log 872 = 2.9405
Log 93 = 1.9685
Log of quotient = 0.9720 ∴ The quotient is 9.376.

Example 4

Divide 42.75 by 752.3.

SOLUTION. Log 42.75 = 1.6309
Log 752.3 = 2.8764
Log of quotient = $\bar{2}$.7545 ∴ The quotient is .05682.

103. Cologarithms

The cologarithm of a number is found by subtracting the log of the number from 10 and then annexing −10 to the remainder. This is done mentally by beginning at the left

and subtracting each figure from 9 until the last significant figure is reached, which must be subtracted from 10. Thus we have

$$\log 234 = 2.3692$$

$$\operatorname{colog} 234 = 7.6308 - 10$$

Instead of *subtracting* the log of the divisor from the log of the dividend, the same result is obtained if we *add* the colog of the divisor to the log of the dividend. This method greatly simplifies and expedites the work.

PROBLEMS

Find the cologarithms corresponding to the following logarithms.

1. 1.6284	**2.** 0.8149	**3.** 2.8762	**4.** 1.3909	**5.** 0.6990
6. 3.4524	**7.** $\bar{3}$.4524	**8.** $\bar{4}$.7356	**9.** 2.4728	**10.** $\bar{2}$.4728
11. 1.8069	**12.** $\bar{1}$.8998	**13.** 4.5391	**14.** 0.6096	**15.** 0.7723
16. 2.6628	**17.** 3.5944	**18.** 1.4757	**19.** 0.5955	**20.** $\bar{2}$.6484
21. 1.8156	**22.** 2.8704	**23.** 3.3856	**24.** $\bar{1}$.5416	**25.** 1.1614

PROBLEMS

Find the cologarithms of the following numbers.

1. 8	**2.** 35	**3.** 864	**4.** 5762	**5.** 400
6. 6.2	**7.** 9.54	**8.** 12.36	**9.** 64.582	**10.** .06
11. .044	**12.** .0069	**13.** .000428	**14.** 382.6	**15.** 42.9
16. 1.602	**17.** 83.4	**18.** 932	**19.** 105	**20.** 600
21. 124	**22.** 23.64	**23.** 8.92	**24.** 6.02	**25.** 58.9

Example

$42.75 \div 752.3$

SOLUTION. $\operatorname{Log} 42.75 = 1.6309$

$\operatorname{Colog} 752.3 = 7.1236 - 10$

$\text{Log of quotient} = 8.7545 - 10 = \bar{2}.7545$

The quotient is .05682.

PROBLEMS

Find the value of the following by means of logarithms.

1. $7856 \div 2.34$	**2.** $.0653 \div .005809$	**3.** $55400 \div 325$
4. $.00448 \div .06925$	**5.** $4.62 \div 889.6$	**6.** $.988 \div 245$
7. $82 \div .0743$	**8.** $62.3 \div 15.7$	**9.** $3266 \div 9452$
10. $.0287 \div 6.3$	**11.** $234 \div 18$	**12.** $652 \div 2.4$
13. $59.6 \div 1.2$	**14.** $842.8 \div 2.86$	**15.** $700 \div 82.4$
16. $6.592 \div 56.88$	**17.** $416 \div 69$	**18.** $2702 \div 3.14$
19. $3.1 \div .026$	**20.** $42.4 \div .00892$	

104. Combined Multiplication and Division

The following example will illustrate the method used in solving problems involving both multiplication and division.

Example

Find the value of $\dfrac{236 \times 8.92}{455 \times .00264}$.

SOLUTION.

$$\begin{aligned} \text{Log } 236 &= 2.3729 \\ \text{Log } 8.92 &= 0.9504 \\ \text{Colog } 455 &= 7.3420 - 10 \\ \text{Colog } .00264 &= 12.5784 - 10 \\ \text{Log of quotient} &= 23.2437 - 20 = 3.2437 \end{aligned}$$

$\therefore$ The quotient is 1752.8.

EXPLANATION. Instead of *subtracting* the *logs* of 455 and .00264 from the logs of 236 and 8.92, we *add* the *cologs* of the numbers in the divisor to the logs of the numbers in the dividend.

PROBLEMS

1. $\dfrac{24.6 \times 563}{32}$	**2.** $\dfrac{2.35 \times .0084}{9.2}$
3. $\dfrac{62.44 \times .056}{3.14}$	**4.** $\dfrac{681.4 \times .065}{8.66}$
5. $\dfrac{24 \times 37}{4.3 \times 61}$	**6.** $\dfrac{72}{83.6 \times 54.9}$

7. $\dfrac{5}{18.3 \times .064}$

8. $\dfrac{42.1 \times 26.5}{52.4}$

9. $\dfrac{10.3 \times 50.9}{30.6 \times 16.8}$

10. $\dfrac{.076 \times .0048}{1.09}$

11. $\dfrac{2.4 \times 3.6}{1.8 \times 4.5}$

12. $\dfrac{16.2 \times 12.8}{45.6 \times 1.16 \times 32.3}$

13. $\dfrac{18 \times 42}{16 \times 13}$

14. $\dfrac{196 \times 2.4}{.56 \times .885}$

15. $\dfrac{.0726 \times .625}{.375 \times .00543}$

16. $\dfrac{432 \times 8.25}{396 \times .055}$

17. $\dfrac{2.32 \times 4546 \times 722}{682.5 \times 300.5 \times .042}$

18. $\dfrac{399 \times 7600 \times 814}{483 \times 8856 \times 212}$

19. $\dfrac{3.26 \times 8.98 \times 6.33}{16.8 \times 9.61 \times 4.52}$

20. $\dfrac{1.23 \times 3.45 \times 4.56 \times 5.67}{.321 \times 5.43 \times 6.45 \times 7.56}$

105. Raising a Number to Any Power

To find the power of a number, multiply the log of the number by the exponent of the power; the result is the log of the power.

Example 1

Find the value of 8.4^2.

SOLUTION. $\log 8.4 = 0.9243$

$\times 2$

$\log 8.4^2 = 1.8486 \quad \therefore 8.4^2 = \mathbf{70.55.}$

Example 2

Find the value of 8.654^3.

SOLUTION. $\log 8.654 = 0.9372$

$\times 3$

$\log 8.654^3 = 2.8116 \quad \therefore 8.654^3 = 648.$

PROBLEMS

1. 18^2
2. 3.5^2
3. 93^2
4. 84.5^2
5. 324.6^2
6. 235^2
7. 36.92^2
8. $.463^2$
9. 6.52^3

10. $.00656^2$ **11.** 3.412^2 **12.** 1.692^2
13. $.866 \times (1.463)^2$ **14.** $(.854)^2 \times (.692)^2$ **15.** $(3.14)^3 \times (1.75)^2$
16. $11\frac{1}{4} \times (6\frac{1}{4})^2$ **17.** $3.14 \times (1\frac{1}{8})^3$ **18.** $.707 \times (7\frac{3}{4})^2$
19. $4.832 \times (3\frac{1}{2})^2$ **20.** $.577 \times (7.22)^2$ **21.** $2.598 \times (3\frac{5}{8})^2$
22. $.828 \times (11\frac{1}{2})^2$ **23.** $1.57 \times (3\frac{3}{4})^3$ **24.** $.828 \times (6\frac{1}{4})^2$
25. $(1\frac{1}{4})^2 \times (2\frac{1}{2})^3$

106. To Extract Any Root of a Number

To find the root of a number, divide the log of the number by the index of the root; the result is the log of the root.

Example 1

Find $\sqrt{256}$.

SOLUTION. Log 256 = 2.4082

$$\text{Log } \sqrt{256} = \tfrac{1}{2} \log 256 = \frac{2.4082}{2} = 1.2041$$

$\therefore \sqrt{256} = 16.$

Example 2

Find $\sqrt{1.89}$.

SOLUTION. Log 1.89 = 0.2765

$$\text{Log } \sqrt{1.89} = \tfrac{1}{2} \log 1.89 = \frac{0.2765}{2} = 0.1382$$

$\therefore \sqrt{1.89} = 1.375.$

Example 3

Find $\sqrt[3]{343}$.

SOLUTION. Log 343 = 2.5353

$$\text{Log } \sqrt[3]{343} = \frac{1}{3} \log 343 = \frac{2.5353}{3} = .8451$$

$\therefore \sqrt[3]{343} = 7.$

Example 4

Find $\sqrt{.7652}$.

SOLUTION. $\text{Log } .7652 = \bar{1}.8838$

$= 9.8838 - 10$

$\frac{1}{2} \log .7652 = 4.9419 - 5 = \bar{1}.9419$

$\therefore \sqrt{.7652} = .8748.$

Before dividing by 2, we add 10 and subtract 10 to eliminate the negative characteristic $\bar{1}$. Likewise, in extracting the cube root of a decimal, we add and subtract 30 to make it possible to divide by 3.

Example 5

Find the value of $\sqrt{\dfrac{6.62 \times 48.75 \times 12.8}{18.4 \times 33.42}}$.

SOLUTION.

$\text{Log } 6.62 = 0.8209$

$\text{Log } 48.75 = 1.6880$

$\text{Log } 12.8 = 1.1072$

$\text{Colog } 18.4 = 8.7352 - 10$

$\text{Colog } 33.42 = 8.4761 - 10$

$$\text{Log} \frac{6.62 \times 48.75 \times 12.8}{18.4 \times 33.42} = 20.8274 - 20 = .8274$$

$$\text{Log} \sqrt{\frac{6.62 \times 48.75 \times 12.8}{18.4 \times 33.42}} = \frac{1}{2} \log \frac{6.62 \times 48.75 \times 12.8}{18.4 \times 33.42}$$

$= \dfrac{.8274}{2} = .4137$, the log of the answer

$$\therefore \sqrt{\frac{6.62 \times 48.75 \times 12.8}{18.4 \times 33.42}} = 2.594.$$

PROBLEMS

Solve the following problems by means of logarithms.

1. $\sqrt{196}$
2. $\sqrt{2.89}$
3. $\sqrt{361}$
4. $\sqrt{.0324}$
5. $\sqrt{.000225}$
6. $\sqrt{6954}$
7. $\sqrt{7.216}$
8. $\sqrt{.8143}$
9. $\sqrt[3]{15}$
10. $\sqrt[3]{82.16}$
11. $\sqrt{.06952}$
12. $\sqrt{.0004882}$
13. $\sqrt[3]{28.69}$
14. $\sqrt{116.7}$
15. $\sqrt{8.72}$
16. $\sqrt{9.82} \times \sqrt[3]{4.12}$
17. $\sqrt{5.12 \times 7.64 \times 8.19}$
18. $\sqrt{.00986 \times 5280}$
19. $\sqrt{.0587 \times 6032}$
20. $4 \times 3.14 \times \sqrt{36.24}$
21. $21.92 \times \sqrt{\dfrac{278.45}{44.18}}$
22. $\sqrt{\dfrac{1029 \times 672 \times 408}{1852}}$
23. $\sqrt{\dfrac{8.84 \times 36.45 \times 15.6}{19.7 \times 44.52}}$
24. $\dfrac{11.76 \times \sqrt{516.9}}{146.5}$
25. $6 \times 2.73 \times \sqrt{86.66}$

26. The volume of a sphere $= \frac{4}{3}\pi R^3$. Find the volume of a sphere whose radius is 14.8″.

27. Find the radius of a sphere whose volume is 1,000 cu. in. $R = \sqrt{\dfrac{3 \times \text{Vol.}}{4\pi}}$.

28. The area of a sphere is $4\pi R^2$. Find the area of a sphere whose radius is 12.5″.

29. The volume of a cylinder is $\pi R^2 H$ where H is the altitude. Find the volume of a cylinder 16.5″ high if the radius of the base is 9.25″.

30. Find the radius of a cylinder 15.5″ high, having a volume of 2,400 cu. in. $R = \sqrt{\dfrac{\text{Vol.}}{\pi H}}$.

8 The Slide Rule

107. Description

There are many types of slide rules, some of which are constructed for special work, such as electrical or hydraulic computations. The simplest and most generally useful instrument is the common ten-inch Mannheim slide rule illustrated in Fig. 148.

The slide rule consists of three parts: the ***rule*** or body of the instrument, the ***slide,*** and the glass ***runner*** or ***indicator,*** with a hair line scratched on its lower surface. Note that there are four scales on the instrument: scales A and D on the rule, and scales B and C on the slide. Note also that scale A is identical with scale B, and that scale C is identical with scale D.

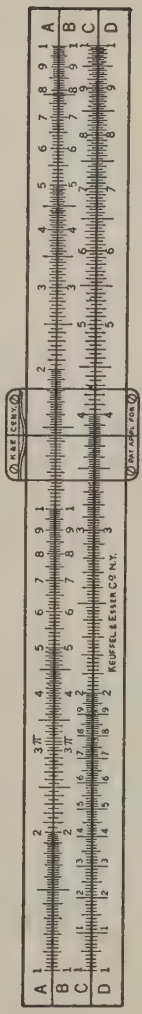


Fig. 148

108. Reading the Slide Rule, First Section

In studying a slide rule for the first time, the student should remove the slide and examine the lower rule scale, scale D. The first part of this scale, from 1 to 2 (a length of about 3 inches), Fig. 149, is divided into ten parts, numbered 1, 2, 3, etc. Each of

these ten parts is further divided into ten parts, with the middle or fifth division emphasized by an increased length of line. If we call the first number 1 (large figure 1), 1000, and the large 2, 2000, the numbers between these two figures will represent 1100, 1200, 1300, etc.* Since the space from 1200 to 1300, for example, is divided into ten parts, each part represents $\frac{1}{10}$ of 100, or 10. The first mark after

Fig. 149

the 1200 is 1210, the second mark is 1220, the fifth or half-way mark is 1250, the last mark before the 1300 is 1290, etc. Similarly the first mark after the 1700 is 1710, the third mark after the 1000 is 1030, the last mark before the 2000 is 1990, etc.

Reading between the marks is a matter of estimating with the eye, and becomes easier and more accurate with practice. If the hair line on the indicator appears exactly half-way between 1340 and 1350, we call it 1345. If it is between 1340 and 1350, but nearer to 1340, the reading might be 1341 or 1342 or 1343 or 1344, depending on the position of the indicator with reference to the 1340 and 1350 lines.

PROBLEMS

Set the following numbers; that is, cover them exactly with the indicator line.

1.	1500,	1300,	1900,	1100,	2000,	1400,	1200
2.	1279,	1680,	1310,	1050,	1220,	1530,	1010
3.	1915,	1325,	1525,	1075,	1035,	1010,	1005
4.	1443,	1221,	1018,	1117,	1933,	1628,	1497

* The student should move the glass runner until the hair line or indicator exactly covers the readings as they are mentioned in the text.

109. Second or Middle Section of Scale *D*

The student should now study the middle section, from 2 to 4, Fig. 150, calling the figures, 2000, 3000, and 4000. Since there are ten major spaces between 2000 and 3000, each major division represents 100. The middle line, which is larger than the others, represents 2500; the next long line, 2600, etc. The length from 2500 to 2600 is divided into 5 spaces; each space therefore represents $\frac{1}{5}$ of 100, or 20. The first line after the 2500 is therefore 2520; the second line is

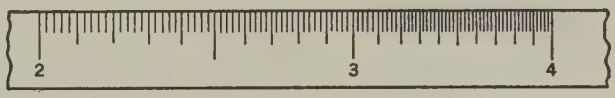


Fig. 150

2540; then 2560, 2580, and 2600. If the indicator appears exactly half-way between 2540 and 2560, the reading is 2550; if it is one-fourth of the distance from 2540, we call the reading 2545; if the distance from 2540 appears about one-third of the entire space, the reading is 2547, etc. We must guide ourselves by the proportional part of the distance traversed.

PROBLEMS

Set the following numbers.

1.	2500,	2600,	2800,	2100,	3100,	3800,	3300,	2400
2.	2520,	2060,	2080,	2860,	3240,	3020,	3660,	2010
3.	2420,	2440,	2430,	2450,	2470,	2490,	2410,	2950
4.	3010,	3815,	3625,	2005,	2790,	3330,	3350,	3175
5.	2716,	3133,	3318,	2017,	2913,	3156,	3977,	2936

110. Third Section of Scale *D*

The third or right section, from 4 to 10, Fig. 151, should now be studied, calling the figures 4000, 5000, 6000, etc. We see that the length from 4000 to 5000 is divided into ten

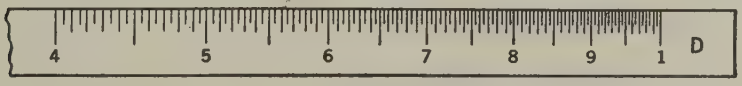


Fig. 151

major spaces; each space therefore represents 100. The middle line representing 4500 is made more prominent than the others; the next long line is 4600; then 4700, 4800, 4900, and 5000. Since the space from 4500 to 4600 is divided into two parts, each part represents $\frac{1}{2}$ of 100, or 50. The line between 4500 and 4600 is 4550; between 6200 and 6300, 6250; etc.

To read between the lines, we must again approximate with the eye judging the relative distance of the indicator from the two adjacent lines. Thus, if the indicator appears half-way between 4800 and 4850, the reading is 4825; if it is nearer to the 4800 line, the reading may be 4805 or 4810 or 4815 or 4820 or whatever proportional part of the entire space of 50 appears to have been traversed.

PROBLEMS

Set the following numbers.

1.	8000,	6000,	4000,	5000,	9000,	7000		
2.	6500,	7200,	3100,	9900,	5300,	7500,	4100,	9300
3.	4500,	4600,	4550,	4650,	6350,	9250,	8150,	4350
4.	7250,	7200,	7225,	8100,	8150,	8125,	9375,	5525
5.	8650,	8645,	8640,	8610,	4130,	5320,	9160,	5570

Summary

The smallest divisions of the three sections of the slide rule scale *D* are as follows:

In the first section the smallest division is 10.
In the second section the smallest division is 20.
In the third section the smallest division is 50.

Slide rule results are only approximate, owing to the difficulty of reading. However, it is always possible to get the first three or four figures correct, a degree of accuracy which is quite sufficient for the great majority of computations required in ordinary work.

NOTE. To get the first significant figure of any number, go to the left of the indicator to the *first large figure.*

111. The Decimal Point

The values 1000, 2000, etc., adopted for the figures 1, 2, etc., were assumed only to facilitate learning to read the rule. The slide rule merely shows the significant figures of any number in their relation to one another. The actual value of any set of significant figures as determined by the location of the decimal point is not shown. Thus the numbers 1655, 16.55, 1.655, and .1655, consisting of the same set of significant figures, have the same location on the slide rule; 18.25 has the same location as 18,250; 6545 the same as 6.545; 1005 the same as 1.005. In the operation of the slide rule no distinction is made between the various values of these sets of significant figures, but rules will be given for interpreting the results.

PROBLEMS

Set the following numbers.

2460, 1.515, 38.61, 650000, .96, 2.415, 1.005, 4.08, .000615, .01885, 2.74.

112. Multiplication

In the use of the slide rule for multiplication, we add lengths instead of multiplying numbers.

Example 1

Multiply 2 by 3.

SOLUTION. Using scales C and D, place the left index (large 1) of C over the 2 in D, and move the indicator until the hair line is exactly over the 3 of scale C. Reading the line on scale D covered by the indicator, we see that it is 6. We have added the length of 3 on the slide to the length of 2 on the scale, giving a total length on the scale representing the result, 6.

Example 2

Multiply 15 by 16.

SOLUTION. On the slide rule 15 is the same as 1500, and 16 as 1600. Place left index of slide over 1500 on scale D and move the indicator to slide 1600 on scale C. On scale D under the indicator

line we see 2400. For interpreting the result, we know that 15×16 is approximately equal to 10×20, or 200. The answer is therefore 240.

Example 3

19.5×2.75

SOLUTION. Place the left index of the slide C over 1950 of scale D and move the indicator to slide 2750 on C. Under the indicator on scale D, we see 5370. The answer is 53.7 because 19.5×2.75 is approximately equal to 20×3, or 60. The approximate answer 60 shows that there will be two figures before the decimal point.

The method of approximate answers is the simplest and quickest way of determining the location of the decimal point in the result.

Example 4

26.75×199.5

SOLUTION. Approximate answer $= 30 \times 200 = 6000$ (four figures before decimal point).

Slide rule, $2675 \times 1995 = 5340$. The answer is 5340.

Example 5

3.5×46

SOLUTION. Approximate answer $= 4 \times 40 = 160$ (three places before decimal point).

Placing the left C index over 3500 on scale D, we see that 4600 on the slide is beyond the end of the scale. When this occurs, we use the right C index instead of the left. Placing the right C index over 3500 on D, we move the indicator to slide 4600 on scale C. We see 1610 on the scale D under the indicator. Since three places are required before the decimal point, the answer is 161.

Example 6

4.388×171.5

SOLUTION. Approximate answer $= 4 \times 180 = 720$ (three places before decimal point).

Slide rule, $4388 \times 1715 = 7530$. The answer is 753.

Example 7

438.8×17.15

SOLUTION. Approximate answer $= 400 \times 18 = 7200$ (four places before decimal point).

Slide rule, $4388 \times 1715 = 7530$. The answer is 7530.

113. Fractions

In multiplying fractions or mixed numbers, the fraction must be replaced by its decimal equivalent. Thus, to multiply $8\frac{1}{2}$ by $19\frac{3}{4}$, we multiply 8.5 by 19.75, the approximate answer being 10×20, or 200. The slide rule work is $8500 \times 1975 = 1679$. The answer is 167.9.

Rule

In using the slide rule, ***first get the approximate answer*** *in order to locate the decimal point in the result; then multiply the numbers without reference to the location of their decimal points and point off the indicated number of decimal places.*

PROBLEMS

Perform the following multiplications on the slide rule.

1. 2×2	**2.** 3×6	**3.** 5×8
4. 3×3	**5.** 6×75	**6.** 14×55
7. 340×65	**8.** 146×25	**9.** 208×305
10. 128×935	**11.** 1.6×31.5	**12.** 28.4×162.3
13. 106×92	**14.** $3.14 \times 6\frac{3}{4}$	**15.** $3.14 \times 11\frac{1}{2}$
16. $.434 \times 245$	**17.** $.866 \times 1.273$	**18.** 1.414×7500
19. 2.304×65	**20.** $.6495 \times 3\frac{7}{8}$	**21.** $108\frac{1}{2} \times 6\frac{3}{4}$
22. $27.3 \times 1\frac{7}{8}$	**23.** $2.598 \times 2\frac{5}{8}$	**24.** $1.06 \times .18$
25. $1\frac{7}{8} \times 1\frac{7}{8}$		

114. To Multiply More than Two Numbers

Example 1

$7.15 \times 3.25 \times 6.75$

SOLUTION. Approximate answer $= 7 \times 3 \times 7 = 140$ (7×3 is approximately 20 and $20 \times 7 = 140$). This approximation must

be performed mentally. With some practice it will be found that such approximations can be obtained very rapidly and with very little effort.

Slide rule, $7150 \times 3250 \times 6750 = 1570$. The answer is 157.

EXPLANATION. Place the right index over scale 7150 on scale D and move the indicator to slide 3250 on C. The location of the indicator on scale D is the product of the first two factors; it is not necessary to read this product. Leaving the indicator in this position, multiply the first product by the third factor, 6750, by moving the right slide index under the indicator and then moving the indicator to slide 6750 on C. The result, 1570, is seen on scale D under the indicator. The answer, allowing three places before the decimal point, is 157.

Example 2

$1.153 \times 3\frac{3}{4} \times 75\frac{1}{2} \times 36\frac{1}{4}$

SOLUTION. Approximate answer $= 1 \times 4 \times 75 \times 40 = 12{,}000$.

Slide rule, $1153 \times 3750 \times 7550 \times 3625 = 1183$.

Since five places are required before the decimal point, the answer is 11,830.

OPERATION. Left index over scale 1153, indicator to slide 3750; right index under indicator, indicator to slide 7550; right index under indicator, indicator to slide 3625; read answer on scale under indicator, 1183. Since the approximate answer shows five places before the decimal point, the answer is 11,830.

Example 3

$96\frac{1}{4} \times .0678 \times 2.314 \times 50 \times .1135$

SOLUTION. Approximate answer $= 100 \times .07 \times 2 \times 50 \times .1 = 70$; ($100 \times .07 = 7$; $2 \times 50 = 100$; $100 \times .1 = 10$; $10 \times 7 = 70$). Hence two places are required before the decimal point.

Slide rule, $9625 \times 6780 \times 2314 \times 5000 \times 1135 = 8570$.

The answer, allowing two places before the decimal point, is 85.7.

PROBLEMS

Perform the following multiplications on the slide rule.

1. $2 \times 16 \times 5$
2. $3.1 \times 4\frac{3}{4} \times 55$
3. $3.1416 \times 5\frac{1}{4} \times 6.83$
4. $3.1416 \times \frac{3}{4} \times \frac{3}{4}$
5. $.866 \times 2\frac{3}{4} \times 2\frac{3}{4}$
6. $.7854 \times 62\frac{1}{2} \times .433$
7. $.815 \times 7\frac{3}{4} \times 9.63$
8. $4\frac{1}{8} \times 3\frac{3}{4} \times 7\frac{1}{2} \times .31$
9. $.516 \times 8.143 \times \frac{7}{8} \times \frac{7}{8}$
10. $.866 \times 3.12 \times 3.12 \times \frac{7}{8}$
11. $.651 \times .39 \times 1\frac{3}{4} \times 3\frac{5}{8}$
12. $150 \times \frac{3}{4} \times 116 \times .92 \times 480 \times .26$
13. $9\frac{1}{4} \times 7\frac{3}{8} \times 5\frac{1}{2} \times 6\frac{3}{4}$
14. $108\frac{1}{2} \times 3.36 \times 7.1 \times 55$
15. $9\frac{7}{8} \times 3.47 \times 5\frac{1}{4} \times 674.3$
16. $\frac{1}{2} \times 27\frac{1}{4} \times 16\frac{3}{8} \times 490 \times .26$
17. $3.1416 \times 28\frac{1}{4} \times \frac{3}{4} \times 62\frac{1}{2}$
18. $3.1416 \times 6\frac{1}{4} \times 6\frac{1}{4} \times \frac{5}{8} \times 43\frac{1}{2}$
19. $\frac{1}{2} \times 2\frac{3}{4} \times 4\frac{1}{4} \times 17 \times .6$
20. $2146 \times 3.1416 \times 9\frac{7}{8} \times 3\frac{1}{4}$
21. $.93 \times 746 \times 8.6 \times 3\frac{3}{4} \times \frac{1}{8}$

115. Division

In division the operation of multiplication is reversed. We subtract lengths instead of dividing numbers.

Example 1

Divide 6 by 2.

SOLUTION. Place 2 of the slide (scale C) over 6 on scale D and read the answer 3 on scale D under the left index. We have subtracted the length representing 2 from the length representing 6, and the remaining length represents the quotient, 3.

Example 2

Divide 1650 by 286.

SOLUTION. Approximation, $\frac{1800}{300} = 6$ (one figure before the decimal point).

Slide rule, $\frac{1650}{2860} = 5770$. The answer is 5.77.

Example 3

$$\frac{9\frac{1}{4}}{36.8}$$

SOLUTION. Approximate answer $= \frac{9}{36} = \frac{1}{4} = .25$ (the first significant figure is in the first place after the decimal point).

Slide rule, $\frac{9250}{3680} = 2510$. The answer is .251.

EXPLANATION. After getting the approximate answer, the decimal point may be disregarded until the result is read.

PROBLEMS

Perform the following divisions on the slide rule.

1. $8 \div 4$	**2.** $6 \div 15$	**3.** $65 \div 13$
4. $27 \div 4\frac{1}{2}$	**5.** $160 \div 12$	**6.** $216 \div 72$
7. $2960 \div 18$	**8.** $1.4 \div 3.5$	**9.** $81 \div 173$
10. $.217 \div .82$	**11.** $9.21 \div 8.16$	**12.** $.86 \div 9.1$
13. $1614.5 \div 8.45$	**14.** $27.03 \div .083$	**15.** $.7 \div .016$
16. $.0371 \div .36$	**17.** $.621 \div 82.6$	**18.** $8\frac{1}{2} \div 3\frac{3}{4}$
19. $9\frac{1}{4} \div 62\frac{1}{2}$	**20.** $24\frac{3}{4} \div 68\frac{1}{2}$	**21.** $100\frac{1}{2} \div .365$
22. $3.1416 \div 6.4$	**23.** $4\frac{3}{4} \div .602$	**24.** $.92 \div 11\frac{1}{4}$

116. Combined Multiplication and Division

Example 1

Find the value of $\dfrac{18 \times 56}{73}$.

SOLUTION. Approximate answer = 14 (18 goes into 73 approximately 4 times; 4 into 56 = 14).

Slide rule, $\dfrac{18 \times 56}{73} = 13.81$

OPERATION. Slide 73 over scale 18, indicator to slide 56; result on the scale under the indicator, 1381. Since 14 is the approximate answer, the slide rule answer is 13.81.

EXPLANATION. Place slide 73 over scale 18 to divide 18 by 73; the result of this operation is under the right index and is multiplied by 56 by moving the indicator to slide 56.

Example 2

$$\frac{2\frac{3}{4} \times 71.6}{.866}$$

SOLUTION. Approximate answer $= \dfrac{3 \times 70}{1} = 210.$

Slide rule, $\dfrac{2.75 \times 71.6}{.866} = 2273.$ The answer is 227.3.

OPERATION. Slide 8660 over scale 2750, indicator to slide 7160; answer on scale under indicator, 2273. Since the approximate answer is 210, the slide rule answer is 227.3.

Example 3

$$\frac{114.5 \times 3.36 \times 7\frac{1}{2}}{368 \times 2.31}$$

SOLUTION. Approximate answer = 3 (114.5 into 368, approximately 3; this 3 cancels the 3.36; 2.31 into $7\frac{1}{2}$ about 3).

$$\frac{\overset{3}{\cancel{114.5}} \times \cancel{3.36} \times \cancel{7.5}}{\cancel{368} \times \cancel{2.31}_{\,3}} = 3, \text{ approximately}$$

Slide rule, $\dfrac{114.5 \times 3.36 \times 7.5}{368 \times 2.31} = 3393$. The answer is 3.393.

OPERATION. Slide 368 over scale 114.5, indicator to slide 3.36; slide 231 under indicator, indicator to slide 7.5; result on the scale under the indicator, 3393. Since the approximate answer is 3, the slide rule answer is 3.393.

Example 4

$$\frac{3.1416 \times 7\frac{3}{8} \times 7\frac{3}{8} \times 2.615}{1728 \times .534}$$

SOLUTION. Approximate answer = .5 (.534 × 1728 = 900; 3.1416 into 900 = 300; $7\frac{3}{8} \times 7\frac{3}{8} = 50$; 50 into 300 = 6; 2.6 into 6 = 2; result $\frac{1}{2}$ or .5, that is, first place after decimal point).

Slide rule, $\dfrac{3.1416 \times 7.375 \times 7.375 \times 2.615}{1728 \times .534} = 4840.$

The answer is .484.

OPERATION. Slide 1728 over scale 3.1416, indicator to slide 2615; slide 5340 under indicator, indicator to slide 7375; right index under indicator, indicator to 7375; result on the scale under the indicator = 4840. Since the approximate answer is .5, the slide rule answer is .484.

Example 5

$$\frac{21.05 \times 27\frac{1}{2}}{75.3}$$

SOLUTION. Approximate answer $= \dfrac{\overset{7}{\cancel{21.05}} \times 27\frac{1}{2}}{\underset{4}{\cancel{75.3}}} = 7$

Slide rule, $\dfrac{2105 \times 2750}{7530} = 7.68$

OPERATION. Slide 75.3 over scale 21.05. The next operation is to move the indicator to 27.5, but this is not possible because the 27.5 is beyond the end of the scale. When this occurs, we shift the indexes, that is, change the right index for the left. The operation is: slide 75.3 over scale 21.05, indicator to right index, left index under indicator, indicator to 27.5; result on scale under indicator, 7680. Since the approximate answer is 7, the slide rule answer is 7.68.

PROBLEMS

Find the value of the following.

1. $\dfrac{8 \times 16}{17}$
2. $\dfrac{16 \times 345}{136}$
3. $\dfrac{2.94 \times 1.11}{8.603}$
4. $\dfrac{9.1 \times 8.65}{3\frac{1}{4}}$
5. $\dfrac{218 \times 7.35}{.618}$
6. $\dfrac{3\frac{3}{4} \times 7\frac{1}{2}}{9\frac{2}{3}}$
7. $\dfrac{11.5 \times 9\frac{7}{8}}{8\frac{1}{4}}$
8. $\frac{22}{7} \times 8.63$
9. $\dfrac{.866 \times 43.5}{7\frac{1}{2}}$
10. $\dfrac{106 \times 304}{834}$
11. $\dfrac{.21 \times 6.18}{619}$
12. $\frac{22}{7} \times 5\frac{5}{8}$
13. $\dfrac{3 \times 7 \times 8 \times 4}{5 \times 11 \times 17}$
14. $\dfrac{19 \times 1.4 \times 3}{1.76 \times 21}$
15. $\dfrac{21 \times 18 \times 19}{1728}$
16. $\dfrac{16 \times 18 \times 35 \times 1728}{231}$
17. $\frac{22}{7} \times 3\frac{1}{2} \times \frac{5}{8}$
18. $18\frac{3}{4} \times 5\frac{3}{8} \times \frac{1}{8}$
19. $\dfrac{211 \times 8.3}{231 \times .35}$
20. $\frac{22}{7} \times 3\frac{3}{8} \times 3\frac{3}{8} \times \frac{53}{1728}$

117. Squares and Square Roots

Note that scale A consists of two complete scales, 1 to 10, and 1 to 10. Since the entire scale is included in half the length of the lower scales, there is not room for as many divisions of the units. Accordingly, from 1 to 2, the smallest division is 20; from 2 to 5, 50; and from 5 to 10, 100. Care must be used to distinguish between numbers in the left and right scales.

Rule

Numbers whose first significant figure is an odd number of places before the decimal point or an even number of places after the decimal point are on the left scale.

Numbers whose first significant figure is an even number of places before the decimal point or an odd number of places after the decimal point are on the right scale.

Thus 9, 900, 90,000, and .09, .0009, .000009, etc., are read on the left scale; 90, 9000, 900,000, and .9, .009, .00009, etc., are read on the right scale; 236.5 is on the left scale; 2623.65 is on the right scale; .864 is on the right scale; .073 is on the left scale.

118. To Find the Square of a Number

Cover the number on scale D with the indicator and read the number covered by the indicator at the same time on scale A.

Example 1

Find the square of 3.

SOLUTION. Indicator over 3 on scale D, read answer, 9, under the indicator on scale A. $3^2 = 9$.

Example 2

Find the square of 3.73.

SOLUTION. Approximate answer, $4^2 = 16$ (two places before decimal point).

Slide rule: cover 3.73 on scale D with indicator, and read on scale A under the indicator, 1393. Since the approximate answer is 16, the slide rule answer is 13.93.

Example 3

Square 10.95.

SOLUTION. Approximate answer $= 11^2 = 121$.
Slide rule answer, $10.95^2 = 1200$; the answer is 120.

PROBLEMS

Square the following numbers.

1. 4	**2.** 16	**3.** 21	**4.** 7.5	**5.** 91.6
6. .316	**7.** $9\frac{3}{4}$	**8.** $17\frac{1}{4}$	**9.** 181	**10.** 218.7
11. 1.695	**12.** 62.3	**13.** .0195	**14.** .00393	**15.** .993
16. 101	**17.** 9.05	**18.** 2.008	**19.** 99	**20.** 1.85
21. 18.5	**22.** 185	**23.** 1850	**24.** 18500	**25.** 9.9

119. To Find the Square Root of a Number

This operation is the reverse of the operation of finding the square.

Example 1

Find the square root of 625.

SOLUTION. Since there are three figures before the decimal point, we use the left scale; placing the indicator over 625 on left scale A, we read the answer, 25, under the indicator on scale D.

Example 2

Find the square root of 2843.

SOLUTION. Since the first significant figure is an even number of places before the decimal point, we use the right scale; placing the indicator over 2843 on right scale A, we read the answer, 5330, under the indicator on scale D. The answer is 53.3 (approximation, $50^2 = 2500$).

Example 3

Find the square root of .0345.

SOLUTION. Since the first significant figure is in the second place after the decimal point, we use the left scale A. The answer under the indicator on scale D is 1857. The actual answer is .1857 (approximation, $.2^2 = .04$).

PROBLEMS

Find the square root of the following numbers.

1. 9	**2.** 49	**3.** 441	**4.** 1000	**5.** 2946
6. 8.16	**7.** 7.45	**8.** 30	**9.** 871	**10.** 7645
11. 21000	**12.** 431400	**13.** .95	**14.** .0365	**15.** .0061
16. 998001	**17.** 234.09	**18.** .000729	**19.** $52\frac{9}{16}$	**20.** $95\frac{1}{16}$
21. 17.3056	**22.** 234.9	**23.** $42\frac{1}{4}$	**24.** 2	**25.** 6.4

120. Cubes and Cube Roots

Some slide rules have an extra scale on the lower edge, enabling us to read cubes and cube roots directly. This scale, which consists of three complete scales from 1 to 10, is used in the same way as scale A is used for squares and square roots. It is possible, however, to obtain cubes and cube roots with the ordinary slide rule illustrated in Fig. 148, using scales A, B, C, and D.

121. Cubes

To find the cube of a number, set the C index to the given number on scale D and read the cube on scale A above the same number on scale B.

Example

Find the cube of 5.2.

SOLUTION. Set the right index of C on 52 on scale D. Cover 52 of scale B with the indicator. Under the indicator on scale A read the answer, 1405. By approximation the cube of $5.2 = 5 \times 5 \times 5 = 125$. Therefore the cube of 5.2 is 140.5.

PROBLEMS

Find the cube of the following numbers.

1. 6	**2.** 2.3	**3.** 16.4	**4.** 11.93	**5.** 8.35
6. .16	**7.** 3.18	**8.** $3\frac{3}{4}$	**9.** .216	**10.** 217
11. $19\frac{1}{2}$	**12.** 1.195	**13.** 3.89	**14.** $1\frac{1}{4}$	**15.** $5\frac{3}{4}$
16. $\frac{2}{3}$	**17.** $\frac{5}{7}$	**18.** $\frac{3}{5}$	**19.** .8	**20.** .008
21. .08	**22.** 1.05	**23.** 2.1	**24.** 1.5	**25.** 3.3

122. Cube Roots

To find the cube root of a number, we proceed as follows: Beginning at the decimal point, divide the number on either side into periods of three figures. In the cube root of a number there is one figure for each period contained in the number.

If the first period has *one significant figure*, set the indicator to the given number on left scale A.

If the first period has *more than one significant figure*, set the indicator to the given number on right scale A.

Then move the slide slowly until the number on D under the index of C is exactly the same as the number on B under the indicator. This number is the cube root of the given number.

Example 1

Find the cube root of 2650.

SOLUTION. Set the indicator to 2650 on left scale A; move slide until number on D under left C index is exactly the same as the number on B under the indicator. That number is 1388. The decimal point is placed after the second figure because there are two periods in the given number. Therefore the cube root of 2650 is 13.88.

Example 2

Find $\sqrt[3]{26500}$.

SOLUTION. Set the indicator to 26,500 on the right scale A. Move the slide until the number on D under the left C index is

the same as the number on B under the indicator. That number is 2980. Place the decimal point after the second figure because there are two periods in the given number. $\sqrt[3]{26500} = 29.8$.

Example 3

Find $\sqrt[3]{.0026}$.

Solution. Pointing off periods of figures, we get $\overline{.002}\,\overline{600}$. The first period contains one significant figure. Set the indicator to 26 on the left scale A. Move the slide until the number on D under the left C index is the same as the number on B under the indicator. That number is 1375. The decimal point is placed in front of the number because, since the first period in the given number is after the decimal point, the first digit in the cube root is after the decimal point. The answer is .1375.

PROBLEMS

Find the cube root of the following numbers.

1. 216	**2.** 2116	**3.** 14.71	**4.** 2000	**5.** 18.5
6. .86	**7.** 48.92	**8.** .072	**9.** .0064	**10.** 7.096
11. 36250	**12.** 242.62	**13.** .00052	**14.** .9	**15.** 3.18
16. .1	**17.** .01	**18.** .001	**19.** $\frac{125}{343}$	**20.** 84.604519
21. $\frac{8}{27}$	**22.** 6.1	**23.** 5.08	**24.** 50.8	**25.** 4.05

123. To Find the Circumference of a Circle

Circumference $= \pi D$. Hence, to find the circumference of a circle when the diameter is given, set the left B index under π in A, move slide to the diameter in B and read the answer on A.

Note: *For problems involving π we use scales A and B instead of C and D for multiplication and division.*

Example

Find the circumference of a circle whose diameter is 6.5″.

Solution. Set left B index under π on A. Move slide to 6.5 on B. Read the answer on A above the 6.5. The answer is 20.4″.

EXPLANATION. To find the circumference of a circle, we multiply the diameter by 3.1416 or π. Since π is marked on scale A, we use scales A and B to obtain the product of π and the diameter.

PROBLEMS

Find the circumference of circles with the following diameters.

1. $12''$	**2.** $4''$	**3.** $3\frac{1}{8}''$	**4.** $4\frac{1}{4}''$	**5.** $6.5''$
6. $18\frac{1}{2}''$	**7.** $16\frac{3}{4}''$	**8.** $2.65''$	**9.** $5.05''$	**10.** $7\frac{3}{8}''$
11. $2\frac{5}{8}''$	**12.** $6\frac{7}{8}''$	**13.** $4.85''$	**14.** $8.25''$	**15.** $2'\,4''$
16. $3'\,6''$	**17.** $2'\,8\frac{1}{2}''$	**18.** $4'\,6\frac{3}{4}''$	**19.** $5'\,1\frac{1}{2}''$	**20.** $3'\,5\frac{5}{8}''$
21. $16''$	**22.** $21''$	**23.** $17\frac{1}{4}''$	**24.** $20\frac{1}{2}''$	**25.** $30\frac{3}{4}''$

124. To Find the Diameter of a Circle Whose Circumference Is Given

To find the diameter, we reverse the process of finding the circumference. The formula for finding the diameter is: $D = \dfrac{\text{Circumference}}{3.1416}$, or $D = \dfrac{C}{\pi}$. That is, to find the diameter, we divide the circumference by π.

Example

Find the diameter of a circle whose circumference is $14''$.

SOLUTION. Set slide on 14 in scale A. Set π in B under 14 and read the answer on A above the B index. The answer is $4.46''$.

PROBLEMS

Find the diameters of circles with the following circumferences.

1. $11''$	**2.** $175'$	**3.** $4.5''$	**4.** $6.75''$	**5.** $10.25''$
6. $24\frac{1}{2}''$	**7.** $20.65''$	**8.** $8\frac{1}{4}''$	**9.** $9\frac{3}{4}''$	**10.** $6\frac{7}{8}''$
11. $3'\,6''$	**12.** $4'\,8\frac{1}{2}''$	**13.** $6'\,7\frac{3}{4}''$	**14.** $42.5'$	**15.** $65.4''$
16. $9'\,2''$	**17.** $4'\,2\frac{1}{2}''$	**18.** $16'\,1''$	**19.** $8'\,8''$	**20.** $7'\,6''$
21. $3'\,4\frac{3}{4}''$	**22.** $280'$	**23.** $120'$	**24.** $1.52'$	**25.** $160'$

125. To Find the Area of a Circle

This can be done with a single setting of the slide. Area $= \pi R^2$. To find the area of a circle whose diameter is known, three operations are required: (1) divide the diam-

eter by 2 in order to get the radius; (2) square the radius; (3) multiply by π. The following example will illustrate how this is done on the slide rule.

Example

Find the area of a circle whose diameter is 2.57″.

SOLUTION. Set 2 of slide C over 2.57 on scale D and read the answer on scale A over 3.1416 (marked π) on scale B. The answer is 5.19 sq. in.

EXPLANATION. Placing slide 2 over the diameter on scale D divides the diameter by 2, giving the radius; the square of the radius appears on scale A over the left index; moving the indicator to π multiplies by π, giving the area.

PROBLEMS

Find the area of circles with the following diameters.

1. 3″	**2.** $4\frac{1}{2}$″	**3.** $8\frac{1}{4}$″	**4.** $2\frac{7}{8}$″	**5.** 10.75″
6. $2\frac{1}{2}$″	**7.** 1.06″	**8.** 3.24″	**9.** $4\frac{3}{4}$″	**10.** 75″
11. 32″	**12.** .09″	**13.** $\frac{3}{4}$″	**14.** .109″	**15.** 2.73″
16. 5.4″	**17.** .54″	**18.** 6.9″	**19.** $1\frac{5}{8}$″	**20.** 2.05″
21. 20.5″	**22.** 205″	**23.** $7\frac{3}{8}$″	**24.** $1\frac{1}{8}$″	**25.** 10.5″

126. To Find the Diameter of a Circle Whose Area Is Known

To find the diameter, we reverse the process of finding the area. The formula for finding the diameter is: $D = 2\sqrt{\frac{\text{Area}}{\pi}}$. Three operations are required: (1) divide the area by π; (2) take the square root of the result; (3) multiply by 2. The following example will illustrate how this is done on the slide rule.

Example

Find the diameter of a circle whose area is .762 sq. in.

SOLUTION. Set π on scale B under 762 on right scale A, and move the indicator to 2 of scale C; read the answer on scale D under the indicator. The answer is .986″.

EXPLANATION. Placing π of scale B under 762 of right scale A divides the area by π, giving the square of the radius; the radius appears under the left index on scale D, and moving the indicator to 2 of scale C, multiplies the radius by 2, giving the diameter, .986″.

PROBLEMS

Find the diameter of circles with the following areas.

1. 12 sq. in.	**2.** 75 sq. in.	**3.** 3 sq. in.	**4.** .6 sq. in.
5. 175 sq. ft.	**6.** 9.3 sq. in.	**7.** .075 sq. in.	**8.** 8.11 sq. in.
9. 2000 sq. ft.	**10.** 90 sq. in.	**11.** 100 sq. in.	**12.** 1 sq. in.
13. 10 sq. in.	**14.** 1.15 sq. in.	**15.** .1 sq. in.	**16.** 35 sq. in.
17. 200 sq. in.	**18.** 84 sq. in.	**19.** 250 sq. ft.	**20.** 348.5 sq. ft.
21. .75 sq. in.	**22.** 1000 sq. ft.	**23.** 4.5 sq. in.	**24.** .45 sq. in.
25. .06 sq. in.			

MISCELLANEOUS PROBLEMS FOR THE SLIDE RULE

1. $1.414 \times 3\frac{7}{8}$

2. $.433 \times (3\frac{1}{4})^2$

3. $.866 \times (4\frac{1}{2})^2 \times 56$

4. $\dfrac{56 \times 32\frac{1}{2} \times 11\frac{1}{2}}{231}$

5. $4.828 \times (1\frac{1}{4})^2 - 3.14 \times (\frac{5}{8})^2$

6. $\frac{22}{7} \times 8\frac{9}{16} \times 8\frac{9}{16} \times \frac{1728}{231}$

7. $135 \times \frac{7}{8} \times \frac{7}{8} \times 61\frac{1}{2} \times \frac{480}{1728}$

8. $\frac{1}{2} \times 19\frac{1}{4} \times 17\frac{3}{4} \times \frac{490}{1728}$

9. $\frac{1}{3} \times \frac{22}{7} \times 9\frac{3}{4} \times 9\frac{3}{4} \times 7\frac{1}{8}$

10. $63\frac{1}{2} \times 47\frac{1}{4} \times 47\frac{1}{4} \times \dfrac{62\frac{1}{2}}{1728}$

11. $\sqrt{\dfrac{1865}{31.6} \times \dfrac{1}{4} \times \dfrac{7}{8}}$

12. $\sqrt{\dfrac{2000}{27\frac{1}{2}} \times \dfrac{1}{480}}$

13. $2\sqrt{\frac{21000}{490} \times \frac{1728}{116} \times \frac{7}{22}}$

14. $12 \times \pi \times 3\frac{3}{4} \times \frac{1}{4} \times 74 \times \frac{450}{1728}$

15. $\pi \times \frac{3}{8} \times \frac{3}{8} \times \pi \times 3\frac{1}{2}$

16. $2\sqrt{\dfrac{400 \times 231}{186} \times \dfrac{7}{22}}$

17. $\sqrt{\dfrac{10.8 \times 276}{.865}}$

18. $\dfrac{81.6 \times 249 \times 7.21 \times .06}{1728 \times 116 \times 9.27}$

19. $2\sqrt{520 \times \frac{7}{22}}$

20. $\frac{1}{3} \times 7.36(27.6 + \sqrt{27.6 \times 61.4} + 61.4)$

21. $\dfrac{25 \times 1.46}{14.3 + \dfrac{25 \times .135}{4}}$

22. $3.14\sqrt[3]{2[(1\frac{1}{2})^2 + (2\frac{1}{4})^2]}$

23. $\sqrt[3]{\dfrac{25}{450} \times \dfrac{1728}{.5236}}$

24. $\sqrt[3]{\dfrac{6 \times 12.5}{\pi}}$

25. $\sqrt{\dfrac{1728 \times 1000}{27\frac{1}{2} \times 450}}$

9 Miscellaneous Problems for Review

1. Find the weight of a cast iron plate $18'' \times 18'' \times 1\frac{3}{4}''$.

2. Find the weight of 150 steel bars $\frac{7}{8}''$ square and $18'\ 6''$ long.

3. Find the weight of 50 sheets of wrought iron $\frac{5}{8}''$ thick, $4'\ 2''$ wide, and $14'\ 10''$ long.

4. How many tons of steel are there in 150 sheets $\frac{3}{4}''$ thick, $6'\ 3''$ wide, and $12'\ 10''$ long?

5. Find the weight of 350 steel bars $\frac{3}{8}'' \times 1\frac{1}{2}'' \times 26'\ 6''$.

6. How many pounds of $\frac{1}{8}''$ thick sheet lead are required to line the bottom and sides of a tank $8'\ 6''$ square and $7'\ 4''$ deep?

7. What must be the height of a cast iron drop hammer $16''$ square to weigh 1 ton?

8. Find the weight of a steel plate $\frac{5}{8}''$ thick, with shape and size as shown in Fig. 152.

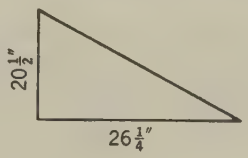


Fig. 152

9. Find the weight of a steel plate $\frac{1}{2}''$ thick with shape and size as shown in Fig. 153.

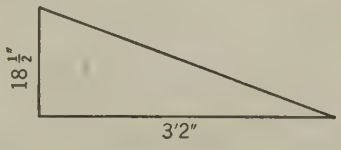


Fig. 153

10. Find the weight of the steel plates shown in Fig. 154(*a*), (*b*), (*c*), and (*d*). The plates are all $\frac{1}{2}''$ thick.

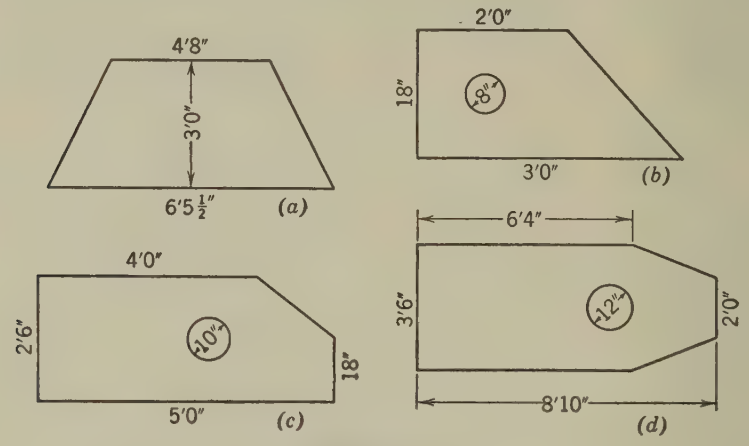


Fig. 154

11. Find the length of the belt around the two pulleys shown in Fig. 155.

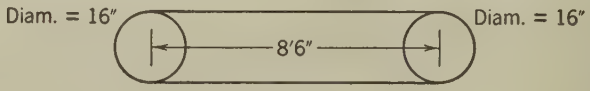


Fig. 155

12. Figure 156(*a*), (*b*), and (*c*) shows belts and pulleys. Assuming in each case that the line AB is a straight line, find the length of the belts.

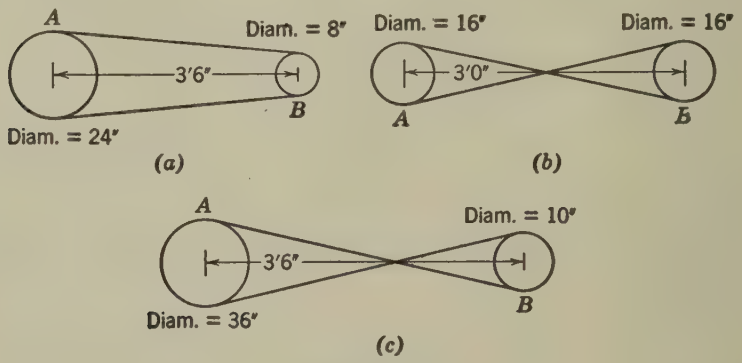


Fig. 156

13. Find the capacity in gallons of a cylindrical tank 1′ 7″ in diameter and 4′ 2″ high.

14. What must be the height of a cylindrical tank 2′ 4″ in diameter to have a capacity of 75 gallons?

15. A cylindrical tank 5′ 6″ high is required to have a capacity of 50 gallons. Find its diameter.

16. Water weighs 62.5 lb. per cubic foot. What is the weight of the water in a tank 7′ 8″ in diameter and 6′ 9″ deep, when full?

17. Figure 157 shows a cross-section of a Sharp V-thread of $\frac{3}{64}$″ pitch. Find the depth of thread. Give the answer in thousandths of an inch.

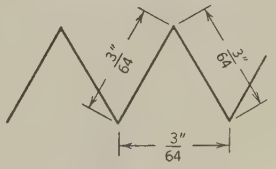


Fig. 157

18. What is the length of a cable which coils 25 times around a drum 4′ in diameter?

19. Find the weight of a load of 150 wrought iron bars $\frac{1}{2}$″ in diameter and 18′ 6″ long.

20. At 36 cents per square yard find the cost of painting the top and sides of a gas tank 72′ high and 50′ in diameter.

21. A copper clothes boiler with semicircular ends, Fig. 158, is 24″ long, 14″ wide, and 16″ deep. How many gallons can it contain?

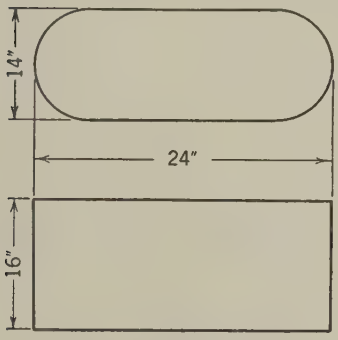


Fig. 158

22. What must be the depth of the boiler in Problem 21 in order to have a capacity of 35 gallons?

23. How many square feet of copper are required to make the boiler in Problem 21? Allow $1\frac{1}{2}$″ on side piece for bottom lock and top roll and $\frac{1}{2}$″ for side seam. On bottom piece allow $\frac{1}{2}$″ all around for lock.

24. A steel bar can safely bear a pull or tension of 16,000 lb. for each square inch of its cross-section. How many pounds can each of the following bars safely carry?

(*a*) A 1″ square steel bar.
(*b*) A $1\frac{1}{4}$″ square steel bar.
(*c*) A steel bar whose cross-section is a rectangle $5\frac{1}{2}$″ × $1\frac{1}{2}$″.
(*d*) A steel bar $1\frac{1}{4}$″ in diameter.
(*e*) A steel bar whose cross-section is a rectangle $3\frac{1}{2}$″ × $\frac{3}{4}$″.

25. Assuming 16,000 lb. per square inch as the maximum safe load, what must be the area of cross-section of a steel bar to carry a tension of:

(*a*) 8,000 lb. (*b*) 22,500 lb. (*c*) 175,000 lb.
(*d*) 20,000 lb. (*e*) 5,000 lb. (*f*) 120,000 lb.

26. Assuming 16,000 lb. per square inch as the maximum safe load, what must be the size of a square steel bar to carry safely a tension of:

(*a*) 22,500 lb. (*b*) 14,500 lb. (*c*) 100,000 lb.
(*d*) 50,000 lb. (*e*) 240,000 lb. (*f*) 150,000 lb.

27. If steel will carry safely a tension of 16,000 lb. per square inch, find the diameter of a round steel bar to carry a load of:

(*a*) 75,000 lb. (*b*) 8,200 lb. (*c*) 48,000 lb.
(*d*) 120,000 lb. (*e*) 200,000 lb. (*f*) 20,000 lb.

28. A steel plate can support safely a crushing or bearing load of 16,000 lb. for each square inch of its area; what must be the area of such a plate to sustain a load of:

(*a*) 250,000 lb. (*b*) 3,000,000 lb. (*c*) 1,000,000 lb.
(*d*) 100,000 lb. (*e*) 300,000 lb. (*f*) 80,000 lb.

29. A column carrying 2,500,000 lb. rests on a square steel plate. What must be the size of the plate, assuming 16,000 lb. per square inch as the safe load?

30. Find the weight of 2000 steel bolts of shape and size as shown in Fig. 159.

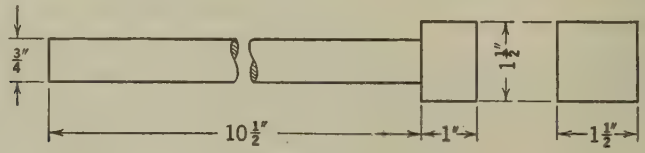


Fig. 159

31. Find the weight of 50 lengths of wrought iron pipe, 9′ 4″ long, the inside diameter being $6\frac{1}{2}$″ and the metal $\frac{3}{4}$″ thick.

32. Find the weight of the cast iron column shown in Fig. 160.

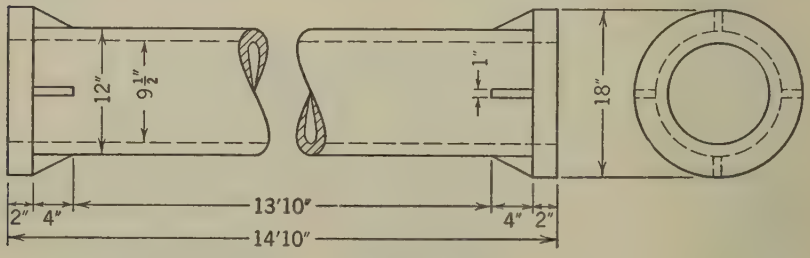


Fig. 160

33. Find the weight of 5000 hexagonal steel nuts $3\frac{3}{4}''$ across flats and $2\frac{1}{8}''$ thick. The hole is $1\frac{7}{8}''$ in diameter.

34. Find the weight of 10,000 steel bolts $1''$ in diameter and $8''$ long under the head. The head is hexagonal, $2''$ across flats, and $1\frac{1}{4}''$ thick.

35. Find the weight of the cast iron flywheel shown in Fig. 161. The hub is $10\frac{1}{2}''$ long. Assume that the spokes are cylindrical and add 1% for fillets.

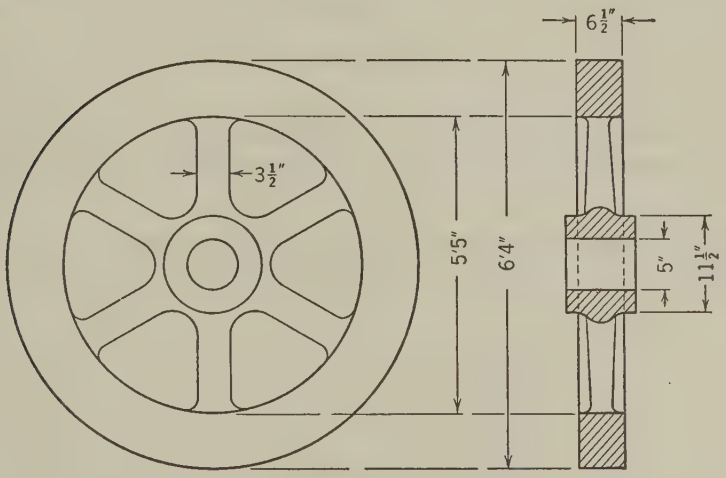


Fig. 161

36. How many pounds of brass are required to make 500 handwheels like those shown in Fig. 162?

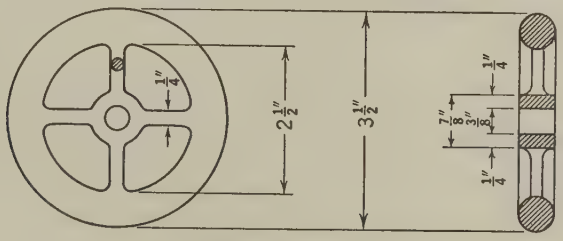


Fig. 162

37. What must be the size of a pipe to have the same carrying capacity as a $1\frac{1}{4}''$ pipe and a $1\frac{3}{4}''$ pipe together?

38. A ladle in the shape of a hemisphere is $14''$ in diameter. How many pounds of lead can it contain?

39. What must be the diameter of a hemispherical ladle to hold 100 lb. of lead?

40. A tubular boiler has a cylindrical dome with hemispherical ends. The cylindrical part is 7′ 1″ long and $16\frac{1}{2}$″ in diameter. Find the capacity of the dome in cubic feet. See Fig. 163.

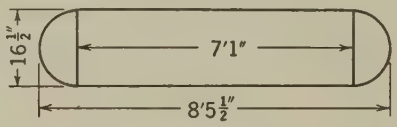


Fig. 163

41. A roof in the shape of a triangle with sides 186′, 110′, and 100′ long is to be covered with patent roofing. Find the cost at 45 cents per square foot.

42. A roof sign is supported by 35 angle iron trusses like that shown in Fig. 164. How many feet of angle iron are required?

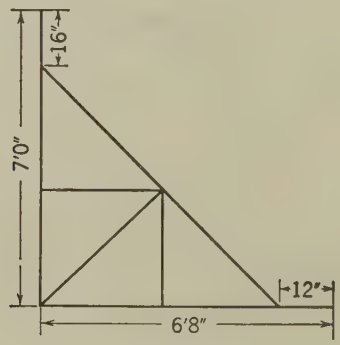


Fig. 164

43. Figure 165 shows a roof truss. Find the length of each of the members of this truss.

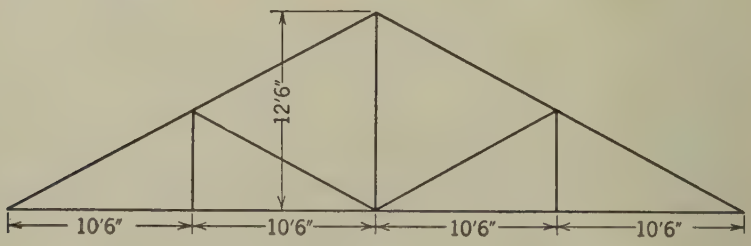


Fig. 165

44. On a construction job the scaffolding is supported by trusses like that shown in Fig. 166, made of 3″ × 6″ timbers. How many linear feet of timber are required for each truss?

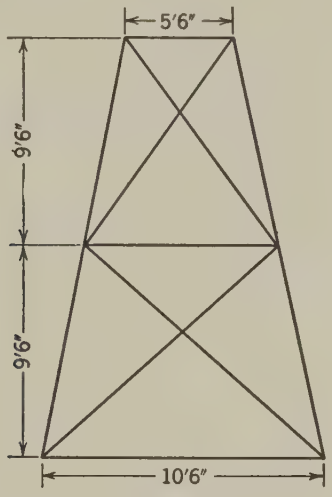


Fig. 166

45. A boiler has 126 tubes each 20′ long and $2\frac{3}{4}$″ in diameter. Find the total heating surface of the tubes in square feet.

46. A casting, when submerged in a cylindrical vessel of water 12″ in diameter, raised the level of the water $7\frac{1}{2}$″. Find the volume of the casting.

47. An elliptical gasoline tank on an automobile is 3′ 4″ long. It has a long diameter of 16″ and a short diameter of 11″. How many gallons can it contain?

48. An elliptical tank whose long diameter is 21″ and short diameter 14″ is required to contain 35 gallons. What must be its length?

49. An elliptical tank is required to have a capacity of 35 gallons. It is to have a length of 3′ 6″ and a long diameter of 22″. Find the short diameter.

50. How many square feet of sheet metal are required to construct an elliptical tank 4′ 2″ long, with a long diameter of 30″ and a short diameter of 21″? The ends are flat and countersunk 1″. See Fig. 167. Allow 1″ on side piece for joint.

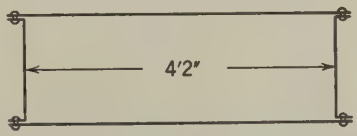


Fig. 167

51. A pail has a top diameter of 12″, a bottom diameter of 8″, and a depth of 11″. Find its capacity in gallons.

52. Find the number of square inches of sheet metal required to make the pail in Problem 51. Allow $1\frac{1}{2}''$ for top roll and bottom lock and $\frac{3}{4}''$ for side lock.

53. Figure 168 shows a fire pail with a hemispherical bottom. Find its capacity in gallons.

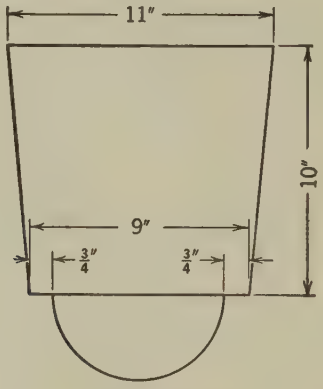


Fig. 168

54. A hopper, Fig. 169, 10′ 0″ square at the top, 4′ 3″ square at the bottom, and 6′ 4″ deep, is made of $\frac{1}{4}''$ sheet steel. Find:

(*a*) Its capacity in bushels (2150 cu. in. = 1 bu.).

(*b*) The weight of sheet steel required to construct it, allowing 5% for riveted joints.

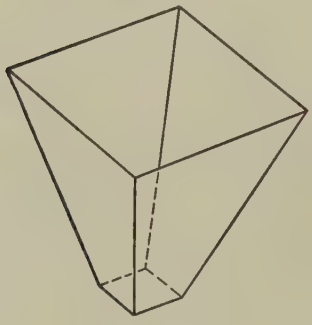


Fig. 169

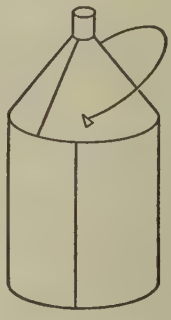


Fig. 170

55. Design an oil can similar to that shown in Fig. 170 to have a capacity of 5 gallons. Make the diameter at the neck $1\frac{1}{2}''$, the height of the cone 7″, and the diameter of the cylindrical part 10″. Compute the area of the metal required to construct it, allowing $\frac{1}{2}''$ around the bottom piece for lock, $1\frac{1}{4}''$ on side piece for top and bottom lock, and $\frac{1}{2}''$ for side lock.

Make the same allowances for the conical part. Disregard the neck and cap.

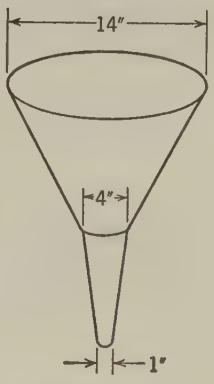


Fig. 171

56. (*a*) How many square inches of sheet metal are required to construct the funnel shown in Fig. 171? The upper part is 12″ high and the lower part 10″ high.

(*b*) What is its capacity?

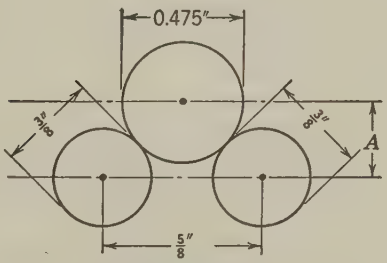


Fig. 172

57. A shell 0.475″ in diameter is to be tested by placing on rollers $\frac{3}{8}$″ in diameter. Find the distance *A* in Fig. 172.

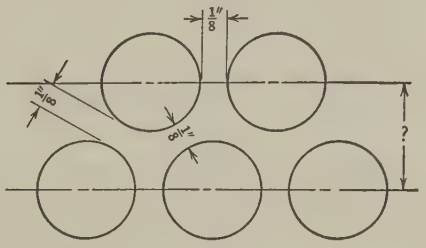


Fig. 173

58. Holes $1\frac{3}{8}$″ in diameter are to be punched in a sheet of brass. Allowing $\frac{1}{8}$″ all around each hole, what is the distance between the center lines of the rows of holes, Fig. 173?

59. A cast iron flywheel requires the removal of 0.17 lb. of metal on one side in order to balance it. This is to be done by boring a hole with a $\frac{3}{4}$″ end mill. Find the depth of hole required.

60. What size drill should be used to remove 0.13 lb. of material from a brass disk $\frac{1}{2}$″ thick if the hole is to go clear through the disk?

61. A concrete culvert, Fig. 174, has a cross-section area of 6.32 sq. ft. How many cubic yards of concrete are there in such a culvert 250 ft. long?

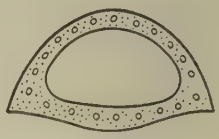


Fig. 174

62. How many cubic yards of concrete in the concrete sewer pipe in Fig. 175? The length is 775 ft.

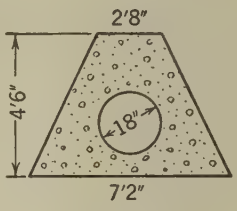


Fig. 175

63. The concrete base of a column has the form and dimensions shown in Fig. 176. How many cubic yards of concrete are there in 25 such pedestals?

64. A steel I beam, Fig. 177, has a cross-section area of 27.94 sq. in. Find the weight of such a beam 24′ 0″ long.

65. A steel angle iron, Fig. 178, has a cross-section area of 6.18 sq. in. Find the weight of 25 such angle irons each 12′ 8″ long.

66. A steel channel beam, Fig. 179, has a cross-section area of 10.29 sq. in. Find the weight of such a beam 12′ 0″ long.

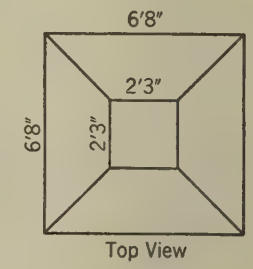


Fig. 176

Fig. 177

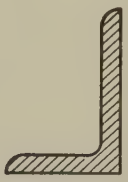


Fig. 178

Fig. 179

Fig. 180

67. A steel T-bar, Fig. 180, has a cross-section area of 3.09 sq. in. Find the weight of such a T-bar 28′ 6″ long.

68. A pattern made of white pine weighs $3\frac{1}{4}$ lb. Find the weight of a brass casting made from this pattern; a steel casting; a cast iron casting.

69. A concrete path around a fountain 28′ in diameter is 5′ 6″ wide and the concrete is 4″ deep. How many cubic yards of concrete were required to build this path?

70. A gas engine crank, Fig. 181, has a throw of $2\frac{1}{4}$″ and the connecting rod is $6\frac{3}{4}$″ long. Find the piston travel for each quarter turn of the crank, *AB*, *BC*, *CD*, *DA*.

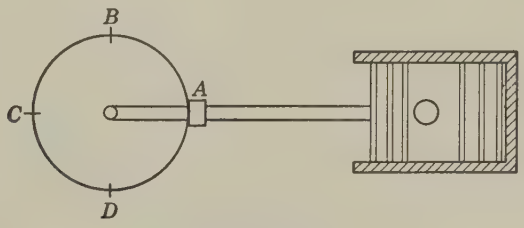


Fig. 181

71. Find the weight of the steel casting shown in Fig. 182. The bolt holes are $\frac{3}{4}$″ diameter.

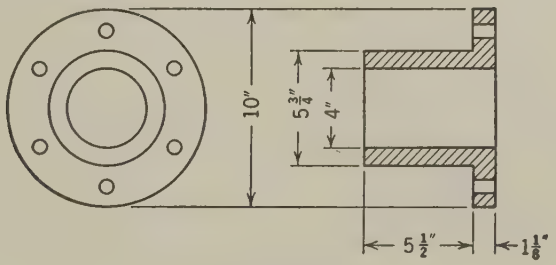


Fig. 182

72. Find the weight of the brass socket, Fig. 183.

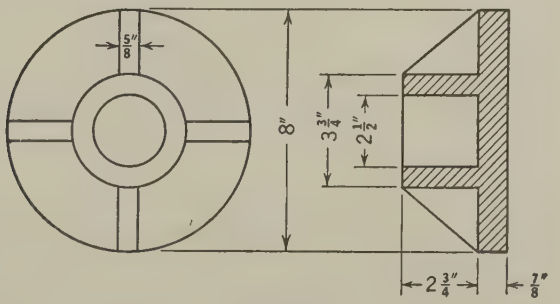


Fig. 183

73. Find the weight of the round steel shaft, Fig. 184.

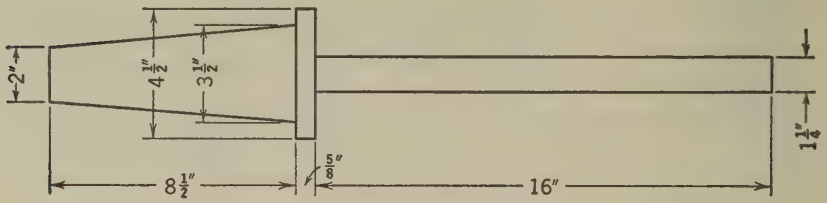


Fig. 184

74. Find the weight of the brass casting shown in Fig. 185.

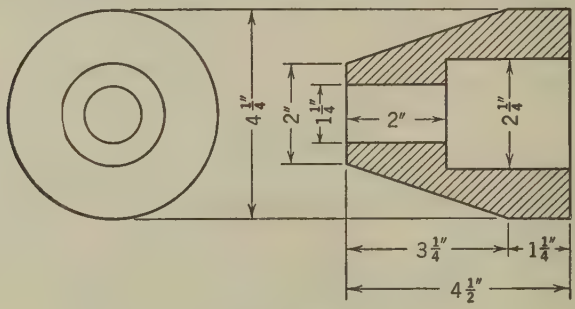


Fig. 185

75. Find the weight of the cast iron dovetail slide 4′ 8″ long shown in Fig. 186. The depth of slot is $1\frac{3}{8}$″.

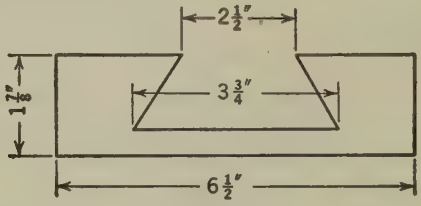


Fig. 186

10 Graphs

127. Definitions

When there are two sets of related facts, the relations between them can often be shown very clearly by means of diagrams, or ***graphs.*** Such graphs are frequently used in all kinds of business and technical work. Squared paper, sometimes called ***cross-section paper,*** is generally used for the purpose, and convenient values are assigned to the horizontal and vertical divisions. Two lines at right angles to each other are selected as ***axes*** or reference lines, the horizontal line being called the ***X-axis*** and the vertical line the ***Y-axis.*** The intersection of these two lines is called the ***origin.*** Points are plotted in relation to the two axes. These points are then connected by straight lines or curves, forming a broken line or a continuous curve, which shows graphically the relations that exist and the changes that take place.

128. Types of Graphs

There are many kinds of graphs, each adapted to a particular purpose. In general, however, they may be divided into two types, as follows:

1. ***Where no causal relation exists,*** that is, where the graph is merely a picture of a changing condition. Graphs showing changes of temperature or of population within a given period are examples of this type. The method of construction and the use of such graphs are illustrated by the following example.

Example

The population of a certain town, taken at intervals of ten years, varied as shown in the following table. Draw a graph showing the growth of this town.

Year	Population
1870	12,000
1880	16,000
1890	22,400
1900	26,000
1910	27,000
1920	27,200

Construction. Draw the horizontal and vertical reference lines as shown in Fig. 187. On the horizontal line, or X-axis,

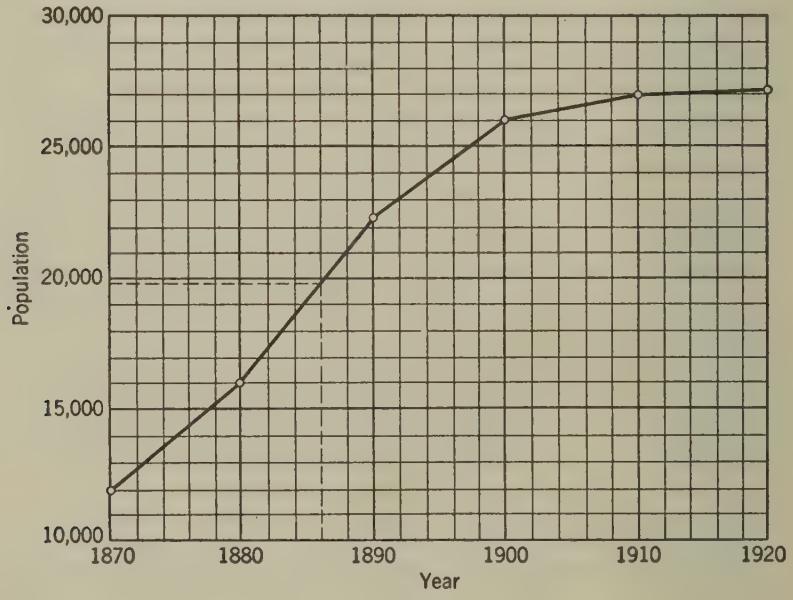


Fig. 187

mark off the years, 1870, 1880, etc.; on the vertical line, or Y-axis, mark off the populations by thousands. Distances on the vertical lines above the X-axis now represent populations, and distances on the horizontal lines to the right of the Y-axis represent years. At the intersection of the 1870 year line with the 12,000

population line, locate the first point. Locate other points at the intersection of the 1880 year line with the 16,000 population line, the 1890 line with the 22,400 line, etc. Connecting successive points with straight lines produces a broken line, which is the required graph.

EXPLANATION. In this case there is no necessary relation between the two sets of figures used in plotting the graph. The values do not depend on any law or rule, and the graph is merely a picture of the increase in the population. Instead of making a study of the tabulated figures, we can see at a glance how the population has varied from 1870 to 1920.

Use of the Graph

In connecting successive points by straight lines, we assume that the change from one point to the next has been uniform. To find the probable population for any year other than those given, we locate a point on the graph corresponding to that year. For example, to find the probable population in the year 1886, we note that the 1886 year line intersects the graph at a point whose height represents a population of approximately 19,700. We therefore say that the population of the town in the year 1886 was about 19,700.

Base Line

Note that the 10,000 line is used as the base line of this graph. This is done merely to save space as nothing is gained by starting at zero, which is the real base line.

2. ***Where causal relation does exist,*** that is, where the figures are related to each other by some definite rule or formula. Graphs showing the electrical resistance of a wire in relation to its diameter and graphs showing the variation in the surface speed of an emery wheel due to variation in the number of revolutions per minute are examples of this type. The following examples will illustrate the construction and use of such graphs.

Example 1

The surface speed in feet per minute of a 6″ emery wheel, at various speeds, is given in the accompanying table. Draw a graph

showing the relation between the surface speed and the revolutions per minute.

Revolutions per Minute	Surface Speed in Feet per Minute
0	0
500	785.4
1000	1570.8
1500	2356.2
2000	3141.6
2500	3927.0
3000	4712.4

Construction. Mark off the r.p.m. on the X-axis and the cutting speed in feet per minute on the Y-axis. The first point is at the origin, zero; the next point is at the intersection of the

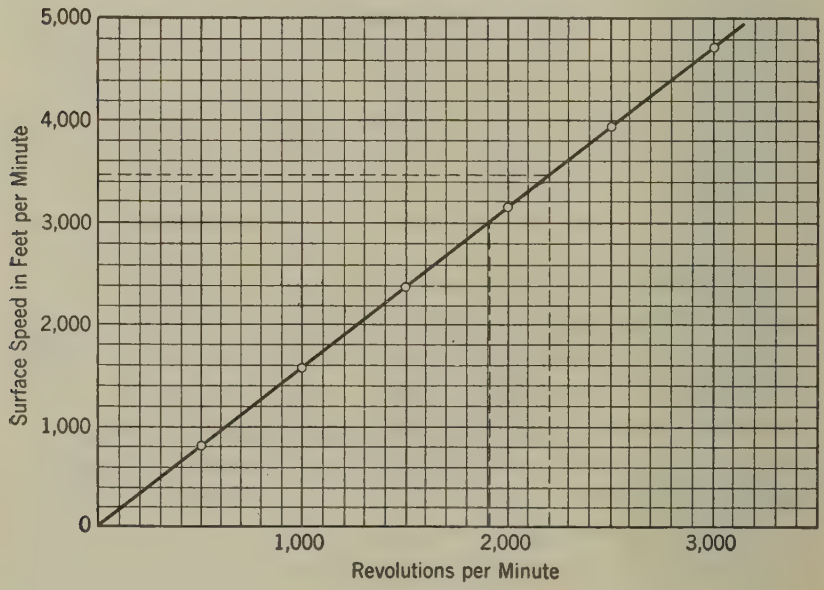


Fig. 188

500 r.p.m. line with the 785.4 ft. per minute line; the third point is at the intersection of the 1000 r.p.m. line with the 1570.8 ft. per minute line; etc. Drawing a line through these points gives the required graph. See Fig. 188.

Explanation. We see that this graph is a straight line beginning at zero and passing through all the plotted points. This is always the case when the two sets of figures used in plotting the graph are related to each other by a simple proportion. The two figures used in plotting any point form the same ratio as those used for any other point; thus 500:785.4::1000:1570.8::1500:2356.2, etc. When we know that such a proportion exists, we can plot the graph by locating only two points and drawing a straight line through them, continuing the line as far as required. All of the points will lie on this line. It is advisable, however, in cases of this sort, to select the two points as far apart as the limits of the graph will permit.

Use of the Graph

This graph may be used for finding the surface speed produced by a given number of revolutions per minute or for finding the revolutions per minute required to produce a given surface speed. To find the surface speed resulting from 2200 r.p.m., for example, we note the intersection of the 2200 r.p.m. line with the graph and read the corresponding value of the surface speed, 3440 ft. per minute, on the Y-axis. To find the number of revolutions per minute required to give a surface speed of 3000 ft. per minute, we note the intersection of the 3000 ft. per minute line with the graph and read the corresponding value of the revolutions per minute, 1910, on the X-axis.

Example 2

The following table shows the electrical resistance in ohms per thousand feet of copper wires of different diameters. Plot a graph showing how the resistance of the wire varies with the diameter.

Diameter of Wire in Mils	Resistance in Ohms per 1000 Ft.
36	8.04
51 "	4.01
72 "	2.00
102	1.00
144	.50
204	.25

Construction. Mark off the diameters in mils on the X-axis and the resistances in ohms per 1000 ft. on the Y-axis. The intersection of the 36-mil line with the 8.04-ohm line is the first point; the second point is the intersection of the 51-mil line with the 4.01-ohm line, etc. The graph is obtained by drawing a *smooth curve* through all the points. See Fig. 189.

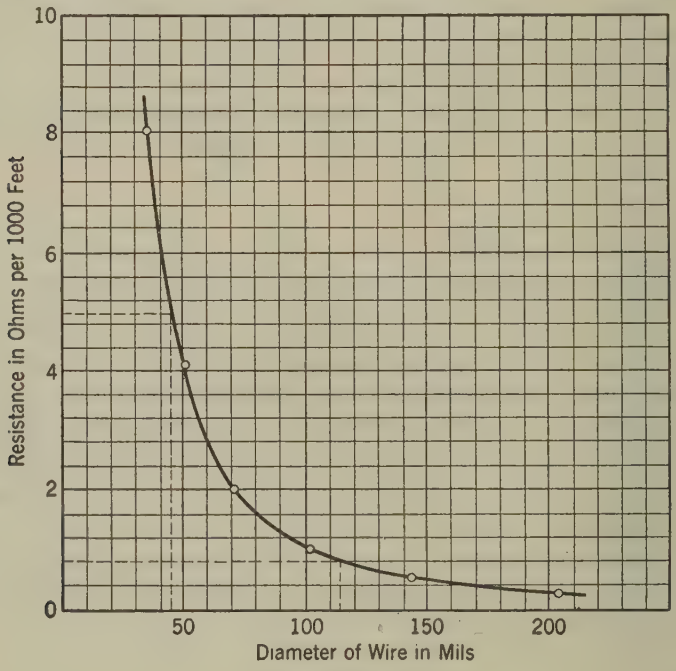


Fig. 189

Explanation. Since the several resistances are related to their respective diameters by a certain rule or formula (in this case the law of inverse squares), there will be no sudden breaks or changes in the direction of the graph. The change will be gradual and always in the direction of the next point, as we can see by plotting a number of intermediate points. It is, however, unnecessary to do this, as a smooth curve through the plotted points will also pass through all the intermediate points. The application of this rule enables us to reduce considerably the labor involved in plotting a graph representing a lengthy tabulation. For plotting we can select every third or fifth point, and by drawing a smooth curve through them obtain the required graph.

Use of the Graph

Values on this graph are found in the same way as on the graph of Example 1. To find the diameter of a wire whose resistance is 5 ohms per 1000 ft., we note the intersection of the 5-ohm line with the graph and read the corresponding value of the diameter, 46 mils, on the X-axis. To find the resistance per 1000 ft. of a wire whose diameter is 115 mils, we note the intersection of the 115-mil line with the graph and read the corresponding value of the resistance, .8 ohm, on the Y-axis.

129. Use of Graphs in Experimental Work

In making tests of materials and in experimental work of various kinds, we frequently find that we get points on the graph which do not seem to lie on the straight line or curve which we know to represent correctly the conditions involved. Such points are accidental, and may be due to inaccuracies in the measuring instruments or in the measurements, or to small imperfections in the material. When this happens, we disregard the "off" points and draw a smooth curve approximating the position of the majority of the points. Let us take, for example, a tension test of a steel wire where we know that, below the elastic limit, the load applied and the elongation are directly proportional to each other. This condition is represented by a straight line. We therefore draw a straight line approximating as nearly as possible the position of the majority of the points, disregarding the "off" points as being accidental and not accurately representing the actual conditions.

130. Two or More Graphs Combined

Such a combination is often of great value in showing the relations existing between two or more series of figures. A number of distinct sets of points may be plotted in reference to the same pair of axes, each set of points determining a separate graph.

Example

The average daily register and attendance of a certain school are given by the following table. Plot a graph showing these statistics.

Year	Register	Attendance
1900	711	683
1903	818	783
1906	901	827
1909	915	895
1912	961	933
1915	972	950
1918	1003	978
1921	1115	1052

Construction. Taking 600 pupils as a convenient base line, Fig. 190, we plot two points on each vertical year line for which

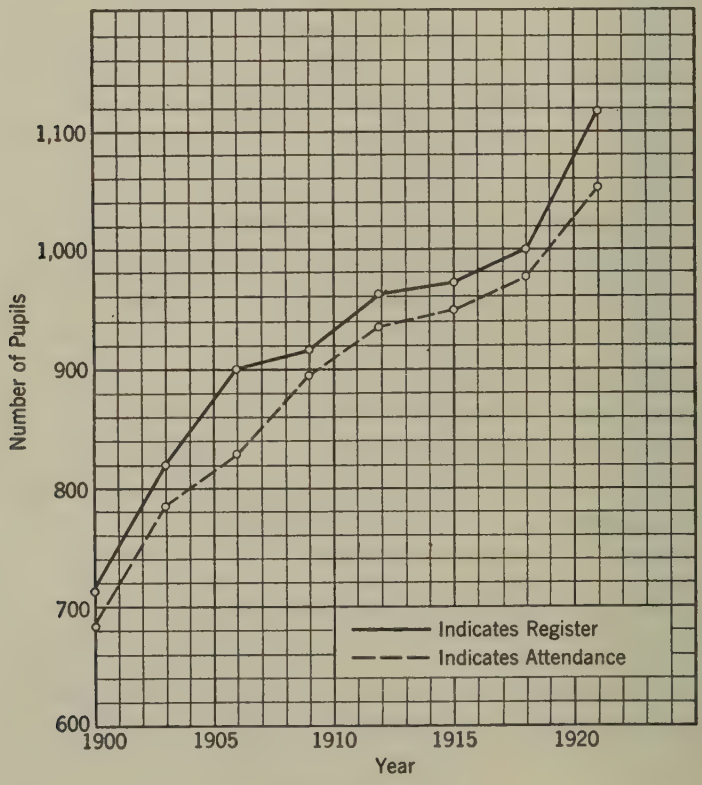


Fig. 190

we have data, one point for the register and one for the attendance. Two distinct sets of points are thus obtained and lines drawn through them. To distinguish the two lines, we use a full line for the register and a dotted line for the attendance with an explanatory note, as in Fig. 190. Sometimes different colors are used to distinguish one graph from another.

PROBLEMS

1. The average temperature in New York City at different times of the year is as follows. Plot a graph showing the variation of temperature during the year.

Month	Temperature, Degrees
January	30
February	31
March	38
April	48
May	59
June	68
July	74
August	72
September	66
October	56
November	44
December	34

2. Illustrate by means of a graph the immigration of aliens to the United States as given in the accompanying table.

Year	Number of Immigrants
1900	448,527
1902	648,743
1904	812,870
1906	1,100,735
1908	782,870
1910	1,041,570
1912	838,172
1914	1,218,480
1916	298,826
1918	110,618
1920	430,000

3. The weather report for a certain town shows the temperature, taken at 3-hour intervals, as follows. Draw a graph showing the variation during the 24-hour period. Find from the graph the probable temperature at 7 A.M., at 2 P.M., and at 8 P.M.

Time	Temperature, Degrees
12 P.M.	27
3 A.M.	24
6 A.M.	22
9 A.M.	25
12 M.	30
3 P.M.	38
6 P.M.	38
9 P.M.	37
12 P.M.	34

4. In a tension test of a bar of wrought iron, the elongation of the bar for applied loads was as follows. Plot a curve showing the relation between the load applied and the elongation of the bar.

Load, Pounds	Elongation, Inches
200	.012
500	.035
1000	.065
1500	.140
2000	.172
2500	.215
2600	.230
2700	.272
2800	.357
2900	.418
3000	.473

5. The horsepower of a 4-cylinder gas engine, as computed by a certain empirical formula, varies with the diameter of the cylinder as shown in the table. Plot a graph showing the relation between the cylinder diameter and the horsepower. From the graph find the cylinder diameter required for a 30-H.P. engine; also the horsepower of an engine whose cylinder diameter is $4\frac{1}{8}''$.

Diameter, Inches	Horsepower
3	14.4
$3\frac{1}{4}$	16.9
$3\frac{1}{2}$	19.6
$3\frac{3}{4}$	22.4
4	25.6
$4\frac{1}{4}$	28.8
$4\frac{1}{2}$	32.6
$4\frac{3}{4}$	36.1
5	40.0

6. Plot a curve showing the relation between angles and their natural sines, from 0 to 90°. Use every tenth degree for plotting. From the graph find the sines of the following angles: 18°, 35°, and 61°. Compare the values found from the graph with those given in the tables.

7. The accompanying table gives the brake horsepower for various speeds of a certain Pelton water wheel as determined by a series of tests. Plot a curve showing the relation between the brake horsepower and the number of revolutions per minute.

Brake Horsepower	Revolutions per Minute
0.625	1120
.668	1360
.673	1500
.658	1750
.640	1980
.590	2100
.530	2340
.475	2500
.390	2700

8. From the following data plot a graph showing the relation between the horsepower of certain engines and the cost of the fuel required to operate them for a week of 48 hours. From the graph find the probable cost of operating similar engines of 25 H.P. and 65 H.P. respectively.

Horsepower of Engine	Cost
20	$ 4.91
40	9.42
60	12.90
80	16.70
100	20.82

9. The following table shows the number of tons of freight carried in the United States during certain years. Draw a graph showing the growth of the freight traffic.

Year	Tons of Freight
1890	631,740,000
1895	686,615,000
1900	1,081,983,000
1905	1,427,732,000
1910	1,849,900,000
1915	1,802,018,000
1920	2,234,548,000

10. The following table gives the normal weight of children from birth until they are 16 years old. Using the same pair of axes, draw a graph showing the weight increase for boys and girls. From the graph find the normal weight of boys and of girls 7 years old; also $13\frac{1}{2}$ years old.

Age	Normal Weight, Pounds	
	Boys	Girls
Birth	7.55	7.16
6 months	16.0	15.5
1 year	20.5	19.8
2 years	26.5	25.5
3 "	31.2	30.0
4 "	35.0	34.0
5 "	41.2	39.8
6 "	45.1	43.8
8 "	54.5	52.9
10 "	66.6	64.1
12 "	79.8	81.4
14 "	99.3	100.3
16 "	123.7	113.0

11. The following results were obtained when ten samples of a certain cement were prepared and tested for tensile strength at different intervals. Draw a graph showing the relation between the age of the cement and its tensile strength.

Age	Tensile Strength, Pounds per Square Inch
1 week	120
2 weeks	150
3 "	175
4 "	195
5 "	210
6 "	223
7 "	233
8 "	242
9 "	250
10 "	255

12. A water wheel was tested for efficiency at different speeds and the following results were obtained:

At 50 r.p.m. the efficiency was 18%
At 100 r.p.m. the efficiency was 37%
At 150 r.p.m. the efficiency was 51%
At 200 r.p.m. the efficiency was 63%
At 250 r.p.m. the efficiency was 68%
At 300 r.p.m. the efficiency was $68\frac{1}{2}$%
At 350 r.p.m. the efficiency was 65%
At 400 r.p.m. the efficiency was 57%

Plot the efficiency curve and find the greatest efficiency and the speed at which it occurs.

131. Circle Graphs

An effective way of showing the percentage distribution of a given quantity is by means of a circle graph. The following example will illustrate the method.

Example

The cost of governing a certain city amounted to $24,652,000, apportioned as follows: education, $4,659,228; police and fire departments, $2,317,288; health, hospital, and sanitation, $2,859,632; pensions, $2,243,322; debt service, $4,511,316; others, $8,061,204. Plot a circle graph showing the percentage allotments.

SOLUTION.

$$\text{Education} = \frac{4{,}659{,}228}{24{,}652{,}000} = 18.9\% \text{ of total} = .189 \times 360° = 68.04°$$

$$\text{Police and fire} = \frac{2{,}317{,}288}{24{,}652{,}000} = 9.4\% \text{ of total} = .094 \times 360° = 33.84°$$

$$\text{Health, etc.} = \frac{2{,}859{,}362}{24{,}652{,}000} = 11.6\% \text{ of total} = .116 \times 360° = 41.76°$$

$$\text{Pensions} = \frac{2{,}243{,}322}{24\ 652{,}000} = 9.1\% \text{ of total} = .091 \times 360° = 32.76°$$

$$\text{Debt service} = \frac{4{,}511{,}316}{24{,}652{,}000} = 18.3\% \text{ of total} = .183 \times 360° = 65.88°$$

$$\text{Others} = \frac{8{,}061{,}204}{24{,}652{,}000} = 32.7\% \text{ of total} = .327 \times 360° = 117.72°$$

$$\text{Total} = 100\% \qquad 360°$$

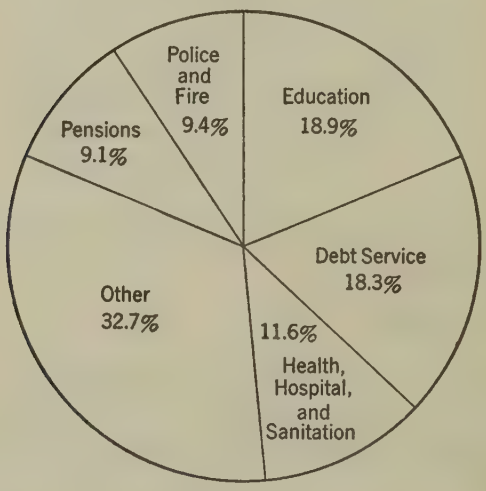


Fig. 191

CONSTRUCTION. Compute the percentage of the total that is spent for each item. Since there are 360° in the circumference of a circle, the amount spent on education, which is 18.9% of the total, is represented by a sector whose arc is 18.9% of 360° or 68.04°. Since 9.4% of the total is allotted to the police and fire departments, that item is represented by a sector whose arc is

9.4% of 360° or 33.84°; etc. With a protractor lay off the respective arcs on the circumference of a circle and draw the radii, forming the sectors as shown in Fig. 191.

PROBLEMS

1. Show by a circle graph the per cent of the total subscribed to the International Bank by the countries listed in the following table:

Country	Per Cent of Total
United States	41.4
United Kingdom	17.0
China	7.8
France	5.9
India	5.2
Canada	4.2
Netherlands	3.6
Other Countries	14.9
	Total = 100.0%

2. In 1951 there were produced 79,124 thousands of short tons of coke, 41,509 thousands of short tons of anthracite and 534,154 thousands of short tons of bituminous coal. Draw a circle graph showing the percentage of each of these items of the total produced.

3. In 1951 there were sold 3,433,000 electric clothes washers, 7,585,000 electric irons, 1,400,000 electric ranges, 2,700,000 electric vacuum cleaners, 4,075,000 electric refrigerators, 9,000,000 home radio sets and 5,337,000 passenger cars. Illustrate these data by means of a circle graph.

4. The following table gives the production of agricultural commodities in 1951 in thousands of bushels. Construct a circle graph showing what per cent of the total each item constituted.

Corn	2,941,423
Wheat	987,474
Rice	48,305
Oats	1,316,396

5. The outlay for education in a city in 1952 was 29.5% of the entire budget; for police and fire, 13.6%; health, hospital, and sanitation, 9.7%; pensions, 5.2%; debt service, 9%; others, 33%. Illustrate the given percentages by means of a circle graph.

132. Bar Graphs

A simple method of comparing quantities is by means of the bar graph.

Example

The number of television sets produced in the U. S. during the years 1947 to 1952, inclusive, was as follows:

Year	Millions of Sets
1947	.21
1948	1.05
1949	3.00
1950	6.50
1951	5.40
1952	6.00

Draw a bar graph showing the number of television sets produced in each of the six years given in the table.

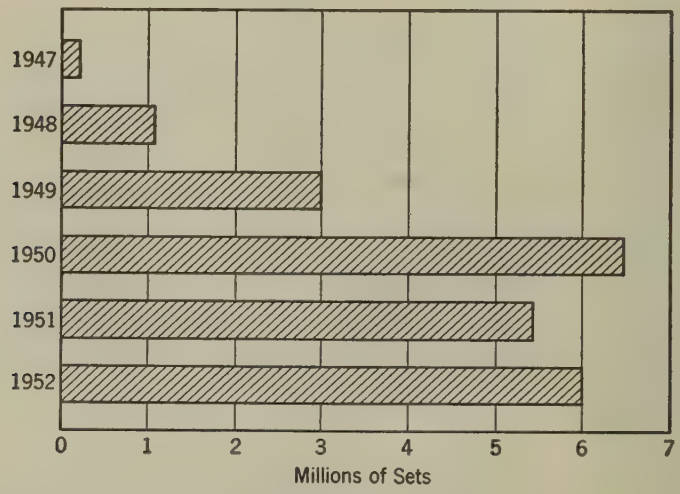


Fig. 192

Explanation. On the horizontal axis mark off the millions of sets, and on the vertical axis mark off the years as shown in the diagram, Fig. 192. The length of each bar represents the millions of sets produced in the year shown on the vertical axis. The

graph shows at a glance how the number of sets produced varied from year to year.

The graph can also be constructed with the bars drawn vertically as shown in Fig. 193. In this case, the years are marked off on the horizontal axis and the millions of sets on the vertical axis.

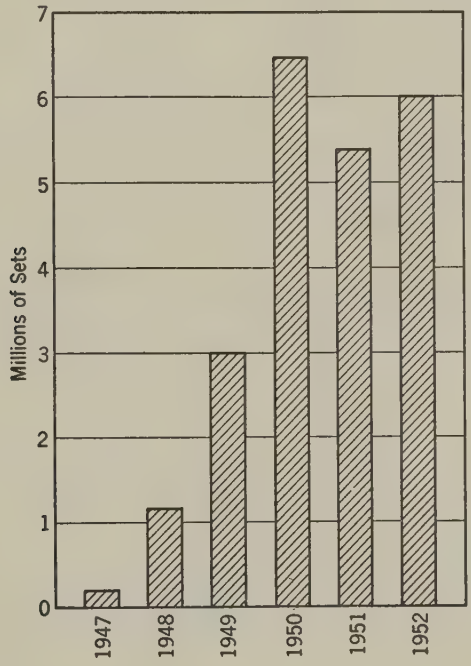


Fig. 193

PROBLEMS

1. Draw a bar graph showing the millions of radio sets produced quarterly as shown in the following table.

Year	Millions of Sets, Quarterly
1946	16.0
1947	20.0
1948	16.4
1949	7.5
1950	14.25
1951	12.5

2. Show by means of a bar graph the amount of anthracite coal produced (in thousands of tons) as given in the following table.

Year	Thousands of Tons
1947	57,190
1948	57,140
1949	42,702
1950	44,392
1951	41,509

3. The following table shows the wool consumption (in millions of pounds) for the years 1947 to 1951 inclusive. Draw a bar graph showing the quantities consumed. Draw the bars vertically.

Year	Wool Consumption, Millions of Pounds
1947	698
1948	693
1949	500
1950	625
1951	479

4. Illustrate the following by means of a bar graph.

Year	Steel Ingots and Castings Produced, Millions of Short Tons
1946	66.6
1947	84.9
1948	88.6
1949	78.0
1950	96.8
1951	105.2

11 Measuring Instruments

133. The Micrometer

For making fine measurements, an instrument called the micrometer is used. This consists essentially of a 40-pitch screw and a thimble whose circumference is divided into 25 equal parts. See Fig. 194.

The pitch of the screw being $\frac{1}{40}$ of an inch, each complete turn of the thimble advances the spindle $\frac{1}{40}''$ or .025″. The

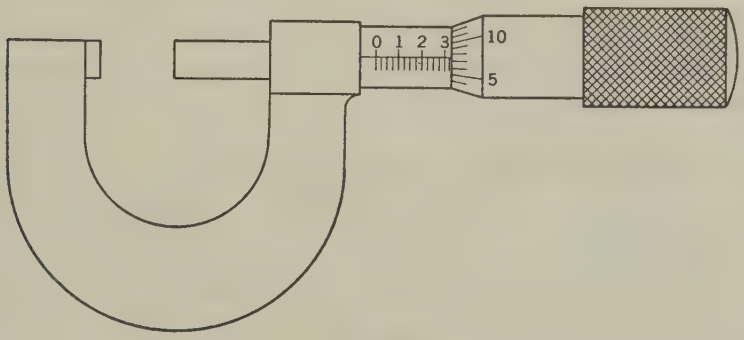


Fig. 194

barrel or body of the instrument is therefore calibrated by dividing the inch into 40 equal parts, each being .025″. For convenience in reading, every fourth division, representing .100″, is numbered; so that number 1 represents .100″, number 2, .200″, etc. The reading is taken at the edge of the thimble. The first mark after the zero represents .025″, the

second mark after the zero, .050″, the first mark after the 4, .425″, the third mark after the 8, .875″, etc. To enable us to read between these figures, the beveled edge is divided into 25 equal parts, each representing $\frac{1}{25}$ of a turn, or $\frac{1}{25}$ of .025″, which is .001″. When the zero on the thimble coincides with the gage line on the barrel, the reading is simply so many complete turns as shown by the barrel reading. If the zero on the thimble does not coincide with the gage line on the barrel, we must see which line does coincide and add so many thousandths to the barrel reading. For example, if the line marked 15 coincides with the gage line, we must add .015″.

Example 1

What is the reading when the edge of the thimble is between the .125 and the .150 lines, and the 18 line on the thimble is the coinciding line?

SOLUTION. Micrometer reading = Barrel + Thimble

$= .125'' + .018'' = .143''$

Example 2

Find the reading if the thimble is between .850 and .875 and the 3 line coincides.

SOLUTION. Micrometer reading = Barrel + Thimble

$= .850'' + .003'' = .853''$

Example 3

Find the reading if the thimble is between .050 and .075 and the gage line is between the 13 and 14 thimble lines, but nearer the 14.

SOLUTION. Micrometer reading = Barrel + Thimble

$= .050'' + .014'' = .064''$

EXPLANATION. When the gage line falls between two of the thimble lines without exactly coinciding with either of them, we select the nearest one.

PROBLEMS

Read the following micrometer settings.

	Thimble Is Between	Coinciding Line on Thimble
1.	0 and .025	8
2.	.125 and .150	18
3.	.275 and .300	17
4.	.850 and .875	16
5.	.125 and .150	5
6.	.975 and 1.000	1
7.	.025 and .050	22
8.	.375 and .400	4
9.	.150 and .175	2
10.	.650 and .675	16

Set the following micrometer readings.

1. .109″ 2. .073″ 3. .218″ 4. .813″ 5. .401″
6. $\frac{3}{8}$″ 7. $\frac{1}{4}$″ 8. $\frac{53}{64}$″ 9. $\frac{21}{32}$″ 10. $\frac{9}{16}$″

134. The Ten-Thousandths Micrometer

When very accurate measurements are required, we employ a micrometer which has an extra scale added to the barrel, enabling us to read to ten-thousandths of an inch.

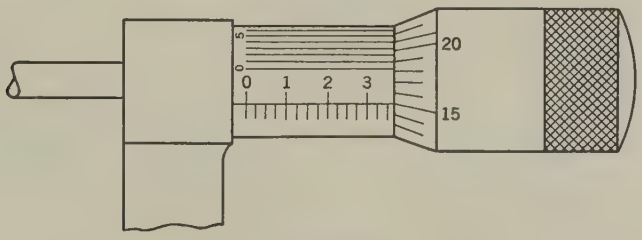


Fig. 195

This scale consists of a series of lines on the barrel, parallel to the axis of the barrel. See Fig. 195. A part of the circumference of the barrel equal to 9 of the thimble divisions is divided into 10 equal parts. Each part therefore represents $\frac{1}{10}$ of .009″ or .0009″. The difference between one of

these divisions and a thimble division is .001″ − .0009″ = .0001″. We find the number of ten-thousandths of an inch to add to the barrel and thimble readings by looking for the line of this auxiliary scale which exactly coincides with a thimble line. A scale of this sort which depends on the *difference between dimensions* for its principle is called a ***vernier scale.***

The reading on a ten-thousandths micrometer consists of three parts:

1. The barrel, on which each division represents .025″
2. The thimble, on which each division represents .001″
3. The vernier, on which each division represents .0001″

Example

The thimble is between .075 and .100, the gage line on the barrel is between 19 and 20 on the thimble, and line 8 on the vernier coincides with a line on the thimble. What is the reading?

Solution.

$$\text{Micrometer reading} = \text{Barrel} + \text{Thimble} + \text{Vernier}$$
$$= .075'' + .019'' + .0008'' = .0948''$$

PROBLEMS

Read the following micrometer settings.

	Thimble Between	Gage Line Between Thimble Lines	Coinciding Vernier Line
1.	.125 and .150	16 and 17	3
2.	.925 and .950	1 and 2	1
3.	0 and .025	9 and 10	2
4.	0 and .025	24 and 25	9
5.	.025 and .050	14 and 15	7
6.	.800 and .825	11 and 12	8
7.	.625 and .650	9 and 10	2
8.	.975 and 1.000	18 and 19	3
9.	.850 and .875	7 and 8	6
10.	.225 and .250	3 and 4	5

Set the following micrometer readings.

1. .9815″	2. .2713″	3. .1043″	4. .0008″	5. .2108″
6. .0315″	7. .0071″	8. .6835″	9. $\frac{3}{16}$″	10. $\frac{31}{64}$″

135. The Vernier Caliper

Another instrument which depends on the vernier principle is the vernier caliper (see Fig. 196). This consists essentially of a main scale with a fixed jaw and an auxiliary or vernier scale attached to the sliding jaw. On the main scale each inch is divided into 40 parts, so that each part is

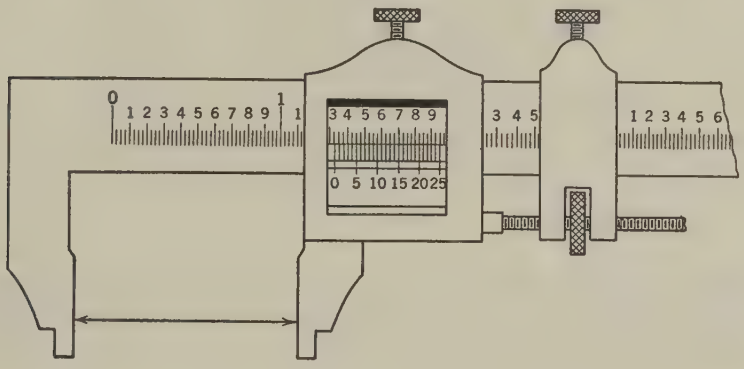


Fig. 196

$\frac{1}{40}$″ or .025″. For convenience in reading, every fourth division is numbered, as 1, 2, 3, etc., indicating .100″, .200″, .300″, etc.

The zero of the sliding scale is the index or gage line, and its location in reference to the main scale determines part of the reading. The remainder of the reading is determined by the vernier scale, which is constructed as follows: A length equal to 24 main scale divisions or .600″ is divided into 25 equal parts on the vernier scale, every fifth division being numbered to facilitate reading. Each space on the vernier is therefore equal to $\frac{1}{25}$ of .600″ or .024″. The difference between a scale division and a vernier division is .025″ − .024″ = .001″. Figure 197 shows an enlarged view of the scale and vernier showing a zero reading, the zero of the vernier

coinciding with the zero of the scale. If the sliding jaw is now moved until the first division of the vernier coincides with the first scale division, we have moved .001″, because that is the difference between a vernier division and a scale division. If we move the sliding jaw until the fifth line coincides, we have moved .005″ and set the caliper for .005″

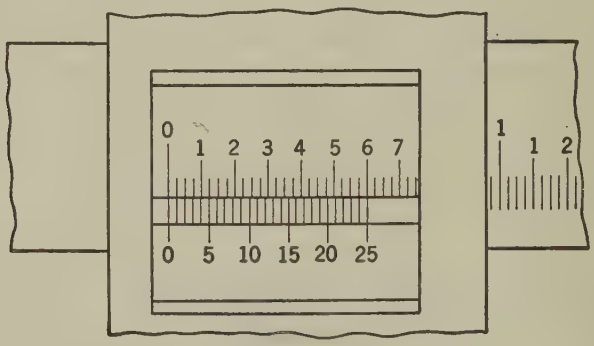


Fig. 197

reading. The vernier number which coincides with a scale division shows the number of thousandths of an inch to add to the scale reading.

Example 1

The index is between .275″ and .300″ on the scale, and line 23 on the vernier coincides with a scale line. What is the reading?

SOLUTION. Reading = Scale + Vernier

$$= .275'' + .023'' = .298''$$

Example 2

Set the vernier caliper for .131″.

SOLUTION. .131 = .125 + .006. Bring the index over .125″, then move the sliding jaw until line 6 on the vernier coincides with a scale line.

PROBLEMS

Read the following settings of the vernier caliper.

	Index Between Lines	Coinciding Vernier Line
1.	.125 and .150	16
2.	.175 and .200	9
3.	.250 and .275	18
4.	.900 and .925	12
5.	.975 and 1.000	6
6.	.350 and .375	1
7.	.100 and .125	13
8.	.000 and .025	5
9.	.075 and .100	24
10.	.050 and .075	11

Set the vernier caliper for the following measurements.

1. .020″ **2.** .218″ **3.** .173″ **4.** .101″ **5.** .908″
6. 1.003″ **7.** .125″ **8.** .099″ **9.** $\frac{3}{4}$″ **10.** $\frac{3}{16}$″

136. The Protractor

Figure 198 shows an ordinary protractor which is used for measuring and laying out angles. In use, the base line 0°–180° is laid on one of the lines with the center of the protrac-

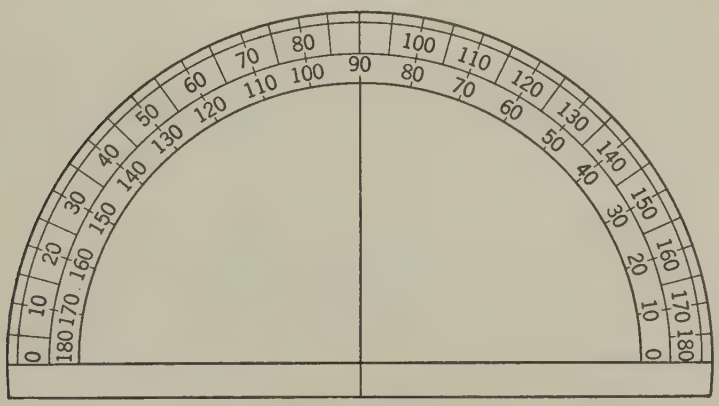


Fig. 198

tor at the point which is to be the vertex of the required angle. A mark is made at the point on the circumference

where the required number of degrees is indicated. A line is then drawn between these two points. The illustration shows a solid 6″ celluloid protractor.

137. The Vernier Protractor

For more accurate work, a steel protractor is used, carrying a vernier scale in addition to the regular scale. See Fig. 199. In this particular instrument the smallest division

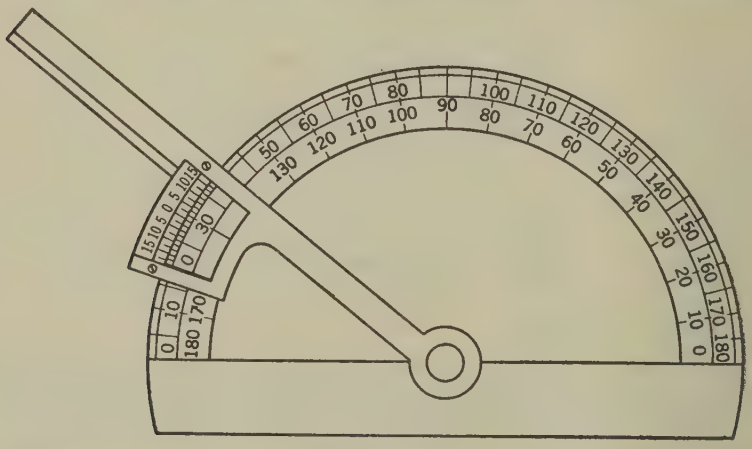


Fig. 199

on the main scale is $\frac{1}{2}°$, or 30′. By means of the vernier scale, we are enabled to divide one of these divisions into 10 parts, enabling us to read to 3′. Figure 200 shows an enlarged view of the vernier scale in relation to the main scale.

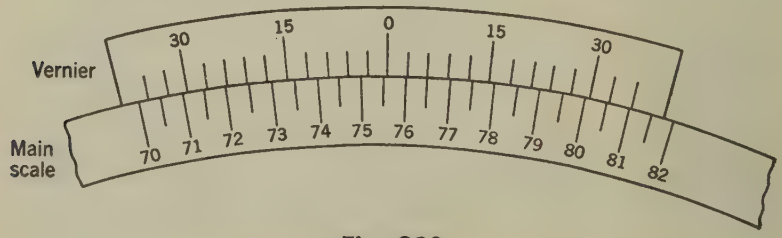


Fig. 200

A length of arc equal to 9 of the smallest scale divisions, or $9 \times 30' = 270'$, is divided into 10 equal parts on the vernier

scale, making each division equal to $\frac{1}{10}$ of 270, or 27′. The difference between a scale division and a vernier division is $30' - 27' = 3'$.

When the vernier zero, which is the index, exactly coincides with a scale division, the scale division shows the reading. But when the index does not exactly coincide with any scale division, we take the lower scale division and add the vernier reading as indicated by the coinciding vernier line.

Example 1

The index is between 27° 30′ and 28°, and the 21 on the vernier is the coinciding line. What is the reading?

SOLUTION. Angle = Scale + Vernier

$$= 27° 30' + 21' = 27° 51'$$

Example 2

Set the vernier protractor for 72° 42′.

SOLUTION. Set the index between 72° 30′ and 73° and move the vernier until the 12 line on the vernier coincides with a scale division.

PROBLEMS

Read the following vernier protractor settings.

	INDEX BETWEEN	COINCIDING VERNIER LINE
1.	12° 00′ and 12° 30′	15
2.	91° 30′ and 92° 00′	21
3.	11° 30′ and 12° 00′	27
4.	60° 00′ and 60° 30′	3
5.	27° 00′ and 27° 30′	0
6.	41° 00′ and 41° 30′	15
7.	125° 30′ and 126° 00′	27
8.	16° 30′ and 17° 00′	6
9.	3° 30′ and 4° 00′	21
10.	75° 00′ and 75° 30′	18

Set the vernier protractor for the following angles.

1. 27° 54′	2. 2° 15′	3. 45° 45′	4. 15° 15′	5. 52° 03′
6. 12° 12′	7. 9° 27′	8. 55° 57′	9. 21° 33′	10. 91° 12′

138. The Planimeter

The planimeter is an instrument used for measuring irregular areas such as indicator card diagrams. Figure 201 shows the Amsler Polar Planimeter, one of the simplest forms of this instrument.

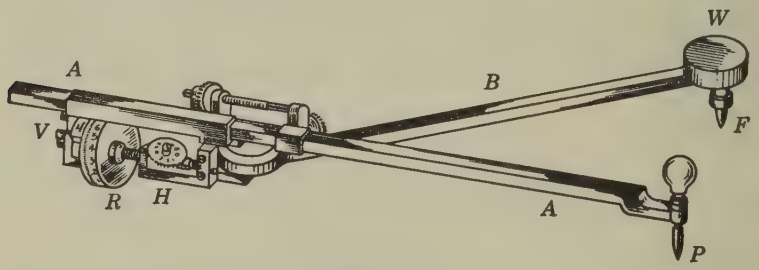


Fig. 201

The following are the essential parts and their functions:

The long beam AA carrying at one end the tracing point P.

The short beam B, pivoted at one end so as to allow free movement at the joint and carrying at the other end a needle point F, which acts as a fixed center around which the entire instrument turns.

A small weight W to hold this needle point in place.

The registering apparatus.

The rolling disk R of the registering apparatus rotates as the entire instrument is dragged around after the tracing point P, which is tracing the outline of the figure whose area is being measured. One complete turn of the rolling disk indicates an area of 10 sq. in. The circumference of the disk, therefore, is divided into 10 parts, each representing 1 sq. in. Each space is further divided into 10 parts, each division representing $\frac{1}{10}$ of a square inch. Hundredths of a square inch are read by means of the vernier V, which is placed next to the roller. The zero of this vernier is the index or gage line of the instrument.

Complete turns of the roller, each representing 10 sq. in., are counted by means of the horizontal disk H, which is connected to the roller by means of a worm and gear. Ten com-

plete turns of the roller are required to produce one complete turn of the counter. Its surface is therefore divided into 10 parts, each representing 10 sq. in.

139. Use of the Planimeter

The planimeter is used as follows: The diagram in Fig. 202 shows a view of the planimeter in reference to the figure whose area is being measured. The fixed point F is located conveniently near the figure. The tracing point P is placed

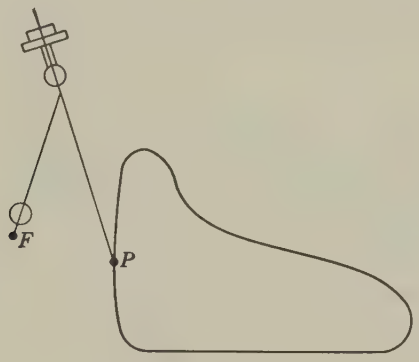


Fig. 202

at a marked point on the perimeter of the figure and carefully drawn over the entire perimeter until it returns to the starting point. This motion of the tracing point causes the roller to rotate and register the area. Readings are taken before and after the operation, and the difference between the readings is the area of the figure. To minimize errors it is customary to repeat the operation several times and take the average of the results.

The initial reading is taken with the tracer P at the starting point, and the final reading when the tracer has returned to the starting point.

The following example will illustrate the method of reading the planimeter. If the gage line of the counter is between 6 and 7, the reading is between 60 and 70 sq. in.; that is, it is 60 plus, each division on the counter representing 10 sq. in.

If the vernier index is between 71 and 72 on the roller, the roller reading is between 7.1 and 7.2 sq. in., that is, 7.1 plus. If line 6 on the vernier coincides with a roller line, the vernier reading is .06 sq. in.

Total reading of instrument

$$= \text{Counter} + \text{Roller} + \text{Vernier}$$

$$= 60 + 7.1 + .06 = 67.16 \text{ sq. in.}$$

If 67.16 sq. in. is the initial reading and 73.81 is the final reading, the area of the figure measured is $73.81 - 67.16 = 6.65$ sq. in.

PROBLEMS

Read the planimeter under the following conditions.

	Counter Between	Roller Between	Coinciding Vernier Line
1.	2 and 3	56 and 57	2
2.	9 and 10	31 and 32	3
3.	5 and 6	00 and 01	1
4.	6 and 7	66 and 67	5
5.	5 and 6	45 and 46	8
6.	0 and 1	98 and 99	9
7.	3 and 4	11 and 12	6
8.	2 and 3	03 and 04	1
9.	7 and 8	33 and 34	4
10.	0 and 1	15 and 16	5

12 Practical Algebra

140. Use of Letters

In algebra numbers are often represented by letters. This is of great advantage to the man in the shop because it enables him to express rules and formulas in concise form. For example, if we wish to express the rule that the area of a rectangle is equal to the length multiplied by the height, we can express it most concisely by the formula

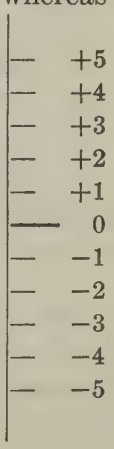
$$A = l \times h$$

where

A = area, l = length, and h = height.

141. Negative Numbers

Negative numbers are numbers less than zero, whereas positive numbers are numbers greater than zero. Negative numbers are distinguished from positive numbers *by the minus sign*. Thus -3 means 3 below zero, while $+3$ or 3 means 3 above zero. In the diagram positive numbers are represented by spaces *above* zero and negative numbers by spaces *below* zero. Hence, to *add* 3 to 1, start at 1, go *up* 3 spaces, and arrive at 4; that is, $1 + 3 = 4$. To add 5 to -2, start at -2, go up 5 spaces, and arrive at $+3$; that is, $-2 + 5 = +3$.



To subtract 3 from 5, start at 5, go down 3 spaces, and arrive at 2; that is, $5 - 3 = 2$. To

subtract 5 from 2, start at 2, go down 5 spaces, and arrive at -3; that is, $2 - 5 = -3$. To subtract 3 from -2, start at -2, go down 3 spaces, and arrive at -5; that is, $-2 - 3 = -5$.

142. Absolute Value

The *absolute value* of a number is its value without reference to its sign. Thus the absolute value of $+6$ is 6; the absolute value of -6 is 6.

143. Factors

The numbers multiplied together to produce a given number are the *factors* of the given number. Thus the factors of 6 are 2 and 3; the factors of 5 are 5 and 1; the factors of $10xy$ are 2, 5, x, and y. The multiplication sign ($\times$) is understood between letters and between a number and a letter; thus $10xy$ means 10 times x times y.

144. Power

A ***power*** is the product of two or more equal factors; thus aa is the second power of a and is written a^2; aaa is the third power of a and is written a^3; $aaaa$ is the fourth power of a and is written a^4.

145. Exponent

The ***exponent*** of a power is a number written above and to the right of a number or letter and shows how many times the number or letter is to be used as a factor. In the expression a^4, 4 is the exponent of a and shows that a is to be multiplied by itself four times; that is, $a^4 = aaaa$. When no exponent is written over a number or letter, the exponent 1 is understood; that is, $a^1 = a$; $2^1 = 2$, etc.

146. Coefficients

In a product any factor or the product of two or more factors is the ***coefficient*** of the remaining factor or of the product of the remaining factors. Thus in the expression

$3a$, 3 is the coefficient of a; in the expression $5abx$, 5 is the coefficient of abx; $5a$ is the coefficient of bx; $5ab$ is the coefficient of x. Where no coefficient is shown, the coefficient 1 is understood; thus a means $1a$; xy means $1xy$.

The student should learn to distinguish between the significance of exponents and that of coefficients. Thus $5^3 = 5 \times 5 \times 5 = 125$, but $3 \times 5 = 15$; likewise $a^3 = aaa$, but $3a = 3$ times a.

147. Roots

A root is one of the equal factors of a number. Since $5 \times 5 = 25$, we say that 5 is the square root of 25; since $2 \times 2 \times 2 = 8$, we say that 2 is the cube root of 8. The root of a number is indicated by the symbol $\sqrt{\ }$. Thus the square root of 25 is written $\sqrt{25}$; the cube root of 8 is written $\sqrt[3]{8}$.

The ***index*** of the root is the number which tells what root is to be taken. It is written to the left and above the root sign. When no index appears, the square root is understood. Thus $\sqrt{16}$ denotes the square root of 16, which is 4; but $\sqrt[4]{16}$ denotes the fourth root of 16, which is 2.

148. Substitution

Example 1

If $a = 3$ and $x = 5$, find the value of $a + x$.

SOLUTION. $a + x = 3 + 5 = 8$

Example 2

If $a = 3$ and $x = 5$, find the value of $4a + 7x$.

SOLUTION. $4a + 7x = 4 \times 3 + 7 \times 5$

$= 12 + 35 = 47$

Example 3

If $a = 3$ and $x = 5$, find the value of $6a^2 + 2x^3$.

SOLUTION. $6a^2 + 2x^3 = 6 \times 3^2 + 2 \times 5^3$

$= 6 \times 9 + 2 \times 125 = 54 + 250$

$= 304$

Example 4

If $a = 4$, $b = 6$, and $y = 9$, find the value of $5(3a + 2b^2)\sqrt{y}$.

SOLUTION. $5(3a + 2b^2)\sqrt{y} = 5(3 \times 4 + 2 \times 6^2)\sqrt{9}$

$= 5(12 + 72)3 = 5 \times 84 \times 3$

$= 1260$

PROBLEMS

If $a = 2$, $b = 5$, $c = 4$, $x = 3$, $y = \frac{1}{2}$, find the numerical value of the following expressions.

1. $5c$	**2.** $4x$	**3.** $2by$
4. $4a^2$	**5.** $5b^2$	**6.** $6abcx$
7. $3a^2b^2c^2$	**8.** $4(b + x^2)$	**9.** $2a(cy + bc)$
10. $4y + 3b$	**11.** $8ac + 5ax$	**12.** $2(a^2 + b^2 + x)$
13. $\sqrt{c}$	**14.** $\sqrt{x^2}$	**15.** $\sqrt[3]{2c}$
16. $\sqrt[3]{b^3}$	**17.** $a\sqrt{b^2}$	**18.** $(a + c)\sqrt{10ab}$
19. $c(b - a)$	**20.** $(c + x)^2$	**21.** $a^2 - y^2$
22. $\frac{c}{a} + \frac{b}{y}$	**23.** $\frac{1}{a} + \frac{1}{c}$	**24.** $\frac{1}{x} + \frac{1}{y}$
25. $4bc - 3ax$	**26.** $2c^2 - x^2$	**27.** $x^2 + y^2$
28. $(x + y)^2$	**29.** $(x + y)^3$	**30.** $(x - y)^2$

149. Addition. Terms

A term is an algebraic expression not separated by the sign $+$ *or* $-$.

Thus x, $5cb$, $3ab \times 5cd$, $\frac{2a}{3c}$ are algebraic expressions of one term each.

The expression $2a - 3bc + d$ consists of the three terms $2a$, $3bc$, and d.

Similar or Like Terms

Similar or like terms are terms having the same letters raised to the same powers; thus $3a^2bc$ and $-5a^2bc$ are like terms.

Dissimilar or Unlike Terms

Unlike terms are terms which are not similar; $3a^2bc$ and $4ab^2c^2$ are not like terms because the exponents of the letters of the two terms are not the same; $2ax$ and $3dy$ are dissimilar terms because they have different letters.

Rules for Addition

I. *To add similar terms having the same sign, add the absolute values of the numerical coefficients, annex the common letters, and prefix the common sign to the result.*

Example 1

Add $3a$, $5a$, and $4a$.

SOLUTION.

$$\begin{array}{r} 3a \\ 5a \\ 4a \\ \hline 12a \end{array}$$

EXPLANATION. Adding the numerical coefficients, we get $3 + 5 + 4 = 12$. We then annex to this sum the common letter a; and since all terms are plus, their sum is plus. Hence the answer is $12a$.

Example 2

Add $-6ab$, $-2ab$, $-9ab$.

SOLUTION.

$$\begin{array}{r} -6ab \\ -2ab \\ -9ab \\ \hline -17ab \end{array}$$

EXPLANATION. The sum of the absolute values of the numerical coefficients is $6 + 2 + 9 = 17$. Annexing the common letters ab gives $17ab$; and since the common sign is minus $(-)$, the sign of the sum is minus. Hence the answer is $-17ab$.

II. *If the terms to be added have unlike signs, proceed as follows:*

(*a*) *Add the plus coefficients.*

(*b*) *Add the minus coefficients.*

(*c*) *Subtract the smaller sum from the larger sum and in front of the result, place the sign of the larger number.*

(*d*) *Annex the common letters.*

Example 1

Add $4x$, $-7x$, $-3x$, $6x$, and $-5x$.

SOLUTION.

$$\begin{array}{r} 4x \\ -7x \\ -3x \\ 6x \\ -5x \\ \hline -5x \end{array}$$

EXPLANATION. The sum of the plus coefficients is $4 + 6 = 10$; the sum of the minus coefficients is $-7 - 3 - 5 = -15$. Subtracting the smaller number 10 from the larger number 15 gives 5. Since the larger number is minus, prefix the minus sign to the 5, giving -5. Finally, annex the common letter x, giving the answer $-5x$.

Example 2

Add $7a - 3b$, $4a + 6b$, $-5a + 4b$.

SOLUTION.

$$\begin{array}{r} 7a - 3b \\ 4a + 6b \\ -5a + 4b \\ \hline 6a + 7b \end{array}$$

EXPLANATION. Write like terms under one another, then add each column separately as in Example 1.

PROBLEMS

Perform the following additions:

1. $7a$, $5a$, $4a$, $3a$

2. $16bx$, $12bx$, $8bx$

3. $-5abc$, $-8abc$, $-10abc$

4. $-2a^2x$, $-9a^2x$, $-4a^2x$

5. $+25by$, $+18by$, $-12by$

6. $-16xy$, $-14xy$, $+15xy$

7. $3b$, $-5b$, $8b$

8. $-mn$, $5mn$, $-12mn$

9. $-15ax^2$, $-18ax^2$, $22ax^2$, $12ax^2$

10. $11b^2y$, $-6b^2y$, $-7b^2y$, $4b^2y$

11. $6ab$, $5ab$, $-8ab$, $-13ab$

12. $-14bc^2$, $6bc^2$, $-18bc^2$, $9bc^2$

13. $4a - 6c$, $-5a - 5c$, $-8a + 4c$

14. $-7x + 4y$, $-9x - 6y$, $5x + 7y$

15. $8b + 3d$, $-3b - 5d$, $-12b - 10d$

16. $-5m - 6n$, $-9m + 5n$, $m - 4n$

150. Subtraction. Rule for Subtraction

Change the sign of the number to be subtracted and then combine the two numbers as in addition.

Example 1

Subtract $5x$ from $12x$.

SOLUTION. $12x$
$5x$
$7x$

EXPLANATION. Changing the sign of $5x$ makes it $-5x$. Adding $12x$ and $-5x$ gives $7x$.

Example 2

Subtract $5x$ from $-12x$.

SOLUTION. $-12x$
$5x$
$-17x$

EXPLANATION. Changing the sign of $5x$ makes it $-5x$. Adding $-12x$ and $-5x$ gives $-17x$.

Example 3

Subtract $-5x$ from $12x$.

SOLUTION. $12x$
$-5x$
$17x$

EXPLANATION. Changing the sign of $-5x$ makes it $5x$. Adding $12x$ and $5x$ gives $17x$.

Example 4

Subtract $-5x$ from $-12x$.

SOLUTION. $-12x$
$-5x$
$-7x$

EXPLANATION. Changing the sign of $-5x$ makes it $5x$. Adding $-12x$ and $5x$ gives $-7x$.

Example 5

From $5x - 2y$ take $8x + 12y$.

SOLUTION. $5x - 2y$
$8x + 12y$
$-3x - 14y$

EXPLANATION. Write like terms under each other and proceed with each pair of like terms as explained in the preceding examples.

PROBLEMS

1. Subtract $12ax$ from $18ax$.
2. Subtract $14b^2y^2$ from $-5b^2y^2$.
3. Subtract $-16a^2b^2$ from $-22a^2b^2$.
4. Subtract $-10mn$ from $8mn$.
5. Subtract $-36x^2y^2$ from $-24x^2y^2$.
6. Subtract $21ac$ from $-9ac$.
7. Subtract $7ay$ from $3ay$.
8. Subtract $-15abc$ from $-18abc$.
9. Subtract $-24xy$ from $32xy$.
10. Subtract $9ac^2$ from $-6ac^2$.
11. Subtract $48z + 14y$ from $36z - 12y$.
12. Subtract $-30abc - 10bx$ from $20abc - 18bx$.
13. Subtract $8a + 9b$ from $6a + 4b$.
14. From $-21x - 25y$ take $33x + 28y$.
15. From $14x + 16y$ take $-7x - 8y$.

151. Signs of Aggregation

The signs of aggregation are the parentheses ( ), the bracket [ ], and the brace { }. The method of removing signs of aggregation and expressing the resulting quantity in its simplest form will be clear from a study of the examples which follow.

Example 1

$5 + (3 + 4)$

SOLUTION. $5 + (3 + 4) = 5 + 7 = 12$

The same result would be obtained if we first dropped the parentheses and then added the numbers; thus $5 + (3 + 4) = 5 + 3 + 4 = 12$.

Rule

Parentheses preceded by the sign $(+)$ *may be dropped without changing the sign of any term.*

Example 2

$15 - (6 + 2)$

SOLUTION. $15 - (6 + 2) = 15 - 8 = 7$

The minus sign before the parentheses means that the value of the numbers within the parentheses is to be subtracted from 15. The rule for subtraction is: change the sign of every term in the subtrahend and combine with the minuend as in addition. Hence we write the above expression thus: $15 - 6 - 2 = 7$.

Rule

Parentheses preceded by the sign $(-)$ *may be dropped if the sign of every term within the parentheses is changed.* When one pair of parentheses appears within another, *remove one pair at a time, beginning with the innermost and proceeding outward.*

Example 3

$6a - [7a - (3a + 2b)]$

SOLUTION.
$$6a - [7a - (3a + 2b)] \quad (1)$$
$$= 6a - [7a - 3a - 2b] \quad (2)$$
$$= 6a - 7a + 3a + 2b \quad (3)$$
$$= 2a + 2b \quad (4)$$

EXPLANATION. Removing the parentheses around $3a + 2b$ gives line (2); we next remove the brackets and get line (3). Combining like terms gives line (4), the answer.

PROBLEMS

Simplify the following expressions:

1. $12 + (5 + 2)$	**2.** $12 + (5 - 2)$	**3.** $12 - (5 + 2)$
4. $12 - (5 - 2)$	**5.** $a + (b + a)$	**6.** $a + (b - 2a)$
7. $a - (b - 2a)$	**8.** $4a + (b - a)$	**9.** $5a - (a - b)$
10. $9 - (2 - 4)$	**11.** $10 - (6 + 3)$	**12.** $(3a - 3b) - (b + c)$
13. $8ab - (2ab - 2a^2)$	**14.** $a - [-(-a)]$	**15.** $a + [b - (a - b)]$

152. Multiplication

In the multiplication of algebraic quantities, two important laws must be followed.

1. The Law of Signs

The product of two numbers having like signs is positive; the product of two numbers having unlike signs is negative. Thus

$$(+3) \times (+5) = +15$$
$$(-3) \times (-5) = +15$$
$$(+3) \times (-5) = -15$$

2. The Law of Exponents

The exponent of a letter in the product is found by adding the exponents of that letter in the factors. Thus

$$a^2 \times a^3 = a^{2+3} = a^5; \quad ab^2c \times a^2b^3c^2 = a^3b^5c^3$$

To multiply one term by another, find the product of their numerical coefficients, prefix the sign according to the rule given above, annex the letters that appear in the factors, and obtain the exponent for each letter in the product by adding the exponents of that letter in the factors.

Example 1

Multiply $6a^2x$ by $-4ax^2y$.

SOLUTION.
$$\begin{array}{r} 6a^2x \\ -4ax^2y \\ \hline -24a^3x^3y \end{array}$$

EXPLANATION. Multiplying 6 by 4 gives 24. Since the signs are unlike, we prefix the minus sign to the product. Annex all the letters that appear in both factors, writing each letter but once. Since the exponent of a in the multiplicand is 2 and in the multiplier 1, the exponent of a in the product is $2 + 1$, or 3. Similarly the exponent of x is $1 + 2$, or 3. The exponent of y is 1 because it appears in the multiplier to the first power and does not appear at all in the multiplicand.

Example 2

Simplify $-3a(2x - 7y)$.

SOLUTION. $-3a(2x - 7y) = -6ax + 21ay$

EXPLANATION. To simplify the given expression means to perform the indicated operations. Each term within the parentheses is multiplied by the multiplier $-3a$. To obtain the first term in the product we have $(-3a) \times (2x) = -6ax$. The second term is equal to $(-3a) \times (-7y) = 21ay$. The complete answer is $-6ax + 21ay$.

Example 3

Multiply $8a - 3b$ by $2a - 5b$.

SOLUTION.

$$\begin{array}{r} 8a - 3b \\ 2a - 5b \\ \hline 16a^2 - 6ab \\ \quad - 40ab + 15b^2 \\ \hline 16a^2 - 46ab + 15b^2 \end{array}$$

EXPLANATION. First multiply each term of the multiplicand by the first term of the multiplier ($2a$), giving the partial product $16a^2 - 6ab$. Next multiply each term of the multiplicand by the second term of the multiplier ($-5b$), giving the partial product, $-40ab + 15b^2$. The term $-40ab$ is written under the similar term $-6ab$ in the first partial product. Adding the two partial products gives the final result, $16a^2 - 46ab + 15b^2$.

PROBLEMS

Perform the following multiplications.

1. $a \times a$
2. $c^2 \times c$
3. $a^2 \times a^4$
4. $2a^2 \times 3a^3$
5. $(-5b) \times (-3b^2)$
6. $(-b^2) \times (-b^3)$
7. $4a \times (-3a^2)$
8. $(-3c) \times (8c^3)$
9. $2a \times 3a^2 \times 5a$
10. $(6a^2) \times (-4a) \times (3a)$
11. $(-x^2) \times (-3x) \times (-2x^3)$
12. $2ab(a + b + c)$
13. $3a^2c(2a - 3b - c)$
14. $-5a(2x^2 - 2y^2)$
15. $(3a - 4b)(5a - 6b)$
16. $(a + b)(a - b)$
17. $(2a + 3b)(4a + 5b)$
18. $(8a - 2b)(2b + 4a)$
19. $(2x + 3y)(2x - 3y)$
20. $(2x + 3y)(2x + 3y)$

153. Division

To perform a division, we reverse the process of multiplication.

Coefficient

Divide the numerical coefficient of the dividend by the numerical coefficient of the divisor; the result is the numerical coefficient of the quotient.

Laws of Signs

Like signs in division give plus, unlike signs give minus.

Law of Exponents

Subtract the exponent of a letter in the divisor from the exponent of the same letter in the dividend; the result is the exponent of the same letter in the quotient.

Example 1

Divide $40a^2x^4$ by $5ax^2$.

SOLUTION. $\dfrac{40a^2x^4}{5ax^2} = 8ax^2$

EXPLANATION. Dividing 40 by 5 gives 8. Since both signs are alike, the sign of the quotient is plus. Write after the numerical quotient 8 the letters that appear in both dividend and divisor, namely a and x. To obtain the exponent of a in the quotient, subtract the exponent of a in the divisor from the exponent of a in the dividend; thus $2 - 1 = 1$, which is not written after the a in the quotient. Similarly the exponent of x in the quotient is found by subtracting the exponent of x in the divisor from the exponent of x in the dividend; thus $4 - 2 = 2$.

If the same number or letter appears with the same exponent in both dividend and divisor, the number or letter in the divisor will cancel the corresponding number or letter in the dividend.

Example 2

Divide $5a^2x^4$ by $5a^2x$.

SOLUTION. $\dfrac{\cancel{5}\cancel{a^2}x^4}{\cancel{5}\cancel{a^2}x} = x^3$

EXPLANATION. The 5 in the dividend contains the 5 in the divisor once; the a^2 in the dividend contains the a^2 in the divisor once. Hence the given example is equivalent to $1 \times 1 \times \dfrac{x^4}{x} = 1 \times 1 \times x^3 = x^3$.

Example 3

Divide $-12a^3x^2y^5$ by $3ay^2$.

SOLUTION. $$\frac{-12a^3x^2y^5}{3ay^2} = -4a^2x^2y^3$$

EXPLANATION. $12 \div 3 = 4$; the signs being unlike, the sign of the quotient is minus. Write in the quotient the letters that appear in both dividend and divisor. The exponent of a is $3 - 1 = 2$; the exponent of x is $2 - 0 = 2$; the exponent of y is $5 - 2 = 3$. The answer is $-4a^2x^2y^3$.

Example 4

Divide $20a^2b^2 - 16a^3b$ by $4ab$.

SOLUTION. $$\frac{20a^2b^2 - 16a^3b}{4ab} = 5ab - 4a^2$$

EXPLANATION. Dividing $4ab$ into the first term of the dividend $20a^2b^2$ gives $5ab$; dividing $4ab$ into the second term of the dividend gives $-4a^2$. The quotient is $5ab - 4a^2$.

Example 5

Divide $a^2 - 8ab + 15b^2$ by $a - 3b$.

SOLUTION.

$$\begin{array}{r|l} a^2 - 8ab + 15b^2 & a - 3b \\ \cline{2-2} a^2 - 3ab & a - 5b \\ \cline{1-1} -5ab + 15b^2 & \\ -5ab + 15b^2 & \\ \cline{1-1} \end{array}$$

EXPLANATION. Divide the first term of the divisor into the first term of the dividend to obtain the first term of the quotient; thus $a^2 \div a = a$. Multiply each term of the divisor by the first term of the quotient; thus $a(a - 3b) = a^2 - 3ab$. Subtract this product from the dividend, leaving $-5ab + 15b^2$. Divide the first term of the divisor into the first term of the remainder to obtain the second term of the quotient; thus $-5ab \div a = -5b$. Multiply the divisor by the second term of the quotient; thus $-5b(a - 3b) = -5ab + 15b^2$. Subtracting this product from the remainder previously obtained leaves 0. The quotient therefore is $a - 5b$.

PROBLEMS

Perform the following divisions.

1. $18x^3 \div 6x$
2. $24x^4y^3 \div 3x^2y^2$
3. $-16x^3y^4 \div (-8xy)$
4. $-12abc \div (-2ab)$
5. $32a^2x^3 \div (-4a^2x)$
6. $27b^5c^4 \div (-9bc)$
7. $-36x^3y^3 \div 4xy^2$
8. $-40a^3d^3 \div 5a^2d$
9. $(24a^2b^2 + 18a^3b^3) \div 2ab$
10. $(16a^3x^2 - 12a^2x^3) \div 4ax$
11. $(21b^4y^5 + 18b^5y^6) \div (-3b^2y^2)$
12. $(14a^4y^4 - 7a^2y^2) \div (-7ay)$
13. $(x^2 + 5x + 6) \div (x + 2)$
14. $(x^2 - 8x + 15) \div (x - 3)$
15. $(x^2 - y^2) \div (x + y)$
16. $(a^2 + 2ab + b^2) \div (a + b)$
17. $(x^2 - 2xy + y^2) \div (x - y)$
18. $\dfrac{48m^7n^7p}{-6mp}$
19. $\dfrac{-14a^2b^3c^4}{-14a^2b^3c^4}$
20. $\dfrac{-12xy^3}{-12y^3}$

154. Simple Equations

An equation is a statement that two quantities are equal. For example, the equation $3x = 15$ states that $3x$ and 15 are equal to each other.

Members

That part of an equation which is to the left of the equality sign is called the *first member*, *left side*, or *left-hand member*. That part of an equation which is on the right of the equality sign is called the *second member*, *right side*, or *right-hand member*. In the equation $3x = 15$, $3x$ is the first or left-hand member, and 15 is the second or right-hand member.

An equation is employed to find the value of an unknown number from its relation to known numbers. The unknown number is usually represented by some letter, as x, y, or z. The first letters of the alphabet, as a, b, or c, usually represent known numbers.

155. Axioms

The solution of equations depends upon the following self-evident truths or axioms:

1. *Equal numbers added to equal numbers give equal sums.* If $x = y$, then $x + 2 = y + 2$.

2. *Equal numbers subtracted from equal numbers give equal remainders.* If $x = y$, then $x - 2 = y - 2$.

3. *Equal numbers multiplied by equal numbers give equal products.* If $x = y$, then $2x = 2y$.

4. *Equal numbers divided by equal numbers give equal quotients.* If $x = y$, then $\frac{x}{2} = \frac{y}{2}$.

156. Transposition of Terms

To *transpose* a term is to transfer it from one side of the equation to the other.

A term may be transposed from one member of an equation to the other provided we change the sign of the term.

Proof 1

Given the equation

$$x - a = b$$

Adding a to each member, we get

$$x - a + a = b + a$$

But $-a + a = 0$; therefore $x = b + a$.

The result is the same as if we had transposed $-a$ from the left side to the right and changed its sign.

Proof 2

Given the equation

$$x + a = b$$

Subtracting a from each member, we get

$$x + a - a = b - a$$

But $a - a = 0$; therefore $x = b - a$.

In this case we could obtain the same result by transposing a from the left side to the right side and changing the sign.

The sign of every term of an equation may be changed without affecting the equality.

Proof

If in the equation

$$a - x = b + c \tag{1}$$

we multiply each term by -1 (axiom 3), we get

$$-a + x = -b - c \tag{2}$$

Equation (2) is the same as equation (1) with the sign of each term changed.

157. Solution of Equations

Example 1

Solve the equation $6x - 4 = 4x + 6$.

Transposing $4x$ to the left side and -4 to the right side,

$$6x - 4x = 6 + 4$$

Combining like terms,

$$2x = 10$$

Dividing both members by the coefficient of x,

$$x = 5$$

CHECK. Substituting 5 for x in both members of the equation, we get

$$6 \times 5 - 4 = 4 \times 5 + 6$$

$$26 = 26$$

Therefore the answer, $x = 5$, is correct.

Example 2

Solve the equation

$$\tfrac{2}{3}(x + 3) = \tfrac{1}{2}(x + 8)$$

Simplifying,

$$\tfrac{2}{3}x + 2 = \tfrac{1}{2}x + 4$$

Transposing,

$$\tfrac{2}{3}x - \tfrac{1}{2}x = 4 - 2$$

Combining,

$$\tfrac{1}{6}x = 2$$

Multiplying both members by 6,

$$x = 12$$

CHECK. Substituting 12 for x in the given equation, we get

$$\tfrac{2}{3}(12 + 3) = \tfrac{1}{2}(12 + 8)$$

$$\tfrac{2}{3} \times 15 = \tfrac{1}{2} \times 20$$

$$10 = 10$$

Example 3

Solve the equation

$$\frac{2x}{a} = b + c$$

Multiplying both members by a,

$$2x = a(b + c)$$

Dividing by the coefficient of x,

$$x = \frac{a(b + c)}{2}$$

Example 4

Solve the equation

$$ax + 2b = bx + c$$

Transposing,

$$ax - bx = c - 2b$$

Combining like terms,

$$(a - b)x = c - 2b$$

Dividing by the coefficient of x,

$$x = \frac{c - 2b}{a - b}$$

From the preceding examples, we formulate the following rule for solving a simple equation:

Rule

Transpose all unknown terms to the left side and all known terms to the right side. Combine like terms and divide both members by the coefficient of the unknown term.

Example 5

$4x^2 - 6 = 2x^2 + 12$

Transposing,

$$4x^2 - 2x^2 = 12 + 6$$

Combining like terms,

$$2x^2 = 18$$

Dividing by the coefficient of x^2,

$$x^2 = 9$$

Taking the square root of both sides,

$$\sqrt{x^2} = \sqrt{9}$$

$$x = 3 \text{ or } -3$$

Since $(+3)^2 = 3 \times 3 = 9$, and $(-3)^2 = (-3) \times (-3) = 9$, the square root of 9 is either 3 or -3. From the conditions of the problem it is possible to tell which of the two values of x is the correct one.

Example 6

$D = \dfrac{N+2}{P}$. Solve for P.

Multiplying by P,

$$PD = N + 2$$

Dividing by D,

$$P = \frac{N+2}{D}$$

PROBLEMS

Solve the following equations.

1. $5x = 10 + 4x$
2. $12x - 4 = 7x + 6$
3. $3x + 3 = 18$
4. $4x - 5 = 3x + 2$
5. $ax = b$
6. $ax + b = c$
7. $ax + b = cx + d$
8. $6(x + 2) = 5(x + 4)$
9. $4(x - 2) = 2(x + 6)$
10. $\dfrac{3x}{b} = a - c$
11. $\dfrac{5x}{a} = b + c$
12. $3x^2 + 3 = 15$
13. $2x^2 - 12 = x^2 + 24$
14. If $C = 2\pi R$, solve for R.
15. $V = lwh$. Solve for h.
16. $N = PD$. Solve for D.
17. $E = IR$. Solve for I and R.
18. Horsepower $= \dfrac{PLAN}{33{,}000}$. Solve for P.
19. Solve formula in Problem 18 for L.
20. Solve formula in Problem 18 for A.
21. $R = \dfrac{10.4 \times L}{CM}$. Solve for L and for CM.
22. $D = \dfrac{N + 2}{P}$. Solve for N.
23. $A = \pi R^2$. Solve for R.
24. $D = M - 3W + 1.732P$. Solve for P.
25. $\dfrac{a}{b} = \dfrac{c}{d}$. Solve for b and c.
26. $V^2 = 2gh$. Solve for g and h.
27. $C = \frac{5}{9}(F - 32)$. Solve for F.
28. $a = \dfrac{V - v}{t}$. Solve for V and v and t.
29. $M = \frac{1}{6}bh^2$. Solve for b and h.
30. $A = \pi ab$. Solve for a and b.
31. $S = 4\pi R^2$. Solve for R.
32. Horsepower $= 0.4D^2N$. Solve for D.
33. $OD = \dfrac{(N + 2)P_c}{\pi}$. Solve for N.
34. $A = lw$. Solve for l and w.
35. $V = \frac{4}{3}\pi r^3$. Solve for r.
36. $P = \dfrac{\pi}{P_c}$. Solve for P_c.
37. $A = .7854D^2$. Solve for D.
38. $N = \dfrac{\pi D}{P_c}$. Solve for D and P_c.

39. Power factor $= \dfrac{\text{Effective power}}{\text{Apparent power}}$. Solve for effective power and for apparent power.

40. Efficiency $= \dfrac{\text{Output}}{\text{Input}}$. Solve for output and for input.

41. $BHP = \dfrac{2\pi LNW}{33{,}000}$. Solve for L and N and W.

42. Cutting speed = Circumference $\times$ r.p.m. Solve for circumference and for r.p.m.

43. Watts = Amperes $\times$ Volts. Solve for amperes and for volts.

44. Area $= \dfrac{ab}{2}$. Solve for a and for b.

45. $A = 0.866S$. Solve for S.

46. $S = 0.707D$. Solve for D.

47. $F = 1.732S$. Solve for S.

48. $C = 1.083F$. Solve for F.

49. $A = 0.433S^2$. Solve for S.

50. $A = 0.866F^2$. Solve for F.

13 Geometrical Constructions

158. Applications of Geometry

Many of the fundamental facts of geometry as related to practical work have already been studied in the chapter on mensuration, and the space available in this book will not permit a more complete discussion. There are, however, certain geometrical constructions which are very common and of great value in the solution of problems. These applications of geometry will be discussed in the following pages.

The work in this chapter requires the use of the following instruments: a compass, a pair of dividers, a rule or straightedge, and a scale.

159. Problem 1. To Bisect or Find the Middle Point of a Line

Example

Bisect the line AB in Fig. 203.

Solution. With a radius more than half of AB, using A as a center, draw the arc CF; with the same radius and B as center, draw the arc ED. Draw a line through the points where these two arcs intersect. This line, crossing AB at H, cuts AB into two equal parts, making H the middle point.

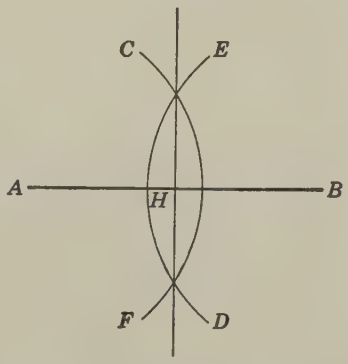


Fig. 203

160. Problem 2. To Bisect an Angle, That Is, To Divide an Angle into Two Equal Parts

Example

Bisect the angle ABC in Fig. 204.

SOLUTION. With any radius as BD and using B as a center, draw the arc DE; with the same radius and E as a center, draw the arc FG; with the same radius and D as a center, draw the arc HI. These arcs intersect at J. Draw a line through J and B, making the angle $ABJ =$ the angle CBJ and bisecting the angle ABC.

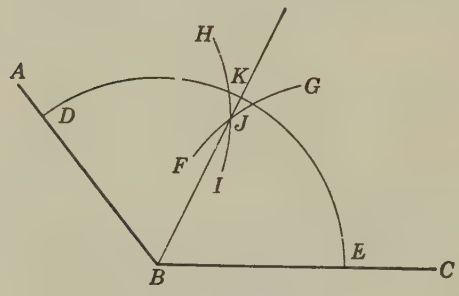


Fig. 204

161. Problem 3. To Bisect an Arc

The method used for bisecting an arc is exactly the same as that used in bisecting an angle. In Fig. 204, Problem 2, the line through J and B cuts the arc DE into two equal parts, DK and EK.

162. Problem 4. To Erect a Perpendicular to a Line at a Given Point on the Line

Example

At the point P draw a line at right angles to the line AB in Fig. 205.

SOLUTION. Using P as a center and any convenient radius, draw the semicircle CD. With a radius somewhat larger than PC and with C as a center, draw the arc EH; with the same radius and D as a center, draw the arc GF. These arcs intersect at K. The line through K and P is the required perpendicular.

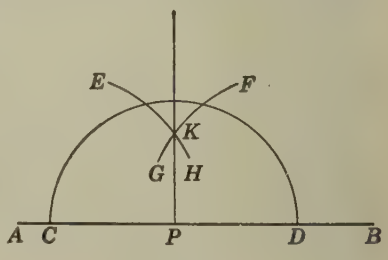


Fig. 205

163. Problem 5. To Erect a Perpendicular to a Line at the End of the Line

Example

Draw a line at right angles to the line AB at A in Fig. 206.

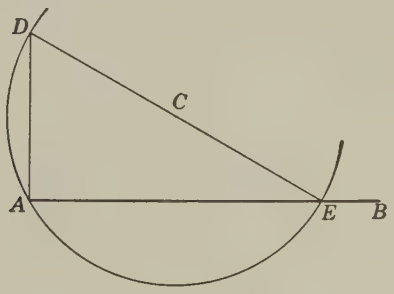


Fig. 206

Solution 1. Take any point as C, not on the line AB, as a center and CA as a radius, and draw the arc EAD. From E draw the line EC and prolong it until it cuts the arc at D. Draw a line through A and D. This is the required perpendicular.

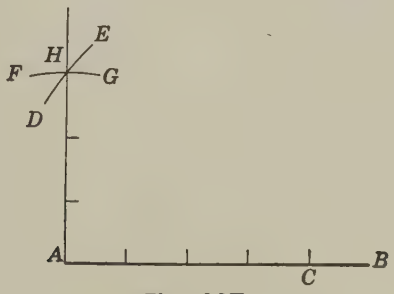


Fig. 207

Solution 2. Lay off on AB, Fig. 207, a distance $AC = \frac{4}{4}'' = 1''$. With a radius of $\frac{5}{4}'' = 1\frac{1}{4}''$ and using C as a center, draw the arc DE. With a radius of $\frac{3}{4}''$ and A as a center, draw the arc FG. These two arcs intersect at H. The line through H and A is the perpendicular required.

164. Problem 6. To Erect a Perpendicular to a Line through a Point Not on the Line

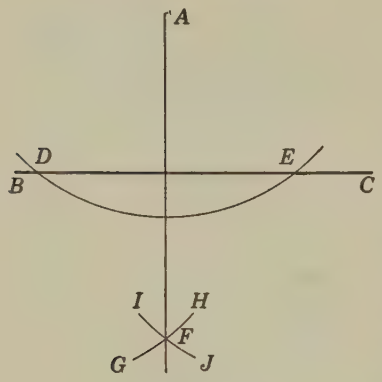


Fig. 208

Example

Draw a line through the point A and at right angles to the line BC, Fig. 208.

Solution. Using A as a center and any convenient radius, draw an arc cutting line BC at D and E. Using the same radius and D as a center, draw the arc GH; using E as a center and the same radius, draw the arc IJ. These arcs intersect at F. The line through A and F is the required perpendicular.

165. Problem 7. To Draw a Line through a Given Point Parallel to a Given Line

Example

Through the point A draw a line parallel to BC. See Fig. 209.

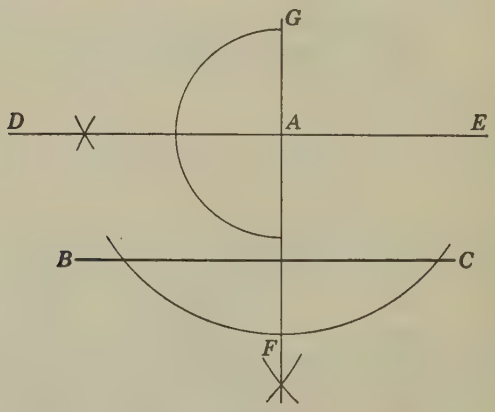


Fig. 209

Solution. By the method of Problem 6 draw the line FG perpendicular to BC. Then by the method of Problem 4 draw

the line DE through A and perpendicular to FG. Then DE is parallel to BC and is the required line.

166. Problem 8. To Divide a Given Line into a Number of Equal Parts

Example

Divide the line AB into seven equal parts. See Fig. 210.

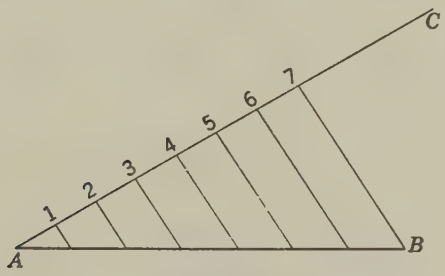


Fig. 210

Solution. Draw the line AC at an angle to AB, not exceeding 45°. Lay off, with the dividers or scale, seven equal parts on the line AC. Connect the last point, 7, with B. Draw lines through the other points parallel to $7B$. The intersections of these lines with AB divide AB into seven equal parts.

167. Problem 9. To Draw an Angle Equal to a Given Angle

Example

Draw an angle equal to the angle ABC, Figs. 211(a) and 211(b).

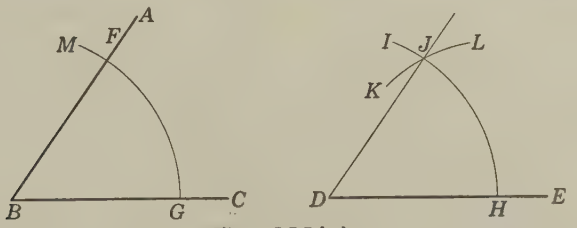


Fig. 211(a)

Solution 1. Draw the line DE. Then, with B as a center and any radius, draw the arc MG cutting the line AB at F. Using

the same radius and D as a center, draw the arc HI. Using FG as a radius and H as a center, draw the arc KL intersecting HI at J. Draw a line through J and D. The angle JDE is equal to ABC and is the required angle [Fig. 211(a)].

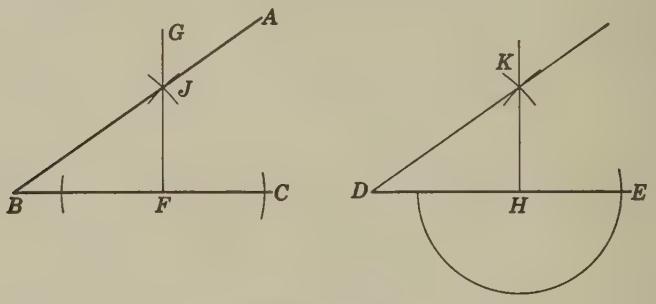


Fig. 211(b)

Solution 2. At any point on the line BC, as F, erect a perpendicular FG intersecting the line BA at J. Draw the line DE and lay off DH equal to BF. Erect a perpendicular to DE at H. On this perpendicular, lay off a distance HK equal to FJ. Draw a line through K and D, giving the required angle KDE [Fig. 211(b)].

168. Problem 10. To Draw an Equilateral Triangle of Given Size

Example

Draw an equilateral triangle with sides equal to AB, Fig. 212.

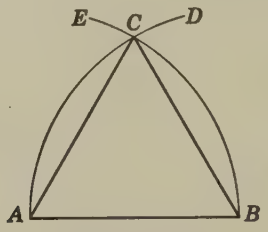


Fig. 212

Solution. With A as a center and AB as a radius, draw the arc BE; with B as a center and the same radius, draw the arc AD intersecting arc EB at C. Join A to C and B to C, giving the required equilateral triangle ABC.

169. Problem 11. To Draw a Circle through Three Given Points

Example

Draw a circle through the points A, B, and C, Fig. 213.

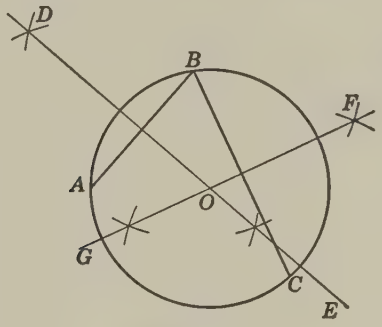


Fig. 213

SOLUTION. Connect the points A and B, and B and C. Draw DE bisecting AB; draw FG bisecting BC. The intersection of these two lines at O is the center of the circle. With O as center and radius OA, draw the required circle through the three given points.

170. Problem 12. To Find the Center of a Circle or Circular Arc

Example

Find the center of the arc in Fig. 214.

SOLUTION. Draw any two chords of the arc as AB and BC. Bisect each chord. The intersection O of the bisecting lines DE and FG is the center of the circle.

NOTE. This construction is similar to that of Problem 11.

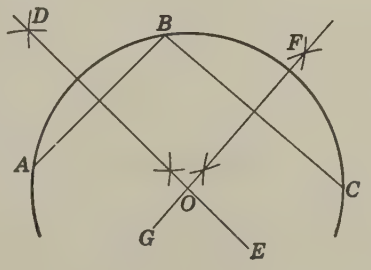


Fig. 214

171. Problem 13. To Inscribe a Square in a Circle

Example

Inscribe a square in the circle in Fig. 215.

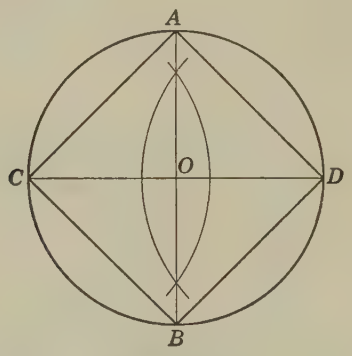


Fig. 215

SOLUTION. Draw the diameters AB and CD at right angles to each other. Connecting the four points A, C, B, and D gives the required square $ACBD$.

172. Problem 14. To Construct a Square of a Given Size

Example

Construct a square with sides equal to length AB in Fig. 216.

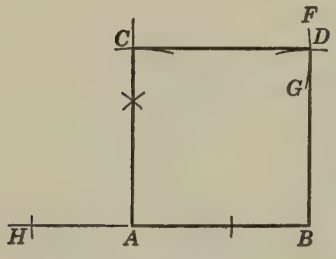


Fig. 216

SOLUTION. Extend line BA to H. Erect a perpendicular to line AB at A. With A as center and AB as radius, draw an arc cutting this perpendicular at C. With the same radius and C as center, draw arc FG. With the same radius and B as center, draw an arc cutting FG at D. Connect points $ACDB$ to produce the required square.

173. Problem 15. To Construct a Square Equal in Area to the Sum or Difference of Two Given Squares

Example 1

Construct a square equal in area to the *sum* of the two squares *ABCD* and *EFGH* (Fig. 217).

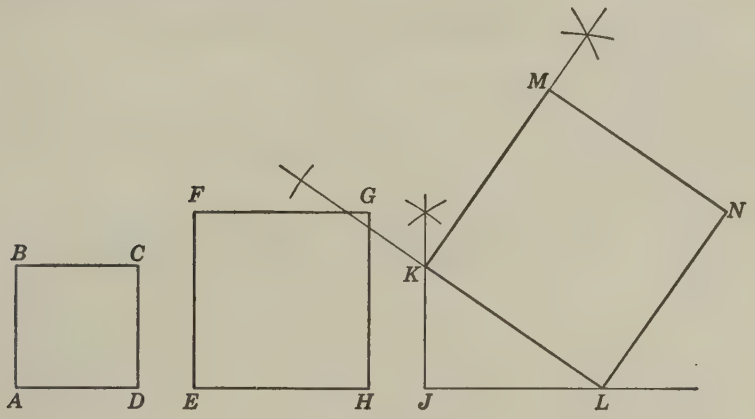


Fig. 217

Solution. Construct a right triangle whose base JL is equal to EH and whose altitude JK is equal to AB. On the hypotenuse KL construct a square $KMNL$, which is the required square.

Example 2

Construct a square equal in area to the *difference* of the two squares *ABCD* and *EFGH* (Fig. 218).

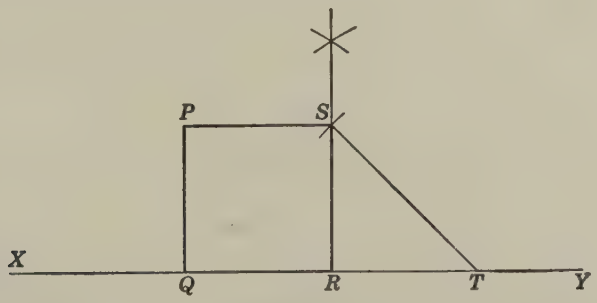


Fig. 218

SOLUTION. Construct the right triangle RST, Fig. 218, with ST equal to EH, the side of the larger square, as hypotenuse, and RT equal to AD, the side of the smaller square, as base. Lay off length RT equal to AD on straight line XY. Draw a perpendicular at R. With radius EH and T as center, draw an arc cutting the perpendicular at S. On the side RS construct the required square $QPSR$.

174. Problem 16. To Draw a Circle Equal in Area to the Sum or Difference of the Areas of Two Given Circles

Example

Draw a circle equal in area to the combined areas of a $\frac{1}{2}''$ circle and a $\frac{3}{4}''$ circle (Fig. 219).

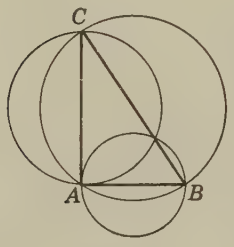


Fig. 219

SOLUTION. Construct the right triangle ABC, making $AB = \frac{1}{2}''$ and $AC = \frac{3}{4}''$. The circle drawn with the hypotenuse BC as diameter is equal in area to the sum of the areas of the two circles drawn with the sides AB and AC as diameters. Also, either of the side circles is equal to the difference between the hypotenuse circle and the other side circle.

175. Problem 17. To Inscribe a Hexagon in a Circle

Example

Inscribe a hexagon in the circle in Fig. 220.

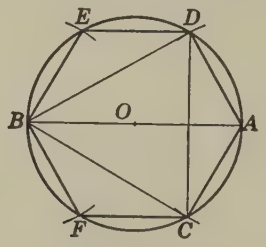


Fig. 220

SOLUTION. Draw the diameter AB. With a radius OA equal to the radius of the circle and with A as a center, draw arcs cut-

ting the circle at C and D; with the same radius and B as a center, draw arcs cutting the circle at E and F. Connect the successive points, producing the hexagon $BEDACF$.

176. Problem 18. To Inscribe an Equilateral Triangle in a Circle

Example

Inscribe an equilateral triangle in the circle in Fig. 220.

SOLUTION. Using the same construction as in Problem 17, draw lines connecting the alternate points B, D, and C, producing the equilateral triangle BDC.

177. Problem 19. To Construct a Hexagon Whose Sides Will Be a Given Length

Example

Construct a hexagon with sides equal to the given length AB in Fig. 221.

SOLUTION. On line XY in Fig. 221 lay off the required length AB. With B as center and AB as radius, draw a circle through A cutting XY at C. With the same radius and A as center, draw arcs cutting the circle at D and E. With the same radius and C as center, draw arcs cutting the circle at F and G. Connecting the successive points gives the required hexagon $AEFCGD$.

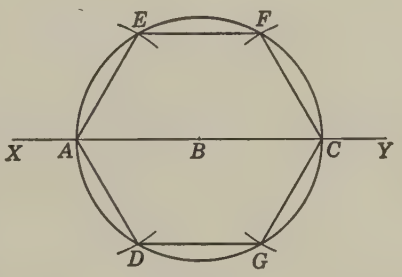


Fig. 221

Problem 19(a). To Construct a Hexagon with One of the Sides on a Given Line

Example

Construct a hexagon on a given line with sides equal to the given length AB in Fig. 222.

Solution. On the line XY in Fig. 222 lay off the length AB. With A as center and AB as radius, draw the arc PQ. With the same radius and B as center, draw an arc cutting the first arc at C. With the same radius and C as center, draw a circle through A and B. Draw the line AM through A and C, cutting the circle at F; draw the line BL through B and C, cutting the circle at E. With the same radius and A as center, draw an arc cutting the circle at D; with the same radius and B as center, draw an arc cutting the circle at G. Connecting the successive points gives the required hexagon $ADEFGB$.

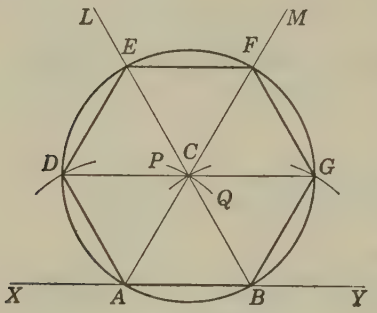


Fig. 222

178. Problem 20. To Inscribe an Octagon in a Circle

Example

Inscribe an octagon in the circle in Fig. 223.

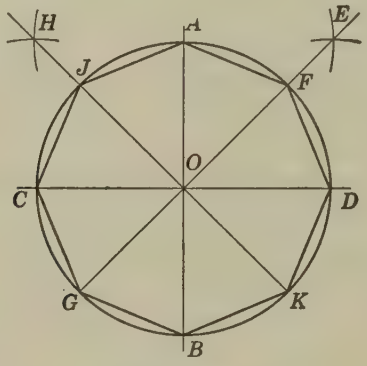


Fig. 223

Solution. Draw the diameters AB and CD at right angles to each other. Bisect angle AOC with line HK, cutting the circle at J and K. Bisect angle AOD with line EG, cutting the circle at F and G. Connecting the successive points gives the octagon $AFDKBGCJ$.

179. Problem 21. To Inscribe an Octagon in a Square

Example

Inscribe an octagon in the square in Fig. 224.

Solution. Draw the diagonals AC and DB, intersecting at O. With radius OA equal to half the diagonal distance, and A as center, draw arcs cutting the sides of the square at F and K. Using the same radius, repeat this operation at the other corners of the square, B, C, and D. Connecting successive points produces the octagon $EFGHIJKL$.

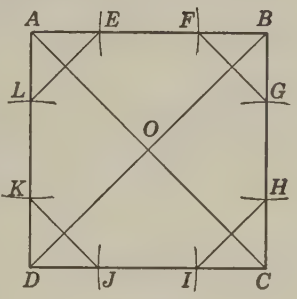


Fig. 224

180. Problem 22. To Construct an Octagon of a Given Size

Example

Construct an octagon with sides equal to AB in Fig. 225.

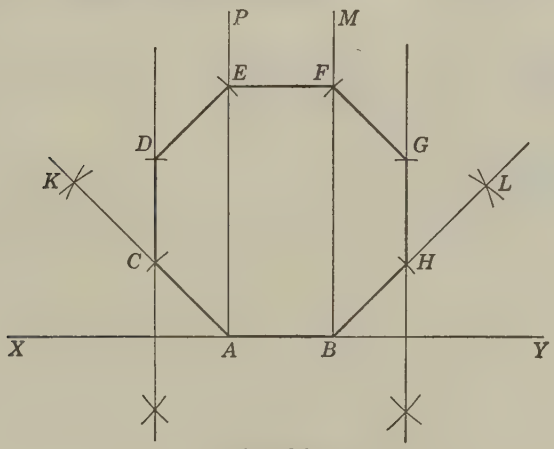


Fig. 225

Solution. Erect perpendiculars to line XY at A and B. Bisect the angles PAX and MBY. With AB as radius and A and B as centers, draw arcs cutting the bisectors at C and H. Draw

perpendiculars to XY through C and H. With the same radius and C and H as centers, draw arcs cutting these perpendiculars at D and G. With the same radius and D and G as centers, draw arcs cutting AP and BM at E and F. Connecting the successive points gives the required octagon $ACDEFGHB$.

181. Problem 23. To Construct a Pentagon

Example

Inscribe a pentagon in the circle in Fig. 226.

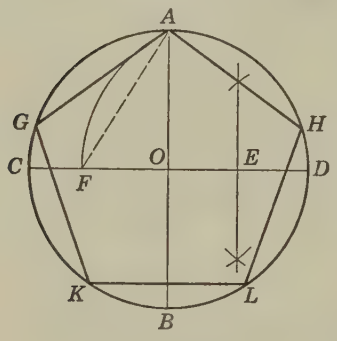


Fig. 226

Solution. Draw the diameters AB and CD at right angles to each other. Bisect OD at E. With E as a center and a radius equal to EA, describe the arc AF. The line AF is the side of the inscribed pentagon. Starting at A with a radius equal to the line AF, strike arcs around the circumference, giving the points A, G, K, L, H. Connecting these points produces the pentagon $AGKLH$.

182. Problem 24. To Draw a Tangent to a Circle at a Point on the Circumference

Example

Draw a tangent to the circle at the point A, Fig. 227.

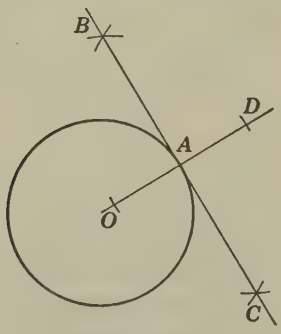


Fig. 227

SOLUTION. Draw the line OD through the center of the circle and point A. Draw BC perpendicular to OD at A by the method previously studied. Then BC is the required tangent.

183. Problem 25. To Draw a Tangent to a Circle through a Point outside the Circle

Example

Draw a tangent to the circle from the point A, Fig. 228.

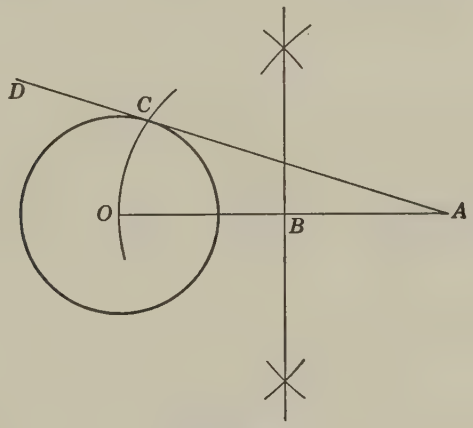


Fig. 228

SOLUTION. Draw the line OA connecting the center of the circle with the point A. Bisect the line OA, obtaining the midpoint B. Using B as a center and BO as a radius, draw the arc OC cutting the given circle at C. The line AD drawn through A and C is the required tangent.

184. Problem 26. To Draw a Tangent to Two Circles of Equal Size

Example

Draw a tangent to the two equal circles shown in Fig. 229.

SOLUTION. Draw the line AB through the centers O and C of the circles. At O and at C draw the lines ND and EF perpendicular to AB. Draw the line KL through the points G and H, where

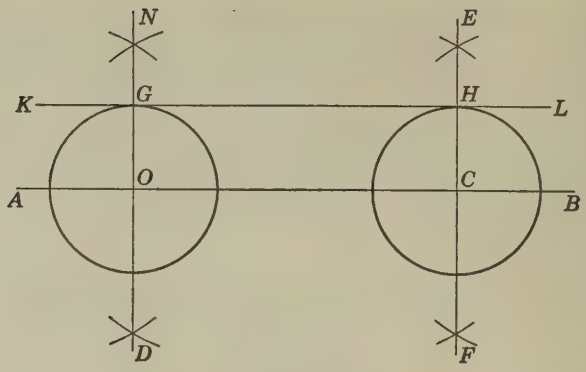


Fig. 229

these perpendiculars intersect the circles. Then KL is the required tangent.

185. Problem 27. To Draw an Internal Tangent to Two Equal Circles

Example

Draw an internal tangent to the two equal circles shown in Fig. 230.

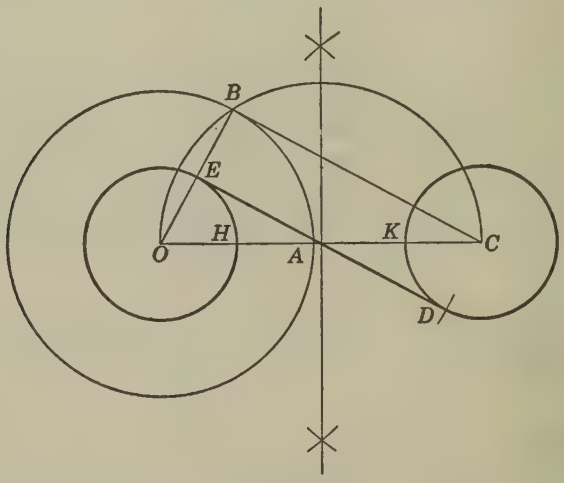


Fig. 230

Solution. Draw the line OC connecting the centers of the two circles. On this line lay off $HA = OH$. With O as a center

and OA as a radius, draw the large circle shown in the figure. By means of the method of Problem 25, draw the tangent CB to the large circle through C. Draw OB intersecting the small circle at E. With E as center and BC as radius, draw an arc cutting circle C at D. Draw ED, the required tangent.

186. Problem 28. To Draw an External Tangent to Two Circles of Unequal Size

Example

Draw an external tangent to the two circles in Fig. 231.

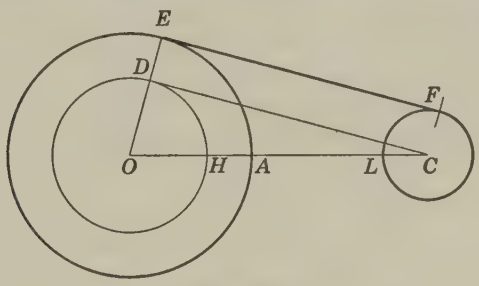


Fig. 231

Solution. Connect the centers of the two circles by the line OC. On OA, the radius of the large circle, lay off $AH = LC$, the radius of the small circle. Using O as a center and OH as a radius, draw the construction circle shown. By the method of Problem 25, draw the tangent CD to the construction circle and through C. Prolong the radius OD until it cuts the outer circle at E. With E as center and DC as radius, draw an arc cutting circle C at F. Connecting E and F gives the required tangent.

187. Problem 29. To Draw an Internal Tangent to Two Unequal Circles

Example

Draw an internal tangent to the two circles shown in Fig. 232.

Solution. Connect the centers by the line OC. Lay off on the line of centers the distance EF equal to the small radius, making OF equal to the sum of the two radii OE and CH. With

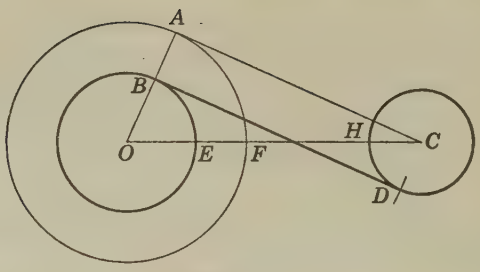


Fig. 232

O as center and radius OF, draw large circle shown in figure. Draw tangent CA to large circle through point C. Connect O and A, cutting the given circle at B. With B as center and AC as radius, draw an arc cutting circle C at D. Connecting points B and D gives the required tangent.

188. Problem 30. To Draw an Ellipse

Example

Draw an ellipse with a long axis of 2″ and a short axis of $1\frac{1}{4}$″.

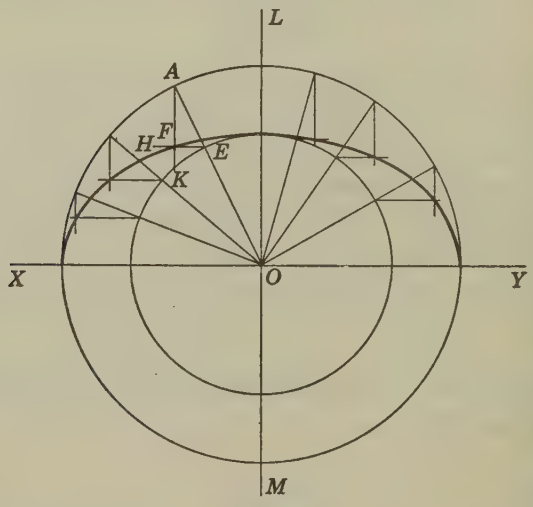


Fig. 233

Solution 1. (Fig. 233.) Draw two concentric circles with diameters equal respectively to the long and the short axes of the ellipse. Draw two axes XY and LM at right angles to each other.

Draw any radius as OA. Through point A, where this radius meets the large circle, draw a line AK parallel to LM. Through point E, where radius OA crosses the small circle, draw a line EH parallel to axis XY. The intersection of these two lines AK and EH at F is a point on the perimeter of the ellipse. Other points on the perimeter may be found in a similar manner, and, by obtaining a sufficiently large number of points, the ellipse may be drawn through these points.

SOLUTION 2. (Fig. 234.) Draw the 2″ and the $1\frac{1}{4}$″ axes at right angles to each other and bisecting each other. Using B as

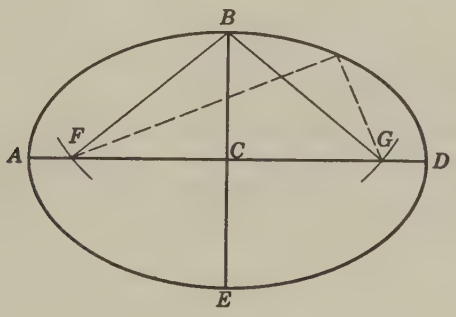


Fig. 234

center and AC, half the major axis, as radius, draw an arc cutting the major axis at two points F and G. Insert thumb tacks into the paper at the points F, G, and B; and stretch a piece of thread so as to be fairly taut around these three points. Now remove the tack from point B, and, keeping the string uniformly taut with the point of a pencil, draw the ellipse.

14 The Essentials of Trigonometry

189. The Right Triangle

Every triangle has three sides and three angles. If we know any three parts, at least one of which is a side, we can construct the triangle. In Fig. 235, ABC is a right triangle, in which C is the right angle. We know from geometry that

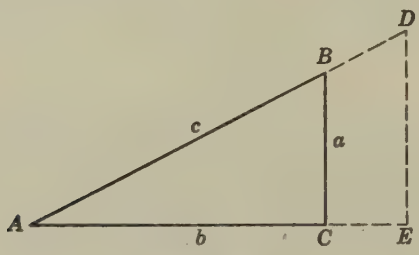


Fig. 235

the sum of the three angles of any triangle is 180°. Since the angle C is a right angle, that is, 90°, the sum of A and B is 90°; hence A and B are acute angles.

For convenience let a represent the side opposite the acute angle A; b the side opposite the angle B; and c the side opposite the right angle C. The side c, opposite the right angle, is called the *hypotenuse;* the side a, opposite the angle A, is *adjacent to the angle* B; the side b, opposite the angle B, is *adjacent to the angle* A.

In Fig. 235, if we measure a, the side opposite the angle A, we find it to be $\frac{3}{4}''$; and the side b opposite the angle B is

found to be $1\frac{1}{2}''$. Dividing $\frac{3}{4}$ by $1\frac{1}{2}$, we get .5. If we prolong the side AB to D and the side AC to E and draw DE, we get the right triangle ADE, in which the acute angle A is the same as in the triangle ABC. If we now measure the side DE, which is opposite the angle A, we find it to be $1\frac{1}{4}''$, and the side AE, which is adjacent to the angle A, measures $2\frac{1}{2}''$. Dividing the length of the opposite side by the length of the adjacent side, we get $\frac{1\frac{1}{4}}{2\frac{1}{2}} = .5$, which is the same value as in the first case. The value of the ratio of the side opposite the angle A to the side adjacent to the angle A will be found to be the same as long as the angle A remains unchanged. If the angle A were to be increased, the ratio of the opposite side to the adjacent side would no longer be .5, but some larger number. If the angle A were to be decreased, the ratio of the two sides would be less than .5. Since the size of the angle determines the value of the ratio of the two sides, the ratio may be taken as a measure of the angle.

The ratio obtained by dividing the side opposite the acute angle A by the side adjacent to the angle A is known as the ***tangent*** of the angle A and is written, $\tan A$. Tables in the back of this book give the value of the natural tangent for every angle from $1'$ to $90°$. If we refer to this table of natural tangents, we find that the angle whose tangent is .5 is $26° 34'$.

Again referring to Fig. 235, suppose we know that the angle $A = 26° 34'$ and the side $b = 6''$, and we are required to find the length of side a. We would proceed as follows: Express the ratio of the side a to the side b, thus, $\frac{a}{b} = \frac{a}{6}$. This ratio of the side opposite A to the side adjacent to A we have just called the tangent of A. From a table of tangents we find that $\tan 26° 34' = .5$. We may therefore write $\frac{a}{6} = .5$, whence $a = 6 \times .5 = 3''$. Similarly, if we know

that $A = 26° 34'$ and $a = 3''$ and are required to find b, we would proceed thus:

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} = \frac{3}{b} = .5$$

Therefore,

$$b = \frac{3}{.5} = 6''$$

Summary

1. Given a and b, to find A.

SOLUTION. $\frac{a}{b} = \tan A$. $\therefore A =$ angle whose tangent is $\frac{a}{b}$.

2. Given A and b, to find a.

SOLUTION. $\frac{a}{b} = \tan A$. $\therefore a = b \times \tan A$.

3. Given A and a, to find b.

SOLUTION. $\frac{a}{b} = \tan A$. $\therefore b = \frac{a}{\tan A}$.

190. Trigonometric Functions

Instead of determining the value of the angle from the ratio of side a to side b, we might equally well divide any side by either of the remaining two and use the value of the ratio thus obtained as a measure of Angle A. There are six possible ratios or *trigonometric functions* of each angle. See Fig. 236.

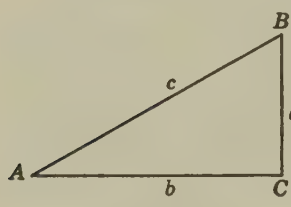


Fig. 236

sine of angle A $= \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c}$; written, $\sin A = \frac{a}{c}$

cosine of angle A $= \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c}$; written, $\cos A = \frac{b}{c}$

tangent of angle A $= \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b}$; written, $\tan A = \frac{a}{b}$

cotangent of angle A $= \frac{\text{adjacent side}}{\text{opposite side}} = \frac{b}{a}$; written, $\cot A = \frac{b}{a}$

secant of angle $A = \dfrac{\text{hypotenuse}}{\text{adjacent side}} = \dfrac{c}{b}$; written, $\sec A = \dfrac{c}{b}$

cosecant of angle $A = \dfrac{\text{hypotenuse}}{\text{opposite side}} = \dfrac{c}{a}$; written, $\operatorname{cosec} A = \dfrac{c}{a}$

For angle B the six functions are as follows:

$$\sin B = \frac{b}{c} \qquad \tan B = \frac{b}{a} \qquad \sec B = \frac{c}{a}$$

$$\cos B = \frac{a}{c} \qquad \cot B = \frac{a}{b} \qquad \operatorname{cosec} B = \frac{c}{b}$$

191. Use of Tables

To find the functions of an angle *up to* 45°, look for the *degrees* at the *top* of the page and for the *minutes* in the *left-hand* column.

Example 1

Find the sine of 39° 50′.

Procedure. On page 538 find 39° at the top in heavy type. Read down the column headed "Sine" until you come to .64056 opposite 50′ in the extreme left-hand column. The decimal point, where not shown, is understood before all numbers in the tables of natural functions. The sine of 39° 50′ = .64056.

To find the functions of an angle from 45° to 90°, look for the *degrees* at the *bottom* of the page and for the *minutes* in the *right-hand* column.

Example 2

Find the tangent of 84° 10′.

On page 541 find 84° at the bottom in heavy type. *Read up* in the column entitled "Tang" at the bottom until you come to the number 9.78817 opposite 10′ in the extreme right-hand column. The tangent of 84° 10′ = 9.78817.

If the function of an angle in degrees, minutes, and seconds is desired, we proceed as in the following example.

Example 3

Find the tangent of 28° 15′ 45″.

In the table of natural tangents we find the tangent of 28° 15′ is .53732, and the tangent of 28° 16′ is .53769. That is, for an angle which is a whole minute more than 28° 15′, the tangent is 37 more. Since 45″ = $\frac{45}{60}$ of a minute, the tangent of 28° 15′ 45′ must be $\frac{45}{60}$ of 37, or 28 more than the tangent of 28° 15′. We must therefore add 28 to the tangent of 28° 15′ to obtain the tangent of 28° 15′ 45″, giving us 53732 + 28 = 53760. Placing the decimal point in front of the number, we get tan 28° 15′ 45″ = .53760.

The operation of obtaining an intermediate value of a function between two given values in the table is called *interpolation.*

PROBLEMS

Find the value of each of the following by means of the tables.

1. sin 48° 03′	**2.** cos 67° 59′	**3.** tan 52° 20′
4. cot 52° 20′	**5.** sin 85° 40′	**6.** cos 85° 40′
7. tan 85° 40′	**8.** cot 85° 40′	**9.** sin 18° 30′
10. cos 18° 30′	**11.** tan 40° 30′ 30″	**12.** sin 65° 20′ 45″
13. cot 72° 36′ 15″	**14.** cos 58° 06′ 20″	**15.** sin 20° 20′ 40″
16. sin 10°	**17.** sin 20° 30′ 18″	**18.** cos 8° 45′
19. cos 50° 15′	**20.** sin 5° 45′	

192. To Find an Angle Corresponding to a Given Function

Example 1

Find the angle whose cosine = .40088.

Turn to the table of natural sines and cosines and look along the columns of numbers until you find .40088. This will be found on page 535. Since .40088 is the cosine of the angle and since the column in which it is located has "Cosin" at the bottom of it, we must take the degrees from the bottom of the page and the minutes from the right-hand column. The degrees at the bottom are 66° and the minutes in the right-hand column opposite .40088 are 22′. Hence the angle whose cosine is .40088 is 66° 22′.

Example 2

Find the angle whose cotangent is 1.36675.

Turning to the table of natural tangents and cotangents, we do not find the number 1.36675. We do, however, find the numbers 1.36716 and 1.36633, between which the given cotangent lies. Since 1.36716 is the cotangent of 36° 11′ and 1.36633 is the cotangent of 36° 12′, it shows that the angle whose cotangent is 1.36675 is between 36° 11′ and 36° 12′. The difference between 1.36633 and 1.36716 is 83. (We can disregard the decimal point in interpolating as long as we remember that all additions to the values in the table and subtractions from them are done to the last figure in the decimal part of the function.) That is, if the cotangent is 83 less than 1.36716, the angle is a whole minute more than 36° 11′. But, since the given tangent 1.36675 is only 41 less than 1.36716, the angle is $\frac{41}{83}$ of a minute more than 36° 11′. $\frac{41}{83} \times 60'' = 30''$. Hence the angle whose cotangent is 1.36675 is 36° 11′ 30″.

PROBLEMS

Find the value of angle A in each of the following.

1. $\sin A = .59131$	**2.** $\cos A = .80644$	**3.** $\tan A = .41592$
4. $\cot A = .48055$	**5.** $\cos A = .13341$	**6.** $\tan A = 2.08094$
7. $\sin A = .89087$	**8.** $\cot A = 1.50512$	**9.** $\tan A = 1.50512$
10. $\sin A = .71121$	**11.** $\cos A = .72927$	**12.** $\sin A = .14800$
13. $\tan A = .49507$	**14.** $\cos A = .43483$	**15.** $\cot A = 2.56597$

193. Solution of Right Triangles

The following examples will illustrate the method to be followed in the solution of right triangles.

Example 1

Given $A = 36° 28'$ and $b = 14''$.

Find B, a, and c, Fig. 237.

SOLUTION. Since $A + B = 90°$

$$B = 90° - A$$
$$= 89° 60' - 36° 28'$$
$$= 53° 32'$$

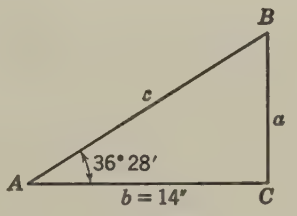


Fig. 237

To find a, we write a function of the given angle A, employing for that purpose the given side b and the required side a. That function is, $\frac{a}{b} = \tan A$.

$$\therefore a = b \times \tan A$$

That is, the length of side a is found by multiplying b by the value of the tangent of A which is given in the table of natural tangents.

$$a = b \times \tan A = 14 \times \tan 36° 28'$$
$$= 14 \times .73906 = 10.3468''$$

Similarly, to find c, we write a function of angle A, employing for that purpose the given side b and the required side c. That function is, $\frac{b}{c} = \cos A$, whence $c = \frac{b}{\cos A}$. That is, the length of side c is found by dividing b by the value of the cosine of A given in the table of natural cosines.

$$c = \frac{b}{\cos A} = \frac{14}{\cos 36° 28'} = \frac{14}{.80420} = 17.41''$$

A very satisfactory way of arranging the work in the preceding example is as follows:

Given $A = 36° 28'$, $b = 14''$.

Find B, a, and c, Fig. 238.

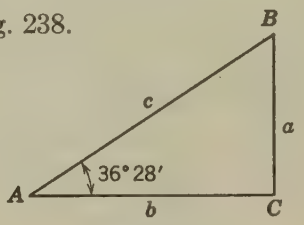


Fig. 238

To find B. $B = 90° - A \quad = 89° 60' - 36° 28'$
$= 53° 32'$

To find a. $\frac{a}{b} = \tan A$. $\therefore a = b \times \tan A$
$= 14 \times \tan 36° 28'$
$= 14 \times .73906$
$= 10.3468''$

To find c. $\dfrac{b}{c} = \cos A$. $\therefore c = \dfrac{b}{\cos A}$

$$= \frac{14}{\cos 36° 28'}$$

$$= \frac{14}{.80420} = 17.41''$$

EXPLANATION. The method of solving a right triangle when an acute angle and one of the sides are given is as follows:

1. To find the unknown acute angle, subtract the given acute angle from 90°.

2. To find an unknown side, write a function of the given angle, employing for that purpose the given side and the required side. From the function so written, get an expression for the required side in terms of the given side and the function of the angle. For example:

I. Given A and a.

To find b. $\dfrac{b}{a} = \cot A$. $\therefore b = a \times \cot A$.

To find c. $\dfrac{a}{c} = \sin A$. $\therefore c = \dfrac{a}{\sin A}$.

II. Given A and c.

To find a. $\dfrac{a}{c} = \sin A$. $\therefore a = c \times \sin A$.

To find b. $\dfrac{b}{c} = \cos A$. $\therefore b = c \times \cos A$.

III. Given A and b.

To find a. $\dfrac{a}{b} = \tan A$. $\therefore a = b \times \tan A$.

To find c. $\dfrac{b}{c} = \cos A$. $\therefore c = \dfrac{b}{\cos A}$.

If angle B is given, derive angle A by subtracting B from 90° and use the formulas I, II, and III to find the missing sides.

PROBLEMS

1.	Given $A = 24° 36'$	and $a = 7''$.	Find B, b, and c.
2.	Given $B = 30° 40'$	and $a = 15''$.	Find A, b, and c.
3.	Given $A = 70° 42'$	and $a = 11.4''$.	Find B, b, and c.
4.	Given $A = 48° 18' 30''$	and $b = 54.8''$.	Find B, a, and c.
5.	Given $A = 8° 48' 40''$	and $b = 45''$.	Find B, a, and c.
6.	Given $B = 16° 08'$	and $b = 90'$.	Find A, a, and c.
7.	Given $B = 66° 24'$	and $c = 60.55''$.	Find A, a, and b.
8.	Given $A = 53° 28' 15''$	and $c = 16.5''$.	Find B, a, and b.
9.	Given $A = 31° 59'$	and $c = 21.7'$.	Find B, a, and b.
10.	Given $B = 45° 45'$	and $a = 32.5'$.	Find A, b, and c.

Example 2

Given $a = 8''$ and $b = 15''$.

Find A, B, and c, Fig. 239.

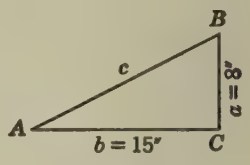


Fig. 239

SOLUTION.

To find A.

$$\tan A = \frac{a}{b} = \frac{8}{15} = .53333. \quad \therefore A = 28° 04'$$

To find B.

$$B = 90° - A = 90° - 28° 04' = 61° 56'$$

To find c.

$$c = \sqrt{a^2 + b^2} = \sqrt{8^2 + 15^2} = 17.00''$$

EXPLANATION. Since we know a, the side opposite angle A, and b, the side adjacent to angle A, we can find the tangent of angle A, namely $\frac{8}{15}$. Reducing this to a decimal gives $\tan A = .53333$. Referring to a table of natural tangents, we find that the angle whose tangent is .53333 is 28° 04′.

Having found angle A, we find angle B by subtracting angle A from 90°.

To find c we write the sine function of angle A, thus:

$$\frac{a}{c} = \sin A. \quad \therefore c = \frac{a}{\sin A};$$

or we might write

$$\frac{b}{c} = \cos A. \quad \therefore c = \frac{b}{\cos A}.$$

But we can also get c by employing the fact learned in geometry, that the square of the hypotenuse is equal to the sum of the squares of the other two sides. We have made use of this method in this solution.

PROBLEMS

1. Given $a = 7.5''$ and $b = 8.75''$. Find A, B, and c.
2. Given $a = 9.4''$ and $b = 12.8''$. Find A, B, and c.
3. Given $a = 12.8''$ and $b = 16''$. Find A, B, and c.
4. Given $a = 6.6''$ and $b = 14.48''$. Find A, B, and c.
5. Given $a = 40'$ and $b = 28'$. Find A, B, and c.
6. Given $a = 38.3'$ and $b = 45.6'$. Find A, B, and c.
7. Given $a = 20''$ and $b = 28''$. Find A, B, and c.
8. Given $a = 102'$ and $b = 126'$. Find A, B, and c.
9. Given $a = 16.75''$ and $b = 24.5''$. Find A, B, and c.
10. Given $a = 3'\,4''$ and $b = 4'\,0''$. Find A, B, and c.

Example 3

Given $a = 4.5''$ and $c = 7.25''$.

Find A, B, and b, Fig. 240.

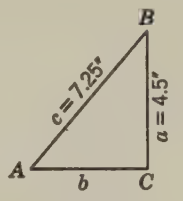


Fig. 240

SOLUTION.

To find A.

$$\sin A = \frac{a}{c} = \frac{4.5}{7.25} = .62069. \quad \therefore A = 38° 22'$$

To find B.

$$B = 90° - A = 90° - 38° 22' = 51° 38'$$

To find b.

$$b = \sqrt{c^2 - a^2} = \sqrt{7.25^2 - 4.5^2} = 5.68''$$

PROBLEMS

1. Given $a = 3.8''$ and $c = 5.2''$. Find A, B, and b.
2. Given $b = 18''$ and $c = 22''$. Find A, B, and a.
3. Given $b = 92'$ and $c = 124'$. Find A, B, and a.
4. Given $a = 15''$ and $c = 25''$. Find A, B, and b.
5. Given $b = 7.1''$ and $c = 15.84''$. Find A, B, and a.
6. Given $a = 21''$ and $c = 38''$. Find A, B, and b.
7. Given $a = 14.3''$ and $c = 23.4''$. Find A, B, and b.
8. Given $b = 10.6''$ and $c = 16.2''$. Find A, B, and a.
9. Given $a = 45'$ and $c = 70'$. Find A, B, and b.
10. Given $b = 25.5''$ and $c = 40''$. Find A. B, and a.

194. Isosceles Triangles

A perpendicular dropped from the vertex of an isosceles triangle to the base divides the isosceles triangle into two

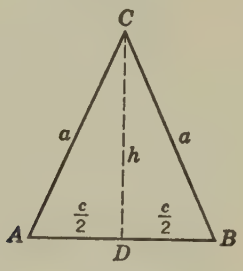


Fig. 241

equal right triangles which can be solved by the methods shown in the preceding section. See Fig. 241.

Example 1

Given $a = 16''$, $c = 12''$.

Find A, B, C, and h, Fig. 242.

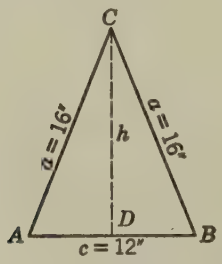


Fig. 242

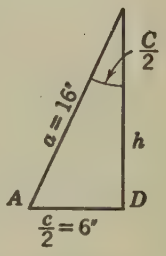


Fig. 243

Solution. See Fig. 243.

To find A.

$$\cos A = \frac{\frac{c}{2}}{a} = \frac{6}{16} = .37500. \quad \therefore A = 67^\circ\,59'$$

$$B = A = 67^\circ\,59'$$

To find C. First find $\frac{C}{2}$.

$$\frac{C}{2} = 90° - A = 90° - 67° 59' = 22° 01'$$

$$C = 2 \times 22° 01' = 44° 02'$$

Or

$$C = 180° - (A + B)$$

$$= 180° - (67° 59' + 67° 59')$$

$$= 180° - 135° 58' = 44° 02'$$

To find h.

$$h = \sqrt{a^2 - \left(\frac{c}{2}\right)^2} = \sqrt{16^2 - 6^2} = 14.83''$$

Example 2

Given $C = 54° 42'$, $a = 25''$.

Find A, B, c, and h, Fig. 244.

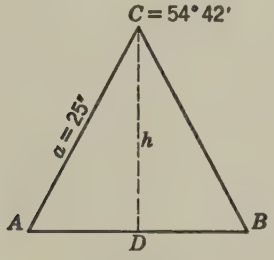


Fig. 244

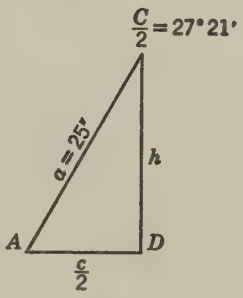


Fig. 245

Solution. See Fig. 245.

To find A.

$$A = 90° - \frac{C}{2} = 90° - 27° 21' = 62° 39'$$

$$B = A = 62° 39'$$

To find c. First find $\frac{c}{2}$.

$$\frac{\frac{c}{2}}{a} = \sin 27° 21'$$

$$\frac{c}{2} = a \times \sin 27° 21'$$

$$= 25 \times .45942 = 11.48''$$

$$\therefore c = 2 \times 11.48'' = 22.96''$$

To find h.

$$\frac{h}{a} = \cos 27° 21'$$

$$\therefore h = a \times \cos 27° 21'$$

$$= 25 \times .88822 = 22.20''$$

PROBLEMS

Compute the missing dimensions in the following isosceles triangles.

	Base	Sides	Altitude	Base Angles	Vertex Angle
1.	?	10.8″	?	64° 15′	?
2.	?	14.6″	?	?	85° 30′
3.	18.2″	?	?	?	71° 40′
4.	16″	?	?	69° 45′	?
5.	24.5″	?	15.6″	?	?
6.	?	?	28″	82° 30′	?
7.	?	32.8″	25.4″	?	?
8.	?	?	19.3″	?	100°

MISCELLANEOUS PROBLEMS

1. The equal sides of an isosceles triangle are 25″ long and the altitude is 14″. Compute the base and the angles.

2. An isosceles triangle has sides of 13″, 13″, and 5″. Compute the angles and the altitude.

3. An isosceles triangle has a base of 4.34″ and an altitude of 10″. Compute the angles and the length of the equal sides.

4. The base angles of an isosceles triangle are each 72° 46′ and the altitude is 7.5″. Compute the vertex angle, the base, and the length of the equal sides.

5. The vertex angle of an isosceles triangle is 38° 22′, and the equal sides are each 2.5″ long. Find the base angles, the base, and the altitude.

6. Find the perpendicular distance from the center to the side of a hexagon with 5″ sides. (*Suggestion:* Draw OA, Fig. 246. Angle $OAB = 60°$. Solve for OB.)

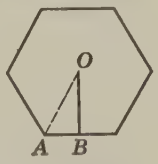


Fig. 246

7. Find the included taper angle in a piece of work having a taper of $\frac{3}{4}$″ per foot.

8. Find the included taper angle in a piece of work having a taper of .6″ per foot.

9. Figure 247 shows a 29° Acme thread. Find the bottom flat of a 1″ pitch thread if the depth is $\frac{1}{2}$ pitch + .01″.

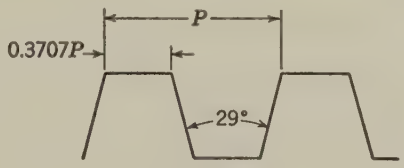


Fig. 247

10. In the Whitworth thread in Fig. 248 the sides make an angle of 55° with each other. The depth of the thread is two-thirds of the depth of the triangle formed by prolonging the sides. Find the depth of a thread of 1″ pitch.

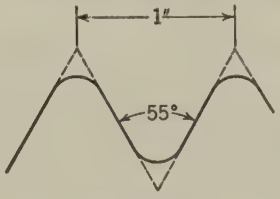


Fig. 248

11. A road has a rise of 4 ft. in 100 ft. What is the angle of slope with the horizontal?

12. A road has a slope of 1° 50′ with the horizontal. What is the rise in 100 ft.?

13. What is the largest square that can be milled from a circular disk 6″ in diameter?

14. Find the radius of a circle circumscribed about an octagon with 10″ sides.

15. Find the angle of slope of the rafter in Fig. 249.

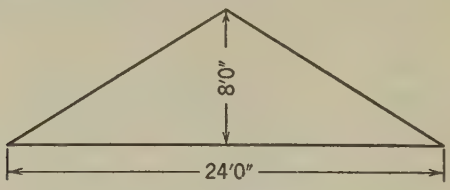


Fig. 249

16. A chord of a $2\frac{1}{2}$″ circle is 1.028″ from the center. Find its length.

17. The chord of a $19\frac{1}{2}$″ circle is $11\frac{1}{4}$″ long. Find the central angle and the distance from the center.

18. Lay out an angle of 12° 40′ by means of the value of its natural tangent.

19. Compute the missing dimension in the sketch in Fig. 250.

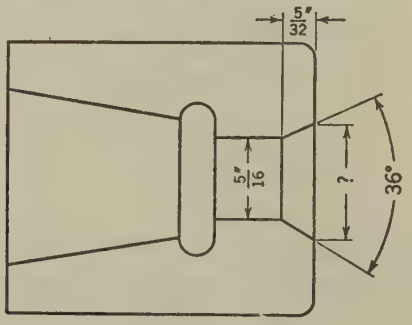


Fig. 250

20. Compute the missing dimension in the flat-headed screw shown in Fig. 251.

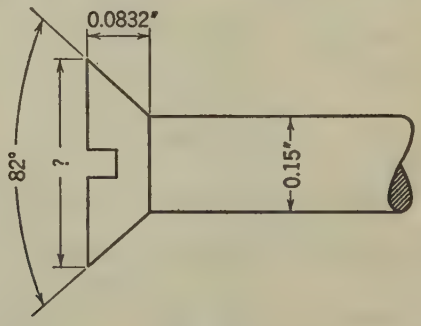


Fig. 251

21. Figure 252 shows a taper gib with a taper of $\frac{3}{16}''$ per foot. Compute the following dimensions:

(*a*) Thickness at large end.
(*b*) Dimension A at the small end.
(*c*) Width W at the large end.
(*d*) Width w at the small end.

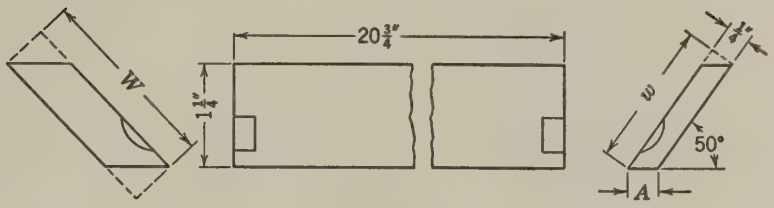


Fig. 252

22. Twelve bolt holes are to be drilled in a cylinder head on the circumference of a 14″ circle. What is the straight line distance between centers of holes?

23. If 15 holes are drilled on the circumference of a 27″ circle, find the distance center to center of holes.

24. Nine holes are to be drilled on the circumference of a circle $7\frac{1}{4}''$ in diameter. What should be the distance center to center of holes?

25. Find the length of the chord joining two points 119° apart on the circumference of a $3\frac{1}{2}''$ circle.

26. Find the distance between two points 37° 30′ apart on the circumference of a circle 2.37″ in diameter.

27. What should be the check measurement over a pair of $\frac{1}{2}''$ wires on the dovetail slide shown in Fig. 253, both inside and outside? The sides

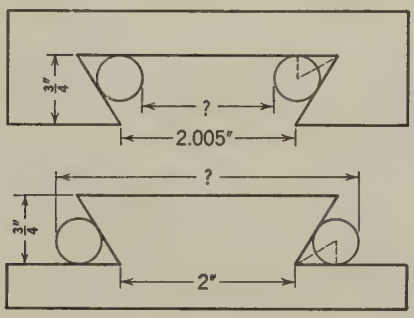


Fig. 253

form an angle of $62\frac{1}{2}°$ with the horizontal. (*Suggestion:* The dotted lines indicate the solution.)

28. Figure 254(*a*) shows the arrangement for gaging screw threads by the 3-wire system. Compute the measurement over the wires for the following sharp V-threads:

(*a*) $\frac{1}{4}''$ diameter, 20 threads per inch, wires .035″ diam.
(*b*) $\frac{9}{16}''$ diameter, 12 threads per inch, wires .050″ diam.

(*a*) (*b*)

Fig. 254

(*Suggestion:* Solution indicated on Fig. 254(*b*). Add twice the distance AB to the root diameter and add half a wire diameter on each side.)

29. Figure the check measurements for the same two screws as in Problem 28 but for American Standard threads. Use the same wires.

30. Compute the height of a $7\frac{1}{2}''$ sine bar for an angle of 27° 55′.

31. A sine bar 10″ long has a difference in elevation of 4.873″ between the two ends. What angle is indicated?

32. A cylindrical automobile gasoline tank 14″ in diameter and 3′ 1″ long is lying in a horizontal position. A gage inserted through the top indicates a depth of gasoline of $11\frac{1}{4}''$. How many gallons of gasoline are there in the tank? (Fig. 255.)

Fig. 255

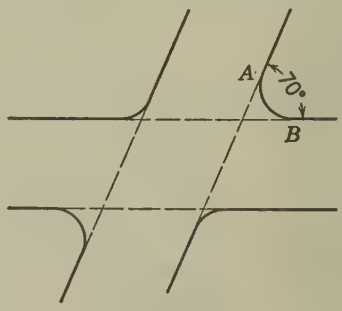


Fig. 256

33. Two roads cross each other at an angle of 70°, Fig. 256. The arc AB is drawn tangent to the main curb lines with a radius of 12′. Find the length of the piece of curbing, AB. If the roadways are 60′ wide, how many square yards of pavement are required for their intersection?

34. In a Brown and Sharpe 29° worm thread, find the diameter of the wire laid in the groove that will lie exactly flush with the top of the thread: (*a*) 4 threads per inch; (*b*) 5 threads per inch; (*c*) 3 threads per inch. See Fig. 257.

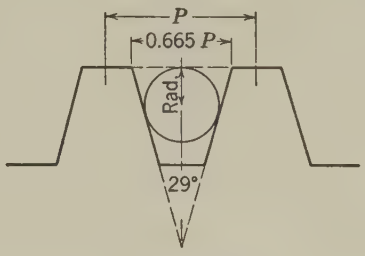


Fig. 257

35. In the 29° Acme tap thread the space $S = 0.6293P + 0.0052''$. Find the diameter of a wire that will lie in the groove flush with the top of the thread if the pitch is: (*a*) $\frac{1}{8}''$; (*b*) $\frac{1}{6}''$; (*c*) $\frac{1}{10}''$. See Fig. 258.

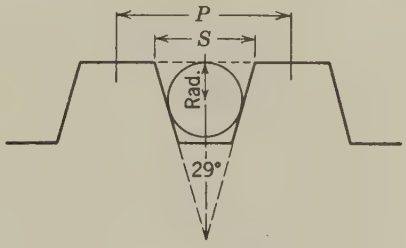


Fig. 258

195. Oblique Triangles

An *oblique triangle* is a triangle that is not a right triangle. The solution of oblique triangles may be grouped into four cases:

Case I. Given two angles and a side.
Case II. Given two sides and the angle between them.
Case III. Given the three sides.
Case IV. Given two sides and an angle opposite one of them.

An oblique triangle may be solved by means of formulas derived from the Law of Sines and the Law of Cosines.

196. Law of Sines

In any triangle the sides are proportional to the sines of the opposite angles. Let ABC in Fig. 259(a) and (b) be

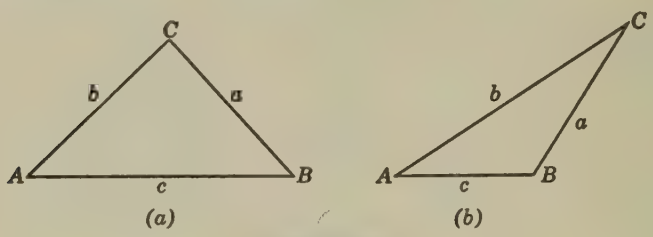


Fig. 259

any triangle. According to the Law of Sines we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} \tag{1}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C} \tag{2}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C} \tag{3}$$

In each of these formulas there are four elements. Hence, if three elements in any formula are known, the fourth one can be found. For example, if we are given B, C, and c, to find b we would use formula (2), $\frac{b}{\sin B} = \frac{c}{\sin C}$, whence we get $b = \frac{c \sin B}{\sin C}$.

197. Law of Cosines

In any triangle the square of any side is equal to the sum of the squares of the other two sides minus twice the product of those two sides into the cosine of their included angle.

In Fig. 260(a) and (b) we have

$$a^2 = b^2 + c^2 - 2bc \cos A \tag{4}$$

$$b^2 = a^2 + c^2 - 2ac \cos B \tag{5}$$

$$c^2 = a^2 + b^2 - 2ab \cos C \tag{6}$$

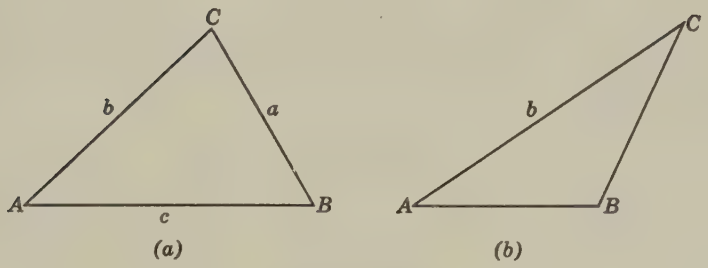


Fig. 260

Solving for $\cos A$, $\cos B$, and $\cos C$, we get

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \tag{7}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \tag{8}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \tag{9}$$

NOTE. When solving triangles, the student should bear in mind the following facts:

1. The sum of the three angles in a triangle equals 180 degrees.
2. The greater side is opposite the greater angle; and, conversely, the greater angle is opposite the greater side.
3. The sine of an angle between 90° and 180° is equal to the sine of its supplement. Example: $\sin 120° 40' = \sin (180° - 120° 40') = \sin 59° 20' = .86015$.
4. The cosine of an angle between 90° and 180° is equal to minus the cosine of its supplement. Example: $\cos 120° 40' = -\cos (180° - 120° 40') = -\cos 59° 20' = -.51004$.

PROBLEMS

Find two values of angle A in each of the following.

1. $\sin A = .41204$	**2.** $\sin A = .91116$
3. $\sin A = .53730$	**4.** $\sin A = .74509$
5. $\sin A = .95372$	**6.** $\sin A = .17365$
7. $\sin A = .30348$	**8.** $\sin A = .70711$
9. $\sin A = .55194$	**10.** $\sin A = .88158$

198. Case I. Given Two Angles and a Side

Example

Given $a = 18.5''$, $A = 56° 36'$, $B = 72° 08'$.

Find C, b, and c, Fig. 261.

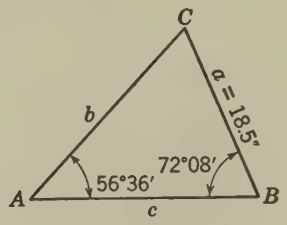


Fig. 261

SOLUTION. $C = 180° - (A + B)$

$= 180° - (56° 36' + 72° 08') = 51° 16'$

By the law of sines we have

$$\frac{a}{\sin A} = \frac{b}{\sin B}. \quad \therefore b = \frac{a \sin B}{\sin A} \qquad \frac{a}{\sin A} = \frac{c}{\sin C}. \quad \therefore c = \frac{a \sin C}{\sin A}$$

$$b = \frac{18.5 \times .95177}{.83485} = 21.09'' \qquad c = \frac{18.5 \times .78007}{.83485} = 17.29''$$

EXPLANATION. Since the three angles add up to 180°, to find C we subtract the sum of A and B from 180°.

To find side b we choose the formula based on the law of sines which contains the three given elements, a, A, and B, and the unknown element b. That formula is $\frac{a}{\sin A} = \frac{b}{\sin B}$. Solving this equation for b, we get $b = \frac{a \sin B}{\sin A}$.

Similarly, to find c we select the formula based on the law of sines which contains the known elements a, A, and C and the unknown element c. That formula is $\frac{a}{\sin A} = \frac{c}{\sin C}$. Solving this formula for c, we get $c = \frac{a \sin C}{\sin A}$. The numerical values of b and c are obtained by substituting in these equations the values of $\sin A$ and $\sin C$ found in the table of natural functions and performing the indicated operations shown above.

PROBLEMS

Solve the following triangles. (To solve a triangle means to find all the missing parts.)

1. Given $a = 2.8''$, $A = 26° 40'$, $B = 68° 24'$
2. Given $a = 8.25''$, $A = 48° 50'$, $B = 70° 38'$
3. Given $b = 6.5''$, $A = 16° 45'$, $B = 54° 15'$
4. Given $b = 14.6''$, $B = 35° 28'$, $C = 95° 12'$
5. Given $c = 10.2''$, $A = 86° 55'$, $C = 48° 32'$
6. Given $c = 3.6''$, $B = 102° 10'$, $C = 38° 24'$
7. Given $c = 12.4''$, $A = 59° 35'$, $B = 63° 46'$
8. Given $b = 7.2''$, $C = 88° 37'$, $A = 17° 23'$
9. Given $b = 22''$, $B = 56° 44'$, $C = 76° 38'$
10. Given $a = 18''$, $A = 62° 52'$, $C = 91° 18'$

199. Case II. Given Two Sides and the Angle between Them

Example

Given $a = 3.7''$, $b = 1.8''$, and $C = 56° 50'$.

Find A, B, and c, Fig. 262.

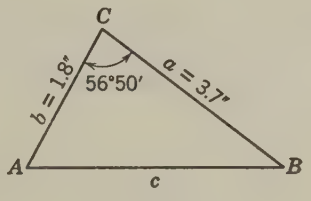


Fig. 262

To find c we use the law of cosines.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= 3.7^2 + 1.8^2 - 2 \times 3.7 \times 1.8 \times .54708$$

$$= 13.69 + 3.24 - 7.2871 = 9.6429$$

$$\therefore c = \sqrt{9.6429} = 3.105''$$

To find B we use the law of sines.

$$\frac{b}{\sin B} = \frac{c}{\sin C}. \quad \therefore \sin B = \frac{b \sin C}{c} = \frac{1.8 \times .83708}{3.105} = .48526$$

$$\therefore \text{Angle } B = 29° 02'$$

$$A = 180° - (B + C)$$

$$= 180° - (29° 02' + 56° 50') = 94° 08'$$

PROBLEMS

Solve the following triangles.

1. Given $a = 8''$, $b = 6''$, $C = 55° 45'$
2. Given $a = 16.5''$, $c = 22''$, $B = 48° 32'$
3. Given $b = 7.2''$, $c = 12.4''$, $A = 72° 20'$
4. Given $a = 18''$, $c = 25''$, $B = 14° 12'$
5. Given $a = 32'$, $b = 48'$, $C = 26° 30'$
6. Given $b = 20''$, $c = 30''$, $A = 92° 18'$
7. Given $a = 8''$, $b = 7''$, $C = 63° 28'$
8. Given $a = 4.8''$, $b = 6.2''$, $C = 34° 40'$
9. Given $a = 1.6''$, $c = 2.4''$, $B = 16° 15'$
10. Given $b = 5.6''$, $c = 7.8''$, $A = 65° 50'$

200. Case III. Given the Three Sides

Example

Given $a = 18''$, $b = 12''$, $c = 15''$.

Find A, B, and C, Fig. 263.

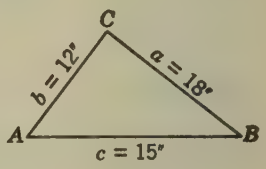


Fig. 263

To find the angles we use the law of cosines.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{12^2 + 15^2 - 18^2}{2 \times 12 \times 15}$$

$$= \frac{144 + 225 - 324}{360} = \frac{45}{360} = .12500$$

$$\therefore \text{Angle } A = 82° 49'$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{18^2 + 15^2 - 12^2}{2 \times 18 \times 15}$$

$$= \frac{324 + 225 - 144}{540} = \frac{405}{540} = .75000$$

$$\therefore \text{Angle } B = 41^\circ 25'$$

$$C = 180^\circ - (A + B)$$

$$= 180^\circ - (82^\circ 49' + 41^\circ 25') = 55^\circ 46'$$

PROBLEMS

Find the angles in the following triangles.

1. $a = 3''$, $b = 4''$, $c = 5''$
2. $a = 6.5''$, $b = 7.4''$, $c = 9.2''$
3. $a = 2.4''$, $b = 1.4''$, $c = 1.6''$
4. $a = 31''$, $b = 25''$, $c = 48''$
5. $a = 19''$, $b = 36''$, $c = 28''$
6. $a = 8''$, $b = 8''$, $c = 8''$
7. $a = 9''$, $b = 9''$, $c = 12''$
8. $a = 10.2''$, $b = 12.6''$, $c = 15.5''$
9. $a = 72'$, $b = 84'$, $c = 96'$
10. $a = 15''$, $b = 20''$, $c = 25''$

201. Case IV. Given Two Sides and an Angle Opposite One of Them

In this case there are three possibilities:

1. No solution, if A is acute and a is less than the perpendicular from the apex to the base, that is, less than $b \sin A$, Fig. 264(a); or if A is obtuse and a is less than b, Fig. 264(b); or if $a = b$.

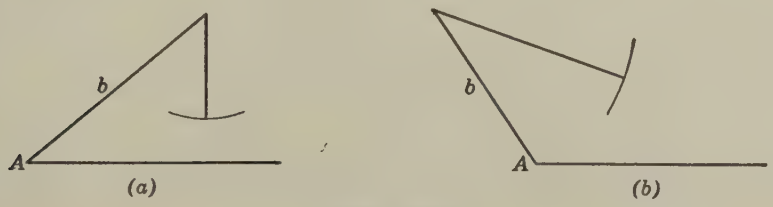


Fig. 264

2. Two solutions, if A is acute and a is less than b and greater than $b \sin A$, Fig. 265.

3. One solution in all other cases.

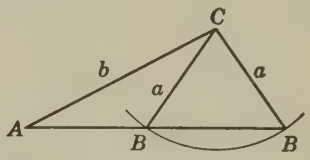


Fig. 265

Usually the number of solutions can be determined by inspection or by constructing the triangle. In case of doubt, find the value of $b \sin A$ and apply the test as given above.

Example 1

Given: $a = 14''$, $b = 18''$, $A = 94°$. Find B, C, and c.

In this case A is obtuse and a is less than b; therefore the triangle is impossible.

Example 2

Given: $a = 40''$, $b = 90''$, $A = 28°$. Find B, C, and c.

In this case $b \sin A = 90 \times .46947 = 42.252$. Since a is less than $b \sin A$, the triangle is impossible.

Example 3

Given: $a = 25''$, $b = 31''$, $A = 32°$.

Find B, C, and c, Fig. 266.

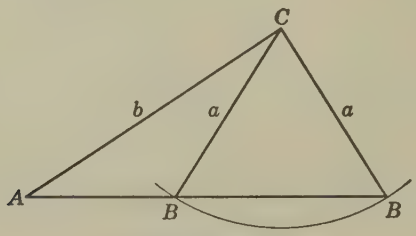


Fig. 266

SOLUTION.

$b \sin A = 31 \times .52992 = 16.4275''$. In this case a lies between b and $b \sin A$; hence there will be two solutions.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\therefore \sin B = \frac{b \times \sin A}{a} = \frac{31 \times .52992}{25} = .65710$$

$$\therefore B = 41^\circ 05'$$

Also $B = 180^\circ - 41^\circ 05' = 138^\circ 55'$, because, when an angle is determined by its sine, it admits of two values, one of which is the angle in the table corresponding to the given sine and the other value is the supplement of the angle in the table. Both values of B, therefore, are to be taken unless the conditions of the problem are such as to exclude one of them. In this case both values are possible.

Since there are two values of B, there will be two values of C, namely:

$$C = 180^\circ - (A + B) = 180^\circ - (32^\circ + 41^\circ 05')$$

$$= 106^\circ 55'$$

or

$$C = 180^\circ - (32^\circ + 138^\circ 55') = 9^\circ 05'$$

To find c, we have

$$\frac{a}{\sin A} = \frac{c}{\sin C}. \quad \therefore c = \frac{a \sin C}{\sin A},$$

$$c = \frac{25 \times \sin 9^\circ 05'}{\sin 32^\circ} = \frac{25 \times .15787}{.52992} = 7.45''$$

or

$$c = \frac{25 \times \sin 106^\circ 55'}{\sin 32^\circ} = \frac{25 \times .95673}{.52992} = 45.14''$$

Example 4

Given: $a = 8.4''$, $b = 3.2''$, $A = 34° 28'$.

Find B, C, and c, Fig. 267.

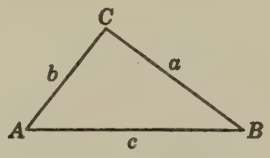


Fig. 267

$$\frac{a}{\sin A} = \frac{b}{\sin B}. \quad \therefore \sin B = \frac{b \sin A}{a} = \frac{3.2 \times .56593}{8.4} = .21559$$

$$\therefore B = 12° 27'$$

$$C = 180° - (A + B)$$

$$= 180° - (34° 28' + 12° 27') = 133° 05'$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}. \quad \therefore c = \frac{a \sin C}{\sin A} = \frac{8.4 \times .73036}{.56593} = 10.84$$

With the above data, we find we can construct but one triangle; that is, the problem has only one solution.

PROBLEMS

Solve the following triangles.

1.	$a = 24''$,	$b = 30''$,	$A = 108° 06'$
2.	$a = 45''$,	$b = 120''$,	$A = 32° 20'$
3.	$a = 8''$,	$b = 10''$,	$A = 91° 30'$
4.	$a = 15''$,	$b = 42''$,	$A = 24° 15'$
5.	$a = 18''$,	$b = 24''$,	$A = 36° 30'$
6.	$a = 50''$,	$b = 60''$,	$A = 42° 40'$
7.	$a = 35''$,	$b = 48''$,	$A = 29° 45'$
8.	$a = 7.5''$,	$b = 12''$,	$A = 52° 10'$
9.	$a = 6.2''$,	$b = 9.1''$,	$A = 18° 45'$
10.	$a = 8.4''$,	$b = 10.8''$,	$A = 24° 12'$
11.	$a = 10.6''$,	$b = 7.9''$,	$A = 65° 30'$
12.	$a = 22''$,	$b = 19''$,	$A = 54° 50'$
13.	$a = 4.8''$,	$b = 3.4''$,	$A = 42° 12'$
14.	$a = 5.4''$,	$b = 5.0''$,	$A = 96° 20'$
15.	$a = 3.2''$,	$b = 2.5''$,	$A = 72° 16'$

202. Area of Triangles

Case I

We know from geometry that the area of a triangle = $\frac{1}{2} \times$ base $\times$ altitude. In Fig. 268(a) area $= \frac{1}{2} \times c \times h$. But $\frac{h}{a} = \sin B$. $\therefore\ h = a \sin B$. In Fig. 268(b) area $= \frac{1}{2} \times c \times h$. But $\frac{h}{a} = \sin (180° - B) = \sin B$. $\therefore\ h = a \sin B$.

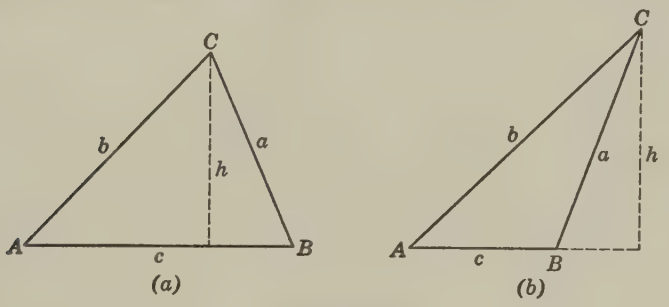


Fig. 268

Substituting for h its value, $a \sin B$, we get

$$\text{Area} = \tfrac{1}{2}ac \sin B$$

Similarly it may be shown that

$$\text{Area} = \tfrac{1}{2}ab \sin C; \quad \text{or,} \quad \text{Area} = \tfrac{1}{2}bc \sin A$$

Example

Find the area of a triangle if $a = 8''$, $b = 6''$, and $C = 63° 10'$.

SOLUTION.

$$\text{Area} = \tfrac{1}{2}ab \sin C = \tfrac{1}{2} \times 8 \times 6 \times .89232 = 21.42 \text{ sq. in.}$$

Case II. Given Two Angles and the Included Side

In this case we find the third angle by subtracting the sum of the two given angles from 180°. Then, by the law of sines, we obtain the length of the other side. We then apply the formula,

$$\text{Area} = \tfrac{1}{2}ab \sin C$$

Case III. Given the Three Sides

The solution in this case is explained in the chapter on Mensuration. See page 79.

PROBLEMS

Find the area of the following triangles.

1. $a = 28''$, $b = 21''$, $C = 92° 16'$
2. $a = 30.6''$, $b = 25''$, $C = 84° 20'$
3. $a = 16''$, $c = 14''$, $B = 65° 42'$
4. $b = 10''$, $c = 12''$, $A = 29° 34'$
5. $b = 6''$, $c = 8''$, $A = 50° 50'$
6. $a = 14.5''$, $c = 17.2''$, $B = 71° 26'$
7. $b = 5.9''$, $c = 6.8''$, $A = 42° 28'$
8. $a = 3.1''$, $b = 4.5''$, $C = 37° 14'$
9. $a = 13.5''$, $c = 19.3''$, $B = 62° 12'$
10. $b = 16.4''$, $c = 18.2''$, $A = 48° 18'$

15 Strength of Materials

203. Stress and Strain

When a load of 100 lb. is suspended by a wire, the load tends to stretch or elongate the wire. This tendency of the load to *distort* or *change* the shape of the wire is called ***strain.*** The wire resists the tendency of the load to pull it apart, and in so doing exerts a force opposite to the load. The internal force by which the wire resists the tendency of the outside load to change its shape is called ***stress.***

Similarly a load on a column tends to compress or crush the column. The column reacts against the tendency of the load to crush it and exerts a force opposite to the load. The tendency of the outside load to change the shape of the column is called *strain,* and the internal force by which the column resists the tendency of the outside load to change its shape is called *stress.*

204. Kinds of Stresses

Three kinds of direct stress may be developed in a body by the application of an outside force or load; namely, ***tension, compression,*** and ***shear.***

Tension

A weight suspended from a rod tends to pull the rod apart. The stress developed in the rod to resist being *pulled* apart is called ***tensile stress.*** The rod is said to be in ***tension.***

Compression

A load on a column tends to crush it. The stress which the column develops to resist being *crushed* is called ***compressive stress.*** The column is said to be in ***compression.***

Shear

A rivet connecting two plates will sometimes give way because the plates push so hard on the rivet in opposite directions as to cause one half of the rivet to slide past the other; that is, the two halves of the rivet are separated as if the rivet were cut by a pair of shears. See Fig. 269. The stress which the rivet develops to resist being cut apart is called ***shearing stress.*** The rivet is said to be in ***shear.***

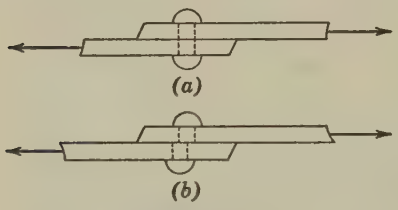


Fig. 269

Bending stress is a combination of tension and compression.

Twisting or ***torsion*** is a form of shearing stress.

205. Unit Stress

The unit stress is the stress per unit of area and is usually expressed in pounds per square inch. Thus, if a column whose cross-section area is 6 sq. in. supports a load of 60,000 lb., the unit stress is $\frac{60000}{6}$ or 10,000 lb. per sq. in.

206. Elastic Limit

If a suspended rod with a cross-section area of 1 sq. in. be loaded with a weight, the latter stretches the rod a certain amount, depending upon the length of the rod and the load applied. If too great a load is not used, the rod will spring back to its original length when the load is removed. When a sufficiently heavy load is hung from the rod, the rod will not spring back to its original length when the load is removed. There remains a permanent set.

The elastic limit is the unit stress beyond which the body will not return to its original shape when the load is removed.

Thus, if a steel rod of 1 sq. in. cross-section area from which a load of 35,000 lb. is suspended springs back to its original length when the load is removed but acquires a permanent set when a load of 36,000 lb. is applied, we say the elastic limit of that rod is 35,000 lb.

207. Ultimate Strength

By the *ultimate* or *breaking strength* of a body is meant the unit stress which causes that body to break. Thus it takes almost 50,000 lb. to pull apart a wrought iron bar with a cross-section area of 1 sq. in. We therefore say that the ultimate tensile strength of wrought iron is 50,000 lb. per sq. in.

It is necessary for the engineer to know the strength of the materials that enter into the machine or structure he is designing. To determine the ultimate strength of the materials in common use, specimens are subjected to tensile, compressive, or shearing stresses in testing machines; and the breaking point in each instance is noted. In this way the average values of the elastic limits and ultimate strengths in pounds per square inch were obtained for the following materials:

ELASTIC LIMIT AND ULTIMATE STRENGTH

Material	Elastic Limit	Ultimate Strength		
		Tension	Compression	Shear
Timber	3,000	8,000	8,000	500 (With grain) 3,000 (Across grain)
Stone			6,000	1,500
Brick		250–300	3,000	1,000
Cast iron	6,000 (Tension) 20,000 (Comp.)	25,000	90,000	18,000
Wrought iron	28,000	50,000	55,000	40,000
Medium steel	36,000	60,000	60,000	50,000
Concrete, Portland 1:2:4		100–150	2,400	300–500

208. Factor of Safety

Though the ultimate compressive strength of steel is 60,000 lb. per sq. in., in designing a machine or structure the engineer will design each member on the assumption that the material can develop a compressive stress of only 15,000 lb. per sq. in. In other words he will make each member of the machine or structure four times as strong as is required in order to keep it from breaking down when loaded. The ratio of the ultimate strength to the actual stress is called the ***safety factor.*** In the example cited above, the safety factor is $\frac{60{,}000}{15{,}000} = 4$. The safety factor ranges from 4 for steel, designed for steady stress, to 35 for brick, designed to withstand shock. A larger safety factor is assumed in the design of a railroad bridge than in the design of an office building, because the moving load on a railroad bridge causes the structure to vibrate, whereas in the case of the building, the load is steady.

209. Working Unit Stresses

As a result of investigation and experience the following values have been adopted as safe working stresses for the materials most commonly employed. In each case it will be noticed that the working unit stress is well within the elastic limit.

SAFE WORKING STRESSES—POUNDS PER SQUARE INCH

Material	Tension	Compression	Shear
Timber	750	750	500
Cast iron	4,000	15,000	3,000
Wrought iron	12,000	12,000	10,000
Medium steel	15,000	15,000	10,000

Example 1

How many pounds can be safely carried in tension by a steel rod whose cross-section is a rectangle $2'' \times \frac{3}{4}''$?

Solution. Area of cross-section $= 2 \times \frac{3}{4} = 1\frac{1}{2}$ sq. in.

Carrying capacity in tension $= 1\frac{1}{2} \times 15{,}000 = 22{,}500$ lb.

Example 2

Allowing a tensile stress of 15,000 lb. per sq. in., find the diameter of a steel rod to carry 100,000 lb.

SOLUTION.

$$\text{Required area of cross-section} = \frac{100{,}000}{15{,}000} = 6.6667 \text{ sq. in.}$$

$$\text{Diameter} = 2\sqrt{\frac{\text{area}}{\pi}} = 2\sqrt{\frac{6.6667}{3.1416}} = 2\sqrt{2.1221}$$

$$= 2 \times 1.46 = 2.92''$$

Example 3

Allowing a tensile stress of 15,000 lb. per sq. in., how many pounds can be safely carried by a steel rod $\frac{7}{8}''$ in diameter?

SOLUTION.

$$\text{Area of cross-section} = .7854 \times .875^2 = .6013 \text{ sq. in.}$$

$$\text{Safe carrying capacity} = 15000 \times .6013 = 9020 \text{ lb.}$$

PROBLEMS

1. A steel rod has a rectangular cross-section $3\frac{1}{2}'' \times 1\frac{3}{4}''$. Allowing a tensile stress of 15,000 lb. per sq. in., how many pounds can it carry?

2. How many pounds can be safely carried in tension by a steel rod $1\frac{7}{8}''$ in diameter?

3. What must be the diameter of a steel bar to carry safely a tensile stress of 65,000 lb.?

4. What should be the diameter of a steel bar to carry a load of 60,000 lb., allowing a tensile stress of 16,000 lb. per sq. in.?

5. What should be the diameter of a steel bar to carry safely a load of 72,000 lb. in tension, allowing 15,000 lb. per sq. in.?

6. A balcony is supported from the ceiling by means of six wrought iron rods. Allowing a tensile stress of 12,000 lb. per sq. in., what size rods must be used if the total weight is 80,000 lb.?

7. Allowing a stress of 10,000 lb. per sq. in., what is the shearing strength of an iron bar $2\frac{1}{2}'' \times \frac{7}{16}''$ in cross-section?

8. The ends of a floor beam carrying a total load of 6400 lb. uniformly distributed rest on cast iron plates placed on the brick. If the plates are $8'' \times 10''$, what is the pressure per square inch on the brick?

9. A loading platform is supported by 6 square sticks of timber. What must be the size of the supports if the platform is to carry 120,000 lb. and the allowable compressive stress is 500 lb. per sq. in.?

10. What is the ultimate compressive strength in pounds per square inch of a brick $8'' \times 4'' \times 2''$, if it gave way in the testing machine under a load of 56,000 lb. when lying flat?

210. Pressure in Pipes

The tendency of steam or water pressure in a pipe is to burst the pipe longitudinally. The material of the pipe is therefore under tension when the pipe is under pressure. The total pressure on a diametral plane is equal to

$$P = p \times L \times D$$

where p = pressure per square inch
L = length of pipe in inches
D = diameter of pipe in inches

The tendency of the steam or water pressure to burst the pipe is resisted by the tension in the walls of the pipe. See Fig. 270. The total resisting stress in the pipe is equal to

$$S = 2 \times s \times L \times t$$

where s = unit tensile stress
t = thickness of pipe

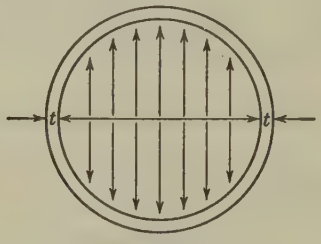


Fig. 270

For the pipe to withstand bursting, the resisting stress must equal the total pressure. We have, therefore,

$$2 \times s \times L \times t = p \times L \times D$$

$$\therefore t = \frac{p \times L \times D}{2 \times s \times L} = \frac{p \times D}{2 \times s}$$

This formula enables us to find the required thickness of pipe to withstand a given pressure.

To find the safe internal unit pressure in a pipe of given thickness, the preceding formula is solved for p, giving,

$$p = \frac{2 \times s \times t}{D}$$

Example

Allowing a tensile stress of 5000 lb. per sq. in., how thick should a 20″ wrought iron steam pipe be to withstand a pressure of 300 lb. per sq. in.?

SOLUTION. $t = \dfrac{p \times D}{2 \times s} = \dfrac{300 \times 20}{2 \times 5000} = .6''$, thickness of pipe required. The next higher standard thickness is used.

PROBLEMS

1. How thick must a 15″ steam pipe be to withstand a steam pressure of 250 lb. per sq. in. with a unit tensile stress of 5000 lb. per sq. in.?
2. What may be the maximum pressure per sq. in. in a steam pipe $\frac{3}{4}''$ thick if its diameter is 16″? Allow a unit tensile stress of 5500 lb. per sq. in.
3. A cast iron water pipe 30″ in diameter is to withstand a pressure of 75 lb. per sq. in. Allowing a tensile stress of 2000 lb. per sq. in. how thick must the pipe be?
4. Allowing a unit tensile stress of 2000 lb. per sq. in., what is the maximum allowable pressure per sq. in. in a cast iron pipe 15″ in diameter and $\frac{5}{8}''$ thick?
5. Find the maximum allowable pressure per square inch in a steam pipe $\frac{7}{8}''$ thick and 20″ in diameter, allowing a tensile stress of 2500 lb. per square inch.
6. What must be the thickness of a steam pipe 18″ in diameter to withstand a pressure of 100 lb. per square inch if the unit tensile stress is 3500 lb. per square inch?

211. Riveted Joints

A riveted joint may fail in one of two ways: either the rivets may be sheared off or the plates may be crushed by the pressure of the rivets. In the former case the area to be

sheared is the cross-section area of the rivet. The safe load on the rivet for this type of stress is called the ***shearing value*** of the rivet.

In order to fail by tearing through the plate, the rivet must crush an area which is a rectangle formed by the thickness of the plate and the diameter of the rivet. The safe load per rivet for this type of stress is called the ***bearing value*** of the rivet. The common unit stresses allowed for steel plates and rivets are: 10,000 lb. per sq. in. for shear and 15,000 lb. per sq. in. for bearing.

Figure 271 shows three types of riveted joints. In (*a*) and (*c*) the rivets are said to be in ***single shear,*** because they

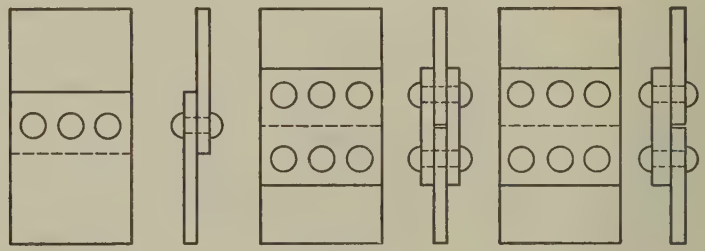


Fig. 271

may be destroyed by shearing at one place only. In (*b*) the rivets are in ***double shear,*** because they must be sheared off in two places at the same time in order to fail.

Example 1

Figure 272(*a*) shows two plates each $\frac{1}{2}''$ thick connected by one rivet $\frac{7}{8}''$ in diameter. Find the strength of the joint.

SOLUTION.

Cross-section area of rivet $= 3.14 \times \frac{7}{16} \times \frac{7}{16} = .601$ sq. in.

Shearing value of rivet $= .601 \times 10{,}000 = 6010$ lb.

Bearing area of rivet $= \frac{7}{8} \times \frac{1}{2} = \frac{7}{16} = .438$ sq. in.

Bearing value of rivet $= .438 \times 15{,}000 = 6570$ lb.

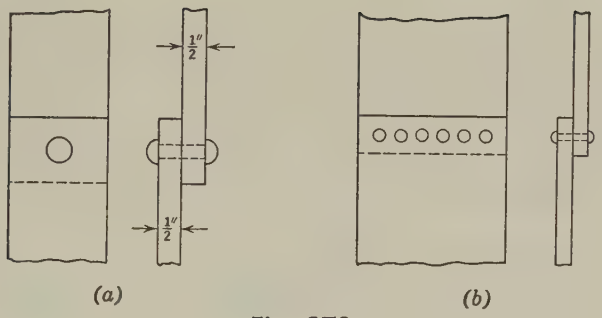


Fig. 272

The smallest value determines the strength of the joint, which is 6010 lb.

Example 2

If the plates in Fig. 272(*b*) are connected by 6 rivets each $\frac{7}{8}''$ in diameter, what is the strength of the joint?

SOLUTION. $6 \times 6010 = 36{,}060$ lb.

EXPLANATION. Since one rivet is good for 6010 lb., 6 rivets will carry 6×6010, or 36,060 lb.

Example 3

How many $\frac{7}{8}''$ rivets are required to carry a stress of 100,000 lb. through the two plates in Fig. 272(*b*)?

SOLUTION. $\dfrac{100{,}000}{6010} = 16+$; or 17 rivets.

EXPLANATION. The number of rivets required is found by dividing the total stress to be carried by the lowest value of one rivet.

Example 4

In Fig. 273 the inside plate is $\frac{3}{4}''$ thick and the outside plates are each $\frac{1}{2}''$ thick. The rivet is $\frac{7}{8}''$ in diameter. What is the strength of the joint?

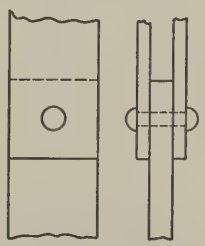


Fig. 273

SOLUTION.

Cross-section area of rivet $= 3.14 \times \frac{7}{16} \times \frac{7}{16} = .601$ sq. in.

Shearing value of rivet (double shear)
$= 2 \times .601 \times 10{,}000 = 12{,}020$ lb.

Bearing area of rivet $= \frac{7}{8} \times \frac{3}{4} = \frac{21}{32} = .656$ sq. in.

Bearing value of rivet $= .656 \times 15{,}000 = 9840$ lb.

The strength of the joint, as determined by the lowest value of the rivet, is 9840 lb.

CROSS-SECTION AREA OF RIVETS

DIAMETER, INCHES	AREA, SQUARE INCHES	DIAMETER, INCHES	AREA, SQUARE INCHES
$\frac{3}{8}''$	.1104	$\frac{3}{4}''$	.4418
$\frac{1}{2}''$	.1963	$\frac{7}{8}''$	.6013
$\frac{5}{8}''$	.3068	$1''$	.7854

PROBLEMS

1. Two $\frac{5}{8}''$ plates are connected by a lap joint with $\frac{7}{8}''$ rivets. What is the strength of the joint if 12 rivets are used?

2. Find the strength of the joint shown in Fig. 274(*a*) if eight $\frac{3}{4}''$ rivets are used.

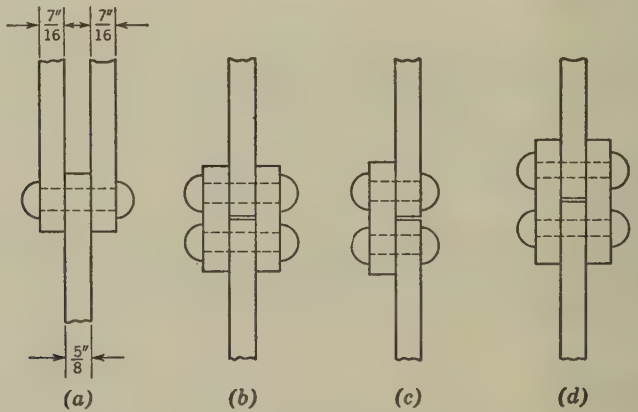


Fig. 274

3. How many $\frac{5}{8}''$ rivets will be required on each side of the joint shown in Fig. 274(*b*) if all plates are $\frac{9}{16}''$ thick and the stress to be developed is 60,000 lb.?

4. Find the strength of the joint shown in Fig. 274(*c*) if ten $\frac{1}{2}''$ rivets are used on each side of the butt joint. All plates are $\frac{1}{2}''$ thick.

5. Find the strength of the joint shown in Fig. 274(*d*) if the plates are $\frac{1}{2}''$ thick and the straps $\frac{9}{16}''$ thick. Eight $\frac{3}{4}''$ rivets are used on each side of the butt joint.

6. Compute the strength of the joint in Problem 1: (*a*) if eighteen $\frac{1}{2}''$ rivets are used; (*b*) sixteen $\frac{5}{8}''$ rivets; (*c*) fourteen $\frac{3}{4}''$ rivets; (*d*) ten $1''$ rivets.

7. (*a*) How many $\frac{7}{8}''$ rivets are required to transmit a stress of 15,000 lb. through a lap joint of $\frac{3}{4}''$ plates? (*b*) How many $\frac{1}{2}''$ rivets? (*c*) How many $\frac{5}{8}''$ rivets? (*d*) How many $\frac{3}{8}''$ rivets? (*e*) How many $1''$ rivets?

8. A single strap butt joint of two $\frac{1}{2}''$ plates is required to transmit 75,000 lb. (*a*) How many $\frac{1}{2}''$ rivets are required if the strap is $\frac{1}{2}''$ thick? (*b*) How many $\frac{5}{8}''$ rivets if the strap is $\frac{9}{16}''$ thick? (*c*) How many $\frac{3}{4}''$ rivets with a $\frac{1}{2}''$ strap? (*d*) How many $\frac{7}{8}''$ rivets with a $\frac{7}{16}''$ strap? (*e*) How many $1''$ rivets with a $\frac{1}{2}''$ strap?

9. Two $1\frac{1}{16}''$ plates are connected by a double strap butt joint with $\frac{1}{2}''$ straps. (*a*) What is the strength of such a joint with eight $\frac{7}{8}''$ rivets each side of the butt joint? (*b*) Compute the strength of this joint with twenty-four $\frac{1}{2}''$ rivets on each side of the butt joint. (*c*) Twenty $\frac{5}{8}''$ rivets on each side. (*d*) Eight $1''$ rivets on each side.

10. (*a*) How many $\frac{3}{4}''$ rivets are required on each side of the butt joint in Problem 9 if a stress of 90,000 lb. is to be transmitted? (*b*) How many $\frac{1}{2}''$ rivets? (*c*) How many $\frac{5}{8}''$ rivets? (*d*) How many $1''$ rivets?

16 Work and Power

212. Work and Power

When we lift a weight of 1 lb. a height of 1 ft., we do *one foot-pound* of work. If we exert a pull of 50 lb. in dragging a load a distance of 10 ft., we do 10×50 or 500 foot-pounds of work.

A ***foot-pound*** is the unit of work. It is the work done in raising 1 lb. a height of 1 ft., or it is the pressure of 1 lb. exerted over a distance of 1 ft. in any direction.

A freight elevator lifting 2000 lb. a height of 40 ft. does 80,000 foot-pounds of work whether it takes 2 minutes or 4 minutes to do it. But an elevator that would lift the load in 2 minutes would have twice the power of one that would take 4 minutes to do the same amount of work.

Power is the rate at which work is done and is measured in foot-pounds per minute. Thus, if an elevator did 80,000 foot-pounds of work in 2 minutes, its power would be $\frac{80000}{2}$ or 40,000 foot-pounds per minute. If it took 4 minutes to do it, its power would be $\frac{80000}{4}$ or 20,000 foot-pounds per minute.

The unit of power is the ***horsepower,*** which is equal to 33,000 foot-pounds per minute.

Example

An engine does 120,000 ft-lb. of work in 3 minutes. Compute the horsepower.

SOLUTION. $\text{Horsepower} = \dfrac{120{,}000}{3 \times 33{,}000} = 1.2.$

EXPLANATION. Since it takes 3 minutes to do 120,000 ft-lb. of work, in 1 minute it will do $\frac{1}{3}$ of 120,000 or 40,000 ft-lb. of work. Since 33,000 ft-lb. = 1 H.P., we divide by 33,000.

PROBLEMS

1. A man carries a load of 50 lb. up a flight of stairs 10 ft. high. Compute the work done in foot-pounds.

2. Compute the horsepower required for an elevator to lift 2400 lb. a height of 120 ft. in $1\frac{3}{4}$ minutes.

3. Find the horsepower of an engine that pumps 30 cu. ft. of water per minute from a depth of 320 ft. Water weighs 62.5 lb. per cubic foot.

4. What must be the horsepower of an engine to lift a girder weighing 7 tons a height of 40 ft. in 5 minutes?

5. A man in pushing a wheelbarrow a distance of 450 ft. exerts a pressure of 45 lb. How much work does he do?

6. An engine does 84,000 ft-lb. of work in $2\frac{1}{2}$ minutes. Compute the horsepower.

213. Horsepower of a Steam Engine

In speaking of the horsepower of a steam engine, the *indicated horsepower* is meant. The formula for finding the indicated horsepower is

$$H.P. = \frac{P \times L \times A \times N}{33{,}000}$$

in which $H.P.$ = indicated horsepower
P = mean effective pressure on the piston in pounds per square inch (P is found by means of an indicator card.)
L = length of stroke in feet
A = area of piston in square inches
N = number of strokes per minute = 2 × r.p.m. for a double-acting engine

Example

Find the horsepower of a $26'' \times 40''$ double-acting engine running at 80 r.p.m. under a mean effective pressure of 75 lb. per square inch.

NOTE. In specifying the dimensions of a cylinder, the first number gives the diameter or bore; the second number, the stroke.

SOLUTION. $H.P. = \dfrac{P \times L \times A \times N}{33{,}000}$

$P = 75$, $L = 40'' = 3\frac{1}{3}'$, $A = 26^2 \times .7854 = 530.93$ sq. in.

$N = 80 \times 2 = 160$

$$H.P. = \frac{75 \times 10 \times 530.93 \times 160}{3 \times 33{,}000} = 642.5 \text{ H.P.}$$

PROBLEMS

1. What is the indicated horsepower of an $18'' \times 28''$ double-acting steam engine making 100 r.p.m. with a mean effective pressure of 48 lb.?

2. Compute the horsepower of a $32'' \times 50''$ double-acting steam engine running at 75 r.p.m. with a mean effective pressure of 55 lb.?

3. A $24'' \times 40''$ double-acting engine is running at 80 r.p.m. under a mean effective pressure of 50 lb. Find the horsepower.

4. Find the horsepower of a $22'' \times 26''$ double-acting steam engine making 75 r.p.m. under a mean pressure of 110 lb.

5. Compute the horsepower of a double-acting steam engine with two $16'' \times 30''$ cylinders, if the engine is making 140 r.p.m. under a mean effective pressure of 50 lb.

214. Horsepower of Gas Engines

There are many formulas for computing the horsepower of a gas engine. The standard formula for the rated horsepower of a 4-cycle gas engine is that of the Society of Automotive Engineers, formerly the American Licensed Automobile Manufacturers. This formula, now known as the

N.A.C.C. (National Automobile Chamber of Commerce) formula, is as follows:

$$H.P. = \frac{D^2 \times N}{2.5} = 0.4 \times D^2 \times N$$

in which D = diameter of cylinders, in inches
N = number of cylinders

This formula is based on a piston speed of 1000 ft. per minute and a mean effective pressure of 90 lb. per square inch.

Example

Find the horsepower of a $4\frac{1}{2}'' \times 6''$, 4-cycle, 4-cylinder gas engine.

SOLUTION.

$$H.P. = 0.4 \times D^2 \times N = .4 \times 4.5^2 \times 4 = 32.4 \text{ H.P.}$$

PROBLEMS

1. Find the rated horsepower of a 6-cylinder automobile engine if the cylinders have a $3\frac{5}{16}''$ bore and a $3\frac{1}{2}''$ stroke.

2. A gas engine with $3\frac{1}{4}''$ bore and $4\frac{1}{2}''$ stroke has 8 cylinders. What is the rated or taxable horsepower?

3. Find the rated horsepower of a 12-cylinder engine with a bore of $3\frac{1}{8}''$.

4. A certain V-8 engine has cylinders $3\frac{1}{16}'' \times 3\frac{3}{4}''$. What is the rated horsepower?

5. If the bore of the engine in Problem 4 is increased by $\frac{1}{8}''$, what increase in horsepower will result?

6. Find the horsepower of a 16-cylinder $3'' \times 4''$ engine.

7. Find the horsepower of a 4-cylinder $3\frac{1}{8}'' \times 4\frac{3}{8}''$ engine.

8. How many additional horsepower will be developed by the engine in Problem 7 if the bore is increased by $\frac{3}{16}''$?

215. Brake Horsepower

The brake horsepower is the *power delivered by an engine at its flywheel,* or the power actually available for use. The brake horsepower of an engine may be determined by the

use of a device known as the ***Prony Brake,*** Fig. 275. This consists of a steel or leather belt carrying a number of wooden blocks or shoes clamped around the pulley by means of the bolt B. The bolt enables us to vary the friction between the shoes and the rim of the pulley. The pull developed by this friction at a distance L from the shaft is measured by

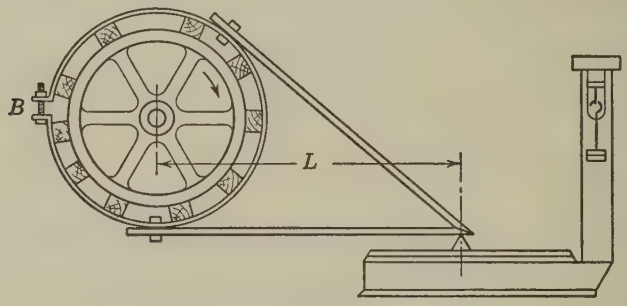


Fig. 275

the platform scale on which the lever rests. Allowance must be made for the weight of the lever itself. The formula for determining brake horsepower is

$$BHP = \frac{2 \times \pi \times L \times N \times W}{33{,}000}$$

in which L = length of lever arm in feet
N = number of revolutions per minute
W = net pressure = scale reading minus weight of lever

Example

Find the brake horsepower of an engine determined by a Prony brake where L is 4 ft., the engine is running at 400 r.p.m., and the net pressure on the scale is 30 lb.

SOLUTION. $$BHP = \frac{2 \times \pi \times L \times N \times W}{33{,}000}$$

$$= \frac{2 \times 3.1416 \times 4 \times 400 \times 30}{33{,}000} = 9.14 \text{ H.P.}$$

PROBLEMS

1. Calculate the brake horsepower of a gas engine running at 450 r.p.m., if the net scale reading is 25 lb. and the lever arm is 4′ 6″ long.

2. What is the brake horsepower of a steam engine making 120 r.p.m. if the Prony brake lever is 5′ 0″ long and the net pressure on the scale is 28 lb.?

3. The brake horsepower of an engine making 60 r.p.m. was determined by a Prony brake having a lever arm of 3′ 4″. Compute the brake horsepower if the net pressure on the scale was 40 lb.

4. Compute the brake horsepower of a gas engine as determined by a Prony brake having a lever arm of 3′ 3″, a net pressure on the scale of 20 lb., and a speed of 400 r.p.m.

5. An engine making 80 r.p.m. is tested with a Prony brake having a lever arm of 3′ 6″. What is the brake horsepower if the net pressure on the scale is 28 lb.?

216. Electrical Power. Watt

The unit of electrical power is the *watt*. A ***watt*** is the power used when 1 volt causes 1 ampere of current to flow. The watts consumed in any circuit are found by multiplying the amperes (current) in the circuit by the volts (pressure) impressed on the circuit.

Kilowatt

The watt being too small a unit for convenience in computation, we use the ***kilowatt,*** which is equal to ***1000*** watts. In electrical power machines ***1 horsepower is equal to 746 watts,*** or in round numbers, 750 watts, which is equal to $\frac{3}{4}$ of a kilowatt.

Electrical Horsepower

The formula for the horsepower of an electric motor is

$$H.P. = \frac{\text{Amperes} \times \text{Volts}}{750}.$$

Example 1

A generator delivers a current of 50 amperes at a pressure of 110 volts. What power in kilowatts does it supply?

SOLUTION. $$KW = \frac{\text{Amperes} \times \text{Volts}}{1000} = \frac{50 \times 110}{1000} = 5.5 \text{ KW}.$$

Example 2

Compute the horsepower input of a motor running on a 110-volt line and taking 20 amperes.

SOLUTION. $H.P. = \dfrac{\text{Amperes} \times \text{Volts}}{750} = \dfrac{20 \times 110}{750} = 2.93$ H.P.

PROBLEMS

1. How many watts are consumed by a 110-volt arc lamp taking $4\frac{1}{2}$ amperes?
2. A motor taking 5 amperes is running on a 220-volt line. How much power does the motor receive?
3. Find the kilowatt capacity of a generator delivering 12 amperes at 220 volts.
4. How many watts are consumed by an electric iron taking 6.4 amperes on a 110-volt line?
5. What must be the horsepower of a motor for a trolley car if the car requires a current of 50 amperes and a voltage of 550 volts?
6. A car heater connected across a 550-volt line takes 2.6 amperes. How much power in watts does it consume?
7. A generator delivers to a motor 200 amperes at 400 volts. What is the horsepower input of the motor?
8. Find the kilowatt capacity of a generator supplying a current of .5 ampere to each of 360 lamps at a pressure of 110 volts.
9. How many horsepower are received by a motor taking 40 amperes from a 550-volt line?
10. Ten per cent of the power delivered to a certain motor is lost by friction. What horsepower will this motor deliver if it receives 25 amperes at 110 volts.

217. Mechanical Efficiency of Machines

Because of friction losses and for other reasons no machine gives out all the power it receives. The power put into a machine is called the ***input.*** The power which the machine delivers is called the ***output.*** The ***efficiency*** of a machine is the ratio of the output to the input and is always expressed in per cent.

$$\textit{Efficiency} = \frac{\textit{Output}}{\textit{Input}}$$

Thus, if a motor receives 10 H.P. and gives out only 8 H.P., its efficiency is $\frac{8}{10} = .80$ or 80%.

The efficiency of a steam engine is the ratio of brake H.P. to indicated H.P. If the indicated H.P. is 7.5 and the brake H.P. is only 6, its efficiency is $\frac{6}{7.5} = .80$ or 80%.

Example 1

A motor receiving 2500 watts has an efficiency of 90%. What H.P. will it deliver?

SOLUTION.

Input = 2500 watts = 2.5 KW.

Output = 90% of input = $.90 \times 2.5 = 2.25$ KW.

$H.P. = KW. \div \frac{3}{4} = 2.25 \div \frac{3}{4} = 2.25 \times \frac{4}{3} = 3$ H.P.

EXPLANATION. One kilowatt = 1000 watts, therefore 2500 watts = $\frac{2500}{1000}$ KW. = 2.5 KW. Since the efficiency is 90%, the motor gives out 90% or .90 of what it gets. Therefore the output = $.90 \times 2.5$ or 2.25 KW. One horsepower = $\frac{3}{4}$ KW.; therefore to convert kilowatt to horsepower we divide KW. by $\frac{3}{4}$ or multiply by $\frac{4}{3}$.

Example 2

What current does a 5-H.P. 220-volt motor take if its efficiency is 80%?

SOLUTION. $\text{Input} = \frac{\text{Output}}{\text{Efficiency}} = \frac{5}{.80} = 6.25$ H.P.

$\text{Watts} = H.P. \times 750 = 6.25 \times 750 = 4687.50$

$\text{Current} = \frac{\text{Watts}}{\text{Volts}} = \frac{4687.5}{220} = 21.31$ amperes

EXPLANATION. If the motor is a 5-H.P. motor with an efficiency of 80%, it means that 5 H.P. is 80% of the input. Hence, to find the input, we divide the output by the efficiency, that is, $\frac{5}{.80} = 6.25$ H.P.

To convert the horsepower to watts, we multiply by 750. We find the current by dividing watts by volts.

Example 3

What is the efficiency of a generator which delivers 60 amperes at 110 volts if the input is 10 H.P.?

SOLUTION. $\text{Efficiency} = \dfrac{\text{Output}}{\text{Input}}$

$$\text{Output} = \frac{60 \times 110}{1000} = 6.6 \text{ KW.}$$

$$\text{Input} = 10 \times \tfrac{3}{4} = 7.5 \text{ KW.}$$

$$\text{Efficiency} = \frac{6.6}{7.5} = .88 = 88\%$$

EXPLANATION. Multiplying 60 amperes by 110 volts gives the output in watts. Dividing watts by 1000 gives the output in kilowatts. To change the horsepower input to kilowatts, we multiply by $\frac{3}{4}$. We then divide the kilowatts output by the kilowatts input to get the efficiency.

NOTE. *Output and input must be in the same units; that is, to find efficiency, we divide kilowatt output by kilowatt input, or horsepower output by horsepower input.*

PROBLEMS

1. A motor having an efficiency of 75% receives 6 KW. What horsepower will it deliver?

2. A steam engine whose indicated horsepower is 8.5 H.P. has an efficiency of 70%. What will be the brake horsepower of the engine?

3. A 110-volt motor taking 20 amperes has an efficiency of 85%. What horsepower will it deliver?

4. An engine supplies 120 H.P. to a generator delivering 125 amperes at 550 volts. What is the efficiency of the generator?

5. Find the efficiency of a steam engine whose indicated horsepower is 6.2 H.P. and whose brake horsepower is 4.8 H.P.

6. What is the efficiency of a 4.5-H.P. motor if it takes 32 amperes on a 110-volt line?

7. On what voltage must a 10-H.P. motor be run if it takes a current of 37.5 amperes and is 86% efficient?

8. What current does a 10-H.P., 220-volt motor take if it is 88% efficient?

9. A 12-H.P., 110-volt motor has an efficiency of 90%. What current does it require?

10. A generator having an efficiency of 80% receives 60 H.P. from the driving engine. What current will it deliver at 110 volts?

11. What is the efficiency of a 12-H.P. motor if it requires 90 amperes at 120 volts?

17 Woodwork

218. Board Measure

Lumber is measured in terms of *board feet.* Feet board measure may be abbreviated fbm. A ***board foot*** of lumber is a piece of wood having an area of 1 sq. ft. and a thickness of 1 in. or less.

From the definition of a board foot we get the following rule:

Rule

To find the number of board feet in a piece of lumber, multiply the length in feet by the width in feet by the thickness in inches, counting a thickness less than 1 in. as 1 in.

Lumber is either rough or dressed.

Rough stock is lumber that is not dressed or planed.

Dressed stock is lumber that is planed on one or more sides.

In measuring lumber, we figure the full size, that is, the rough stock required to make the desired piece.

Allowance for dressing. When lumber is dressed or planed, $\frac{1}{16}''$ is taken off each side if the lumber is less than $1\frac{1}{2}''$ thick. If the lumber is $1\frac{1}{2}''$ or more in thickness, $\frac{1}{8}''$ is taken off each side.

Standard lengths. All building lumber, with few exceptions, is cut in even lengths, as 12′, 14′, 16′, 18′, etc.

In measuring the width of lumber, a fraction of less than $\frac{1}{2}''$ is disregarded, whereas $\frac{1}{2}''$ or more is regarded as $1''$.

Thus a board $5\frac{1}{4}''$ wide is considered as $5''$ wide, whereas a board $7\frac{1}{2}''$ in width is figured as $8''$.

Example 1

Find the number of board feet in a piece of lumber $3'' \times 8'' \times 20'$.

SOLUTION. $20 \times \frac{8}{12} \times 3 = 40$ board feet

EXPLANATION. Convert the width into feet by dividing $8''$ by 12; that is, $8'' = \frac{8}{12}$ of a foot. Multiply the length $20'$ by the width $\frac{8}{12}'$, by the thickness $3''$, and obtain 40 board feet.

Example 2

Find the number of board feet in a piece of lumber $\frac{7}{8}'' \times 14'' \times 22'$.

SOLUTION. $22 \times \frac{14}{12} \times 1 = 25\frac{2}{3}$ board feet

EXPLANATION. Dividing 14 by 12 changes the width to feet. Since the thickness is less than $1''$, it is figured as $1''$. Multiplying the length in feet by the width in feet by the thickness, $1''$, we get the board feet in the piece.

Example 3

Find the number of board feet in a board $2\frac{1}{2}'' \times 3\frac{3}{4}'' \times 16'$.

SOLUTION. $16 \times \frac{4}{12} \times \frac{5}{2} = 13\frac{1}{3}$ board feet

EXPLANATION. Since the width is $3\frac{3}{4}''$ we regard it as $4''$. Dividing $4''$ by 12 reduces it to feet. We then multiply the length in feet by the width in feet by the thickness in inches and obtain the number of board feet in the piece.

PROBLEMS

Find the number of board feet in the following pieces.

1. $1'' \times 8'' \times 12'$
2. $1'' \times 14'' \times 26'$
3. $2'' \times 10'' \times 18'$
4. $3'' \times 6'' \times 14'$
5. $4'' \times 4'' \times 20'$
6. $6'' \times 10'' \times 16'$
7. $8'' \times 12'' \times 24'$
8. $10'' \times 16'' \times 28'$
9. $12'' \times 14'' \times 22'$
10. $16'' \times 16'' \times 24'$
11. $\frac{3}{4}'' \times 6'' \times 10'$
12. $\frac{1}{2}'' \times 4'' \times 12'$

13. $2\frac{1}{2}'' \times 8'' \times 28'$
14. $1\frac{1}{2}'' \times 14'' \times 30'$
15. $3'' \times 5\frac{1}{2}'' \times 18'$
16. $4'' \times 9\frac{3}{4}'' \times 32'$
17. $6'' \times 16\frac{1}{4}'' \times 26'$
18. $8'' \times 14\frac{1}{4}'' \times 18'$
19. $10'' \times 11\frac{3}{4}'' \times 34'$
20. $12'' \times 13\frac{3}{4}'' \times 20'$

21. A truckload of lumber is 16′ long, 5′ 4″ wide, and 4′ 8″ high. How many board feet does it contain?

22. How many board feet are there in 72 boards $2\frac{1}{4}'' \times 16'' \times 28'$?

23. A building lot 120′ long and 30′ wide is to be enclosed by a fence 10′ high. The boards are 6″ wide and 1″ thick. Bracing pieces $3'' \times 4'' \times 12'$ are spaced 10′ apart. Three longitudinal nailing pieces $2'' \times 4''$ are used. How many board feet of lumber are required?

24. How many board feet of lumber are required for a sidewalk 200′ long and 10′ wide if 2″-thick planks are used? The planks are to be nailed to three pieces $3'' \times 4''$ running the full length of the walk.

25. Find the number of board feet required to build a walk a mile long and 12′ wide. The boards are 6″ wide and $1\frac{3}{4}''$ thick, dressed on one side and spaced $\frac{1}{2}''$ apart. Four planks $3'' \times 4''$ run the full length of the walk.

26. How many board feet of lumber are required to floor a loft with $1\frac{1}{2}''$ lumber dressed on one side if the floor is 60′ 6″ long and 22′ 3″ wide, allowing 2% for squaring?

27. Find the number of board feet required for flooring a circular bandstand 24′ in diameter if $1\frac{3}{4}''$ stock dressed on one side is used. Allow 3% for waste.

28. A porch 50′ 6″ long and 8′ 3″ wide is to be floored with 1″ stock planed on one side. If the allowance for squaring is 3%, how many board feet are required?

29. A loading platform is laid with 2″ rough stock. The platform is 42′ 9″ long and 8′ 6″ wide. How many board feet are required if 2% is allowed for waste?

30. A factory floor is laid with $1\frac{5}{8}''$ stock dressed on one side. If the floor is $36' 6'' \times 61' 4''$, how much lumber is required, allowing 3% for waste?

219. Flooring

Hardwood flooring is usually tongued and grooved. Hence, in figuring the amount of lumber required for covering surfaces such as the floor or ceiling of a room, allowance must be made for matching as well as for waste. The allowance for matching varies from 24 to 50% of the surface to be covered, depending on the face width of the lumber used. For waste, the usual allowance is 3 to 5%.

The following table shows the percentages customarily allowed for matching.

Face Width of Boards	Percentage Allowed for Matching
$1\frac{1}{2}''$	50%
$2''$	$37\frac{1}{2}\%$
$2\frac{1}{4}''$	$33\frac{1}{3}\%$
$3\frac{1}{4}''$	24%

The standard thicknesses of hardwood flooring are: $\frac{3}{8}''$, $\frac{1}{2}''$, $\frac{5}{8}''$, and $\frac{25}{32}''$.

Example

Find the number of board feet of flooring required for laying a floor $12'\ 6'' \times 21'\ 3''$ with $2\frac{1}{4}''$ matched flooring $\frac{5}{8}''$ thick. Allow 5% for waste.

Solution.

$12.5 \times 21.25 \times 1$	=	265.62
Allowance for matching, $33\frac{1}{3}\%$ of 265.62	=	88.54
Allowance for waste, 5% of 265.62	=	13.28
		367.44

Total flooring required is 368 board feet.

Explanation. Multiplying the width of the floor by the length gives the area in square feet. Since the thickness of the flooring is less than $1''$, it is figured as $1''$. Hence, multiplying the area of the floor by $1''$ gives 265.62, the number of board feet that would be required if no allowances were made for matching or waste. Since $2\frac{1}{4}''$ matched flooring is used, the allowance for matching, according to the table, is $33\frac{1}{3}\%$ of 265.62, or 88.54 fbm. Five per cent for waste = $.05 \times 265.62$, or 13.28 fbm. Adding these three quantities gives the total of 367.44, or 368 board feet.

PROBLEMS

1. Find the number of board feet of flooring required for a room $14'\ 8'' \times 24'\ 0''$ if $2''$ matched flooring $\frac{1}{2}''$ thick is used. Allow 3% for waste.

2. How many board feet of flooring are needed for a room $13'\ 6'' \times 22'\ 9''$ if $2\frac{1}{4}''$ matched boards $\frac{1}{2}''$ thick are used? Allow 5% for waste.

3. An apartment consists of three rooms having the following dimensions: The living room is 11′ 6″ × 20′ 6″, the bedroom is 10′ 9″ × 12′ 3″, the dinette is 7′ 10″ × 11′ 8″, the foyer is 6′ 3″ × 15′ 6″. Find the number of board feet required for the floors of the apartment, using 2″ boards $\frac{5}{8}$″ thick. Allow 3% for waste.

4. A room 14′ 3″ × 24′ 0″ is to be ceiled with $2\frac{1}{4}$″ matched lumber $\frac{5}{8}$″ thick. Allowing 3% for waste, how many board feet are required?

5. A room 24′ 6″ × 32′ 6″ is to be floored with $3\frac{1}{4}$″ matched boards $\frac{25}{32}$″ thick. How many board feet are required if 5% is allowed for waste?

6. At $75.00 per M board feet (that is, 1000 board feet) find the cost of covering the floor and walls of a room 18′ 3″ × 28′ 6″ and 12′ 3″ high. Matched lumber $2\frac{1}{4}$″ wide and $\frac{5}{8}$″ thick is used. Deduct 25% of the wall surface for doors and windows. Allow 5% for waste.

7. The walls of a dining room 21′ 3″ × 12′ 6″ are to be wainscoted for a height of 4 ft. above the floor. There are two doors 3′ 0″ wide. How many board feet of lumber are required if 2″ boards $\frac{5}{8}$″ thick are used, allowing 2% for waste?

8. A semicircular platform 48′ in diameter is to be floored with $2\frac{1}{4}$″ stock $\frac{25}{32}$″ thick. Allowing 3% for waste, how many board feet are needed?

9. Compute the amount of lumber required to cover the floor of a dance hall 70′ × 42′ if $3\frac{1}{4}$″ boards $\frac{5}{8}$″ thick are used. Allow 2% for waste.

10. How many board feet of hardwood flooring are required for a classroom 32′ 6″ × 28′ 9″ if $2\frac{1}{4}$″ boards $\frac{5}{8}$″ thick are used? Allow 3% for waste.

220. Shingles

Shingles are usually 4″ wide and 16″, 18″, or 24″ long. The butt thickness varies from $\frac{5}{16}$″ to $\frac{1}{2}$″. Shingles are laid 4″, $4\frac{1}{2}$″, 5″, or 6″ to the weather. The number of shingles required to cover a given roof area depends on the amount of shingle exposed to the weather. Shingles are sold in bundles of 250.

The following examples illustrate the methods of finding the number of shingles required to cover a roof.

FIRST METHOD. A 4″ shingle laid $4\frac{1}{2}$″ to the weather covers an area of $4 \times 4\frac{1}{2}$, or 18 sq. in. An area of 100 sq. ft., called a square, contains $100 \times 144 = 14{,}400$ sq. in. The number of shingles required to cover one square $= \dfrac{14{,}400}{18} = 800.$

The following table gives the number of 4″ shingles required per square according to the number of inches exposed to the weather.

Exposure of 4″ Shingle to Weather	Number of Shingles per Square
4″	900
$4\frac{1}{2}$″	800
5″	720
6″	600

Example

A roof 48′ long and 22′ wide is to be covered with 4″ shingles laid $4\frac{1}{2}$″ to the weather. Find the number of shingles required, allowing 10% for waste.

Solution. $\dfrac{48 \times 22 \times 800}{100} = 8448$

10% for waste $= 845$

Total required $= 9293$

Number of bundles $= \dfrac{9293}{250} = 37+$

Therefore 38 bundles would have to be bought.

Explanation. Multiplying the width of the roof by the length gives the area in square feet. Dividing the area by 100 gives the number of squares. Since 800 shingles laid $4\frac{1}{2}$″ to the weather are required to cover one square, multiplying the number of squares by 800 gives the number of shingles required. Adding 10% for waste and dividing the total by 250 gives the number of bundles required.

Second method. Another method of finding the number of shingles required to cover a given area is to find the number in one course and multiply it by the number of courses.

Example

Find the number of 4″ shingles laid 6″ to the weather required to cover a roof 32′ wide and 50′ long.

SOLUTION.

$$\text{Number of shingles in one course} = \frac{50 \times 12}{4} = 150$$

$$\text{Number of courses} = \frac{32 \times 12}{6} = 64$$

$$\text{Number of shingles required} = 150 \times 64 = 9600$$

To this must be added some allowance for waste.

PROBLEMS

1. Find by the first method the number of shingles required to cover a roof 24′ × 36′ with 4″ shingles laid $4\frac{1}{2}$″ to the weather. Allow 10% for waste. Compute the number of bundles required.

2. Find by the second method the number of shingles required to cover the roof in Problem 1.

3. Find by both methods the number of shingles required to cover a roof 25′ × 48′ with 4″ shingles laid 6″ to the weather. Allow 5% for waste. Find the number of bundles.

4. Using 4″ shingles laid 5″ to the weather, compute by the first method the number of shingles required to cover a gable roof whose ridge is 40′ long and whose rafters are 18′ on each side, allowing 5% for waste. Find the number of bundles required.

5. Compute by the second method the number of shingles required for the roof in Problem 4. Find the number of bundles.

6. Find, by each of the two methods, the number of 4″ shingles, laid 6″ to the weather, required for covering a gable roof whose ridge is 35′ 6″ long and whose rafters are 14′ 6″ on each side. Allow 10% for waste. Find the number of bundles in each case.

7. Find by the first method the number of shingles required to cover a roof 21′ 6″ × 34′ 9″, using 4″ shingles laid $4\frac{1}{2}$″ to the weather and allowing 10% for waste. How many bundles are required?

8. Compute by the second method the number of shingles required to cover a roof 37′ 6″ long and 21′ wide, using 4″ shingles laid 6″ to the weather. Allow 5% for waste.

9. Find by the second method the number of 4″ shingles laid 5″ to the weather that are needed for covering a gable roof with a ridge 35′ long and rafters 15′ 6″ on each side. Allow 5% for waste.

10. Find the number of shingles required for the following: (*a*) a gable roof whose ridge is 33′ 6″ long and whose rafters are 16′ 6″ long; (*b*) a lean-to whose roof is 14′ long and 9′ wide; (*c*) a shed whose roof is 24′ long and 7′ wide. Shingles 4″ wide laid $4\frac{1}{2}$″ to the weather are used. Allow 10% for waste. Compute the number of bundles required for the job.

221. Clapboards

For covering the outside walls of a house, either shingles or clapboards are used. Shingles for siding are usually 24″ long and are laid $7\frac{1}{2}$″ to 10″ to the weather. Clapboards are usually 6″ in width, vary in length from 4′ to 16′, and are laid 4″ to 5″ to the weather.

To find the number of clapboards required for a wall, find the area in square inches of the surface to be covered, allowing for openings, and divide the net area by the area of the board exposed to the weather.

Example

Find the number of clapboards 6″ wide and 8′ long, laid 4″ to the weather, to cover a wall 33′ long and 12′ 6″ high. Allow 20% for openings and 3% for waste.

SOLUTION.

Gross area of wall $= 33 \times 12.5 \times 144 = 59{,}400$ sq. in.

Allowance for openings $= .20 \times 59{,}400 = 11{,}880$ sq. in.

Net area of wall $= 47{,}520$ sq. in.

A 6″ × 8′ clapboard laid 4″ to the weather will cover an area of $4 \times 8 \times 12 = 384$ sq. in.

$$\text{Number of clapboards required} = \frac{47{,}520}{384} = 124$$

Allowance for waste $= .03 \times 124 = 3.72 = 4$

Total clapboards required $= 128$

PROBLEMS

1. How many clapboards 6″ × 10′ laid $4\frac{1}{2}$″ to the weather are required to cover a wall 34′ 6″ × 14′ 6″, allowing 25% for openings and 5% for waste?

2. How many 6″ × 14′ clapboards laid 5″ to the weather are required to cover the four sides of a house 28′ 6″ wide, 54′ long, and 23′ high? Deduct 15% for openings and allow 5% for waste.

3. A barn 42′ × 64′ and 16′ 6″ high is to be covered with clapboards 6″ × 12′ laid $4\frac{1}{2}$″ to the weather. Allowing 10% for openings and 3% for waste, how many clapboards will be required?

4. How many 6″ × 24″ shingles are required to cover the walls of a house 25′ 6″ wide, 49′ long, and 22′ 6″ high if the shingles are laid $8\frac{1}{2}$″ to the weather? Deduct 25% for openings and allow 5% for waste. Find the number of bundles.

5. The walls of a house 19′ 9″ wide, 30′ 6″ long, and 11′ 6″ high are to be covered with shingles 5″ × 24″ laid $7\frac{1}{2}$″ to the weather. Allowing 20% for openings and 10% for waste, how many shingles are required? How many bundles?

6. A lean-to is 15′ 6″ long, 10′ 6″ deep, and 9′ high at the inner end and 8′ high at the outer end. How many clapboards 6″ × 8′ laid $4\frac{1}{2}$″ to the weather are required for the three walls if there is a door 4′ 6″ wide and 7′ high? Allow 5% for waste.

7. How many clapboards 6″ × 10′ laid 4″ to the weather are required to cover the walls of a house 28′ 6″ wide, 38′ 6″ long, and 12′ 3″ high, if 20% is deducted for openings and 5% allowed for waste?

8. Allowing 25% for doors and windows, find the number of 6″ × 8′ clapboards laid 5″ to the weather required to cover the four walls of a house 32′ × 46′ and 20′ 3″ high. Allow 5% for waste.

9. The walls of a cottage 18′ 6″ × 27′ 6″ and 11′ 6″ high are to be covered with clapboards 6″ × 10′ laid $4\frac{1}{2}$″ to the weather. How many boards are required if 25% is deducted for doors and windows and 5% allowed for waste?

10. Find the number of 5″ × 24″ shingles laid 9″ to the weather required to cover the four walls of a frame bungalow 24′ × 16′ and 9′ high if 25% of the area is deducted for doors and windows and 10% is allowed for waste.

222. Stairs

The meaning of the following terms will be evident from an inspection of Fig. 276 and Fig. 277.

The ***rise*** of a flight of stairs is the perpendicular distance between the two floor levels. The ***run*** of the stairs is the

horizontal distance between the upper and lower ends of the stairs. The ***tread*** is the horizontal part of each step. The ***riser*** is the vertical part of each step. The *run of a step* is the horizontal distance from the front face of one riser to the

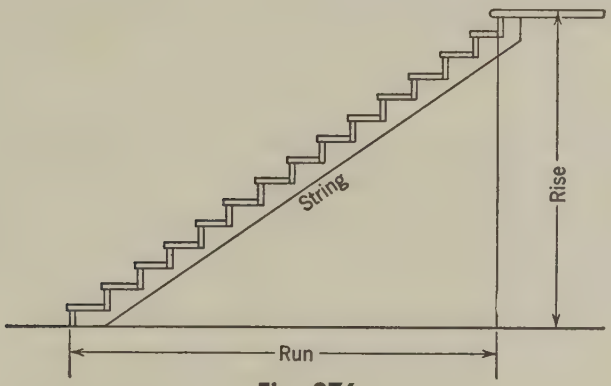


Fig. 276

front face of the next. The *rise of each step* is the perpendicular distance from the upper surface of one tread to the upper surface of the next. The ***nosing*** of a step is the projection of the tread beyond the front of the riser.

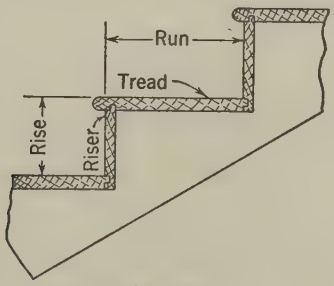


Fig. 277

Lay-out of stairs. The important consideration in laying out a flight of stairs is to proportion the tread and riser so as to make the steps most comfortable. As the result of experience, architects have adopted the following rules:

(1) Rise + run should equal not less than 17″ and not more than 18″.

(2) Rise $\times$ run should equal not less than 70″ and not more than 75″.

The rise, as a rule, is assumed to be about 7″, and the run of the tread is then determined according to the rules given above.

Example

Find the number of steps and the dimensions of the rise and run in a flight of steps having a rise of 10′ 0″.

SOLUTION. Rise of stairs = 10′ 0″ = 120″. Assuming a rise of step of 7″, we have:

$$\text{Approximate number of risers} = \frac{120}{7} = 17\frac{1}{7}$$

But, since all risers must be of the same height, we divide 120 by 17 and obtain $7\frac{1}{17}$″ as the exact height of riser for each of the 17 steps.

Since, according to rule (1), the sum of rise and run should be between 17″ and 18″, the run of the step may be made 10″ or $10\frac{1}{2}$″, to which must be added the nosing of 1″. The run of the entire flight of stairs is found by multiplying the run of each step by the number of steps.

PROBLEMS

NOTE. *Design all steps with a rise as near 7″ as possible.*

1. The rise of a flight of stairs is 9′ 0″. Find the number of steps and the rise of each.

2. How many steps are there in a flight of stairs having a rise of 9′ 3″? Find the exact rise of each step and the run of the flight.

3. Find the number of steps, the rise of each step, and the run of the flight of stairs if the rise of the flight is 10′ 3″.

4. If the rise of step is to be as near 7″ as possible, how many steps will there be and what will be the rise of each step in a flight of stairs whose rise is 9′ 6″?

5. Each of two flights of stairs in a frame bungalow has a rise of 9′ 0″ and a width of 3′ 3″. The rise of the steps is to be as near 7″ as possible. The treads are made of $1\frac{1}{4}$″ stock and the risers of 1″ stock dressed on one side. Allow $1\frac{1}{4}$″ for nosing, 1″ on the length of each piece for squaring, and $\frac{1}{4}$″ on the width of each piece for jointing. Find: (*a*) the number

of steps in each flight; (*b*) the exact rise of each step; (*c*) the run of each flight of stairs; (*d*) the number of board feet of lumber required for the steps.

6. Find the length of the stair strings in Problem 5, allowing 18″ additional length for cutting on a slant. Find the number of board feet of lumber required for the four strings if they are made of $1\frac{1}{4}$″ stock 11″ wide, dressed on one side.

7. How many steps are there in a flight of stairs having a rise of 10′ 6″? What is the run of the flight?

8. Using the same stock as in Problem 5 and making the same allowances for squaring, nosing, and jointing, find the number of board feet of lumber required for the steps in Problem 7 if the stairway is 4′ wide.

9. The rise of a flight of stairs is 10′ 9″, and the width is 3′ 6″. Find: (*a*) the number of steps; (*b*) the exact rise of each; (*c*) the run of the flight; (*d*) the length of the string, allowing 15″ for cutting on slant.

10. The treads and risers in Problem 9 are made of 1″ stock dressed on one side. Allow 1″ for nosing, 1″ on the length of each piece for squaring, and $\frac{1}{4}$″ on the width of each piece for jointing. Find the number of board feet required for the steps.

11. Find the number of board feet required for the strings in Problem 9 if $1\frac{1}{4}$″ stock, 12″ wide, dressed on one side, is used, allowing 15″ for cutting on slant.

12. A flight of stairs has a rise of 9′ 9″. Find: (*a*) the number of steps; (*b*) the exact rise of each step; (*c*) the run of the flight.

18 Tapers

223. Definitions

A piece of work which decreases gradually in diameter so that it assumes a conical shape is said to be *tapered.* Figure 278 shows a piece of tapered work.

When we speak of the ***taper*** on a piece of work, we mean the *difference between the diameters* at the ends of the piece.

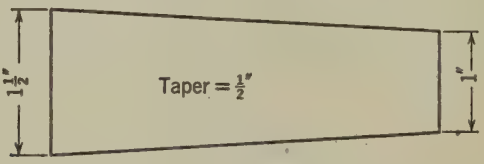


Fig. 278

If, as in Fig. 278, the large diameter is $1\frac{1}{2}''$ and the small diameter $1''$, the taper, or difference between the diameters, is $\frac{1}{2}''$. Therefore, when we say that a piece of work has a taper of $\frac{1}{2}''$, we mean that the diameter at the large end is $\frac{1}{2}''$ greater than the diameter at the small end.

Taper is usually designated as a *fraction of an inch per foot of length,* the length being measured along the axis or center line of the piece of work. When we say, for example, that a piece of work has a taper of $\frac{5}{8}''$ per foot, we mean that there is a difference of $\frac{5}{8}''$ between the diameters 1 ft. apart.

Taper may also be specified as a fraction of an inch per inch of length of the work. Thus we may say that a piece

of work has a taper of $\frac{1}{16}''$ per inch. This means that the diameters 1 in. apart differ by $\frac{1}{16}''$.

224. Computing Taper and Diameter

By referring to the sketch in Fig. 279, we see that the taper in 2″ is twice as much as the taper in 1″; the taper in 3″ is three times as much as the taper in 1″, and so on. Hence,

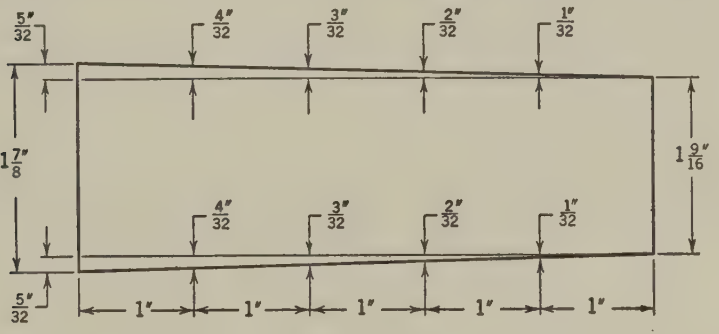


Fig. 279

if we know the taper in a 1″ length of a piece of work, we can find the taper in any other length by multiplying the taper per inch by the number of inches in the given length.

Example 1

The taper per inch on a piece of work is 0.05″; what is the taper in 8″?

SOLUTION. Taper $= 8 \times 0.05 = 0.4''$

If, on the other hand, we are told that the taper in a foot length of a piece of work is $\frac{1}{2}''$, and we are required to find the taper in a 1″ length, we divide by 12, because in 1 in. there is $\frac{1}{12}$ as much taper as there is in 12 in.

Example 2

Find the taper per inch if the taper per foot is $\frac{5}{8}''$.

SOLUTION. $\frac{5}{8} \div 12 = \frac{5}{8} \times \frac{1}{12} = \frac{5}{96} = 0.0521''$

Taper per inch $= 0.0521''$

Example 3

Find the taper per foot on the piece of work shown in Fig. 280.

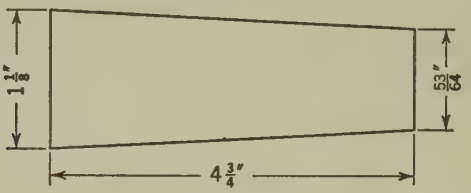


Fig. 280

SOLUTION. Taper in piece $= 1\frac{1}{8}'' - \frac{53}{64}'' = \frac{19}{64}''$

Taper per inch $= \frac{19}{64} \div 4\frac{3}{4}$

$= \frac{19}{64} \times \frac{4}{19} = \frac{1}{16}''$

Taper per foot $= 12 \times \frac{1}{16} = \frac{12}{16} = \frac{3}{4}''$

EXPLANATION. We first find the taper by subtracting the small diameter from the large diameter. We then find the taper per inch by dividing the taper by the length of the piece. Finally we multiply the taper per inch by 12, giving the taper per foot, $\frac{3}{4}''$.

The solution may be abbreviated by combining the steps, thus

Taper $= 1\frac{1}{8} - \frac{53}{64} = \frac{19}{64}''$

Taper per foot $= \frac{19}{64} \div 4\frac{3}{4} \times 12$

$= \frac{19}{64} \times \frac{4}{19} \times 12 = \frac{3}{4}''$

Example 4

The piece of work shown in Fig. 281 has a taper of 0.6″ per foot. Find the small diameter.

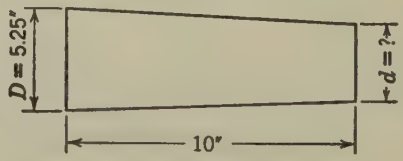


Fig. 281

SOLUTION. Taper per inch $= \frac{1}{12} \times 0.6 = 0.05''$

Taper in $10'' = 10 \times 0.05 = 0.50''$

Small diameter = large diameter − taper

$= 5.25 - 0.50 = 4.75''$

EXPLANATION. To find the small diameter, we must subtract the taper in the piece of work from the large diameter. Dividing the taper per foot by 12 gives $0.05''$, the taper per inch. Multiplying this by 10 gives $0.50''$, the taper. Subtracting the taper from the large diameter gives the small diameter, $4.75''$.

Example 5

Find the large diameter of a piece of work $7\frac{3}{4}''$ long, having a small diameter of $0.658''$ and a taper of $\frac{1}{16}''$ per inch, Fig. 282.

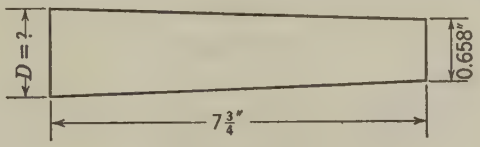


Fig. 282

SOLUTION. Taper $= 7\frac{3}{4} \times \frac{1}{16} = \frac{31}{4} \times \frac{1}{16}$

$= \frac{31}{64} = 0.485''$

Large diameter = small diameter + taper

$= 0.658 + 0.485 = 1.143''$

EXPLANATION. The large diameter is equal to the small diameter plus the taper. The taper in the piece which is $7\frac{3}{4}''$ long is $7\frac{3}{4}$ times the taper per inch, or $7\frac{3}{4} \times \frac{1}{16} = 0.485''$. The large diameter is therefore equal to $0.658 + 0.485 = 1.143''$.

225. American Standard Self-holding (Slow) Taper Series

The taper series most commonly used are the following:

1. Brown and Sharpe.
2. Morse.
3. $\frac{3}{4}''$ per foot series.

1. The *Brown and Sharpe taper* is $\frac{1}{2}''$ per foot.

2. The *Morse taper* is nominally $\frac{5}{8}''$ per foot. The exact taper for each size is as follows:

No. 1. 0.600″ per foot
No. 2. 0.600″ per foot
No. 3. 0.602″ per foot
No. 4. 0.623″ per foot
No. 4½. 0.623″ per foot
No. 5. 0.630″ per foot

3. In the $\frac{3}{4}''$ *per foot series* the taper in all cases is $\frac{3}{4}''$ per foot.

Steep Taper. The most common steep taper has a taper of $3\frac{1}{2}''$ per foot.

PROBLEMS

1. What is the taper in the piece of work in Fig. 283? What is the taper per inch? What is the taper per foot?

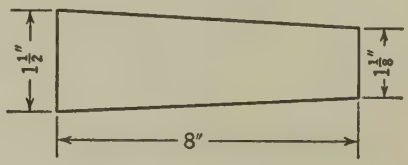


Fig. 283

2. What is the taper per inch in the Brown and Sharpe? Morse No. 1? $\frac{3}{4}''$ per foot series?

3. Compute the small diameter of the lathe center in Fig. 284. The taper is Morse No. 2.

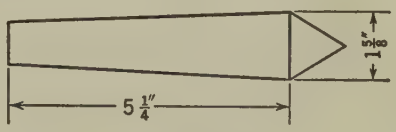


Fig. 284

4. What is the diameter 4″ from the small end of the lathe center in Problem 3?

5. Compute the taper per foot on the taper socket in Fig. 285.

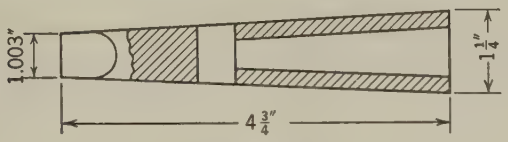


Fig. 285

6. Compute the taper per foot on the bushing in Fig. 286.

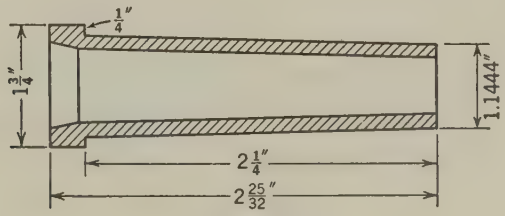


Fig. 286

7. Compute the taper per foot on the reamer in Fig. 287.

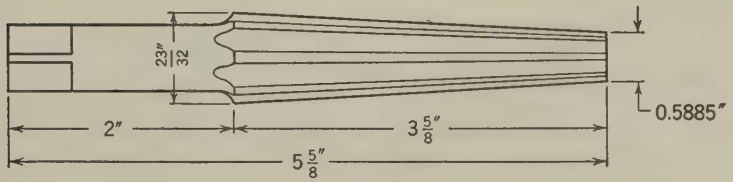


Fig. 287

8. Compute the taper per foot on the collet in Fig. 288.

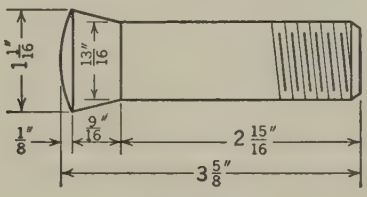


Fig. 288

9. Compute the taper per foot on the milling machine arbor in Fig. 289.

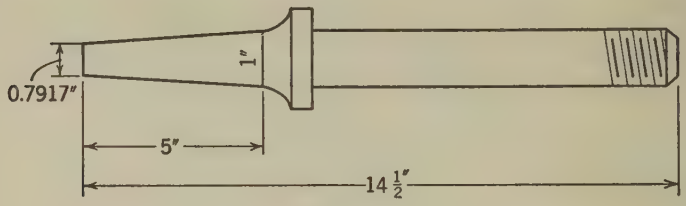


Fig. 289

10. Compute the taper per foot on the drill socket in Fig. 290.

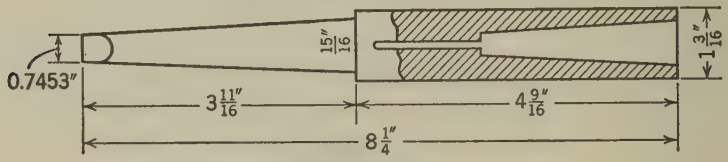


Fig. 290

11. Compute the taper per foot on the drill shank in Fig. 291.

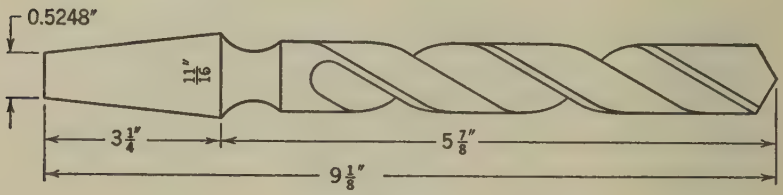


Fig. 291

12. Compute the taper per foot on the mandrel in Fig. 292.

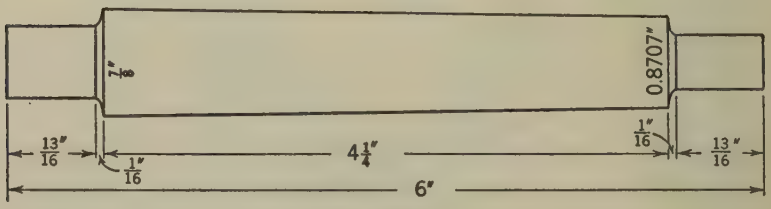


Fig. 292

13. Compute the taper per foot at both ends of the piston rod in Fig. 293.

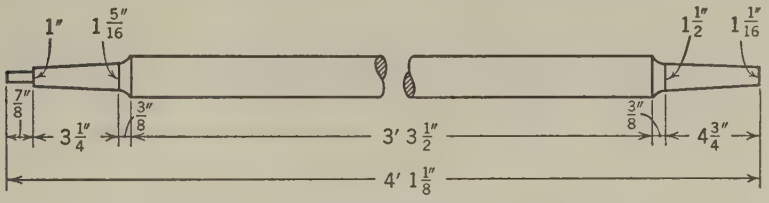


Fig. 293

14. The inside taper of the taper socket in Fig. 294 is Morse No. 5. Compute the small diameter.

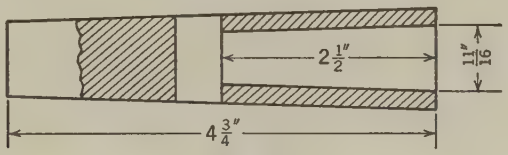


Fig. 294

15. Compute the large diameter of the inside taper (Brown and Sharpe) of the taper bushing in Fig. 295.

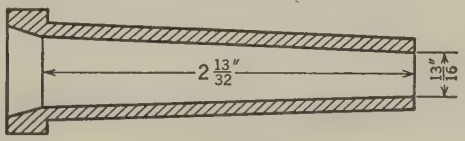


Fig. 295

16. Compute the small end diameter of the inside taper (Brown and Sharpe) of the drill socket in Fig. 296.

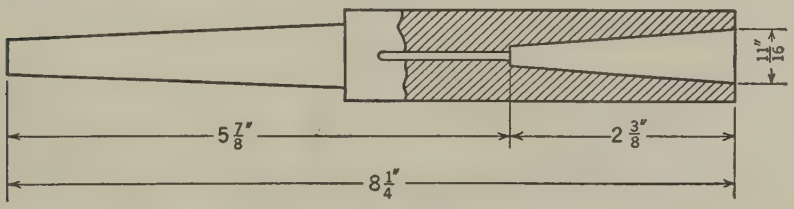


Fig. 296

17. Find the diameter at the large end of the taper pin reamer in Fig. 297. The taper is $\frac{3}{8}''$ per foot.

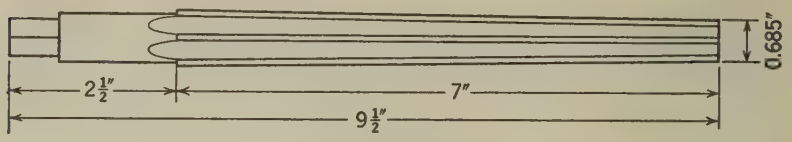


Fig. 297

18. The crank pin in Fig. 298 is to be cut with Morse No. 4 taper. Find the diameter at the small end of the tapered portion.

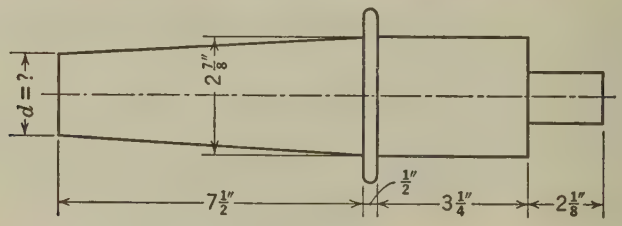


Fig. 298

19. A piece of work $6\frac{1}{2}''$ long has a taper of $\frac{3}{4}''$ per foot. Find the small diameter if the large diameter is $\frac{1}{2}''$.

20. Find the large diameter of a piece of tapered work $4\frac{1}{8}''$ long, having a taper of $\frac{5}{8}''$ per foot and a small diameter of $0.572''$.

21. Find the small diameter of the shaft in Fig. 299. The taper is Brown and Sharpe.

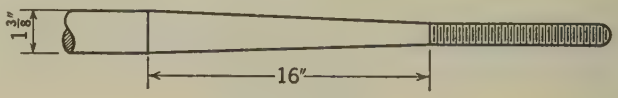


Fig. 299

226. Taper Angle

The taper on a piece of work is sometimes specified by giving the *angle included between the sides* or the prolonga-

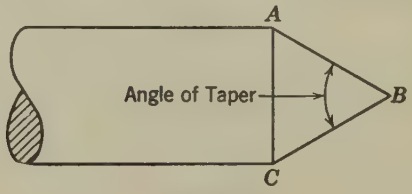


Fig. 300

tion of the sides of the piece of work. Thus in Fig. 300 the sides AB and CB meet at B, forming the angle ABC, which is the *included angle* or the *taper angle.*

In Fig. 301 the sides BA and EF of the piece of tapered work, when prolonged to H, form the angle of taper BHE.

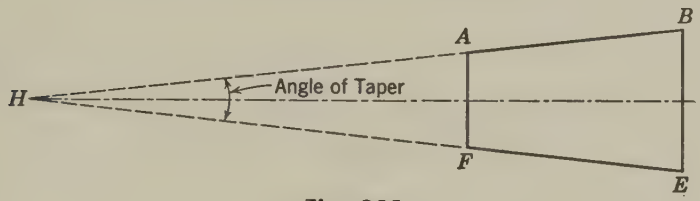


Fig. 301

Example 1

Compute the taper angle in a piece of work 10″ long, whose small diameter is 2″ and large diameter $3\frac{1}{2}$″. See Fig. 302.

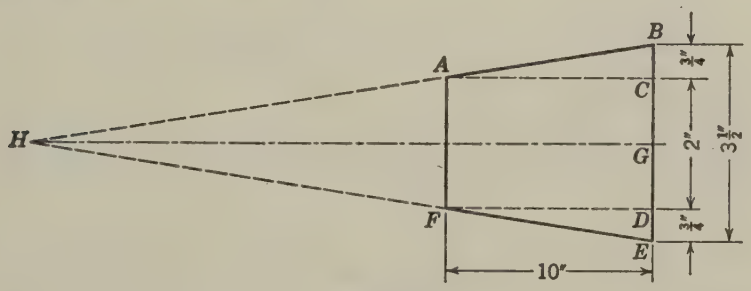


Fig. 302

Solution. Draw AC parallel to the center line of the piece of work, forming the right triangle ABC. Angle BAC = BHG = half of the taper angle. From trigonometry we know that the tangent of an angle $= \dfrac{\text{side opposite}}{\text{side adjacent}}$. Hence tangent $BAC = \dfrac{BC}{AC}$ $= \dfrac{\text{Half of taper}}{\text{Length}} = \dfrac{0.75}{10} = .07500$. By referring to a table of natural functions, we find that this tangent corresponds to an angle of 4° 17′. Since the angle DFE = angle BAC, the included angle at H, which is twice BAC, equals $2 \times 4°\ 17' = 8°\ 34'$.

Example 2

Find the angle of taper in a piece of work having a taper of $\frac{5}{8}''$ per foot. See Fig. 303.

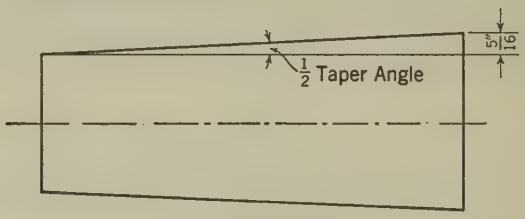


Fig. 303

SOLUTION.

$$\text{Tangent of } \tfrac{1}{2} \text{ taper angle} = \frac{\frac{1}{2} \text{ of } \frac{5}{8}}{12}$$

$$= \tfrac{1}{2} \times \tfrac{5}{8} \times \tfrac{1}{12} = \tfrac{5}{192} = 0.02604$$

$\frac{1}{2}$ taper angle $= 1° 29\frac{1}{2}'$

Taper angle $= 2 \times 1° 29\frac{1}{2}' = 2° 59'$

Example 3

Find the taper per foot if the included angle is 12° 40′. See Fig. 304.

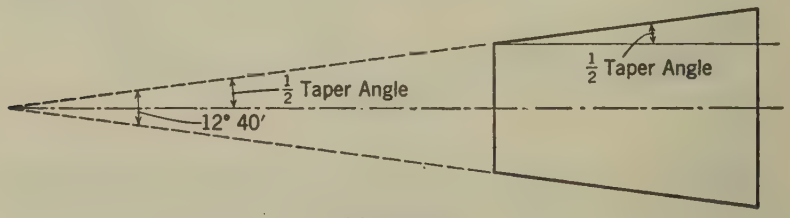


Fig. 304

SOLUTION.

$$\tfrac{1}{2} \text{ of the taper angle} = \frac{12° 40'}{2} = 6° 20'$$

Tangent 6° 20′ $= 0.11099$

Taper per inch $= 2 \times 0.11099 = 0.22198$

Taper per foot $= 12 \times 0.22198 = 2.66396$ or $2.664''$

EXPLANATION. From a table of natural functions get the tangent of half the taper angle. This gives half the taper in one inch of length. To get the taper per inch, multiply this result by 2. The taper per foot is 12 times the taper per inch.

PROBLEMS

1. Compute the taper angle for the following tapers: Brown and Sharpe, Morse No. 5, $\frac{3}{4}''$ per foot series.

2. What is the included angle of the bevel of the piece shown in Fig. 286? The length of the bevel is $\frac{3}{8}''$, the large diameter $1\frac{1}{8}''$, and the small diameter .9253″.

3. Compute the included angle in the tapered portion of the collet in Fig. 288.

4. What is the angle of taper of the left-hand end of the piston rod in Fig. 293?

5. Find the taper per foot if the taper angle is 5°.

6. Find the taper per inch if the taper angle is $3\frac{1}{2}°$.

7. The included angle on a piece of tapered work is 7°. What is the taper per foot?

8. What standard taper has a taper angle of 2° 59′?

9. The angle with the center line in a piece of tapered work is 2° 23′. What is the taper per foot?

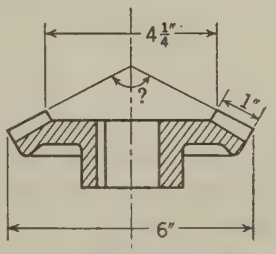


Fig. 305

10. Compute the included angle in the bevel gear blank shown in Fig. 305.

11. Find the included or taper angle in the drill chuck in Fig. 306.

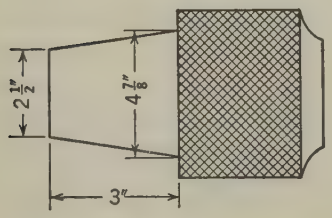


Fig. 306

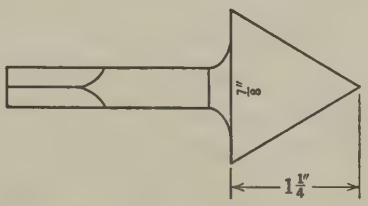


Fig. 307

12. Compute the included angle in the point of the countersink in Fig. 307.

227. Taper Turning by Offsetting the Tail Stock

There are three methods of turning a taper in a lathe. The first method, by offsetting the tail stock, can be used for outside tapers only and when the taper to be turned is not too great. The effect of offsetting the tail stock will be seen from a study of Fig. 308.

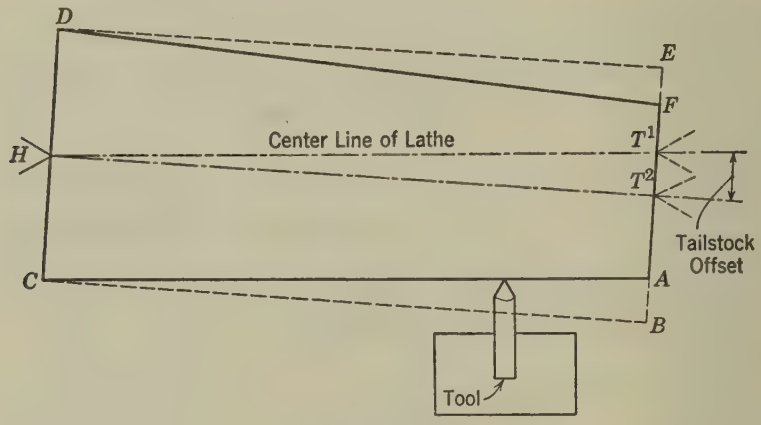


Fig. 308

For cylindrical turning, the tail stock center T^1 is on the center line of the lathe, that is, directly in line with the head stock center H. When the tail stock is offset for cutting a taper, the center assumes the position T^2, and the piece of work assumes the position $DEBC$. The tool, however, travels in a straight line parallel to the center line of the machine from C to A. The radius of the work is therefore reduced at the tail end by the amount AB, which is equal to the tail stock offset from T^1 to T^2. But, since the work revolves, an equal amount is taken off all around. In other words the diameter at the tail end is less than the diameter at the head end by an amount equal to twice the offset of the tail stock. The tail stock offset is therefore equal to half the total taper.

Rule

The amount of set-over of the tail stock is equal to one-half the total length of the work in inches multiplied by the taper per inch.

Example 1

Calculate the tail stock offset for turning the taper on the piece in Fig. 309.

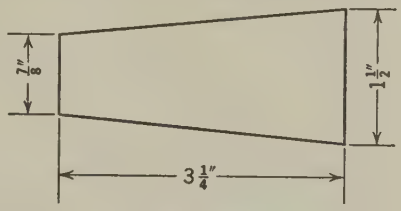


Fig. 309

SOLUTION. Taper $= 1\frac{1}{2} - \frac{7}{8} = \frac{5}{8}''$

Offset $= \frac{1}{2}$ of taper $= \frac{1}{2} \times \frac{5}{8} = \frac{5}{16}''$

Example 2

Calculate the tail stock offset for turning a taper of $\frac{3}{4}''$ per foot for a distance of 4″ on a piece of work 10″ long. See Fig. 310.

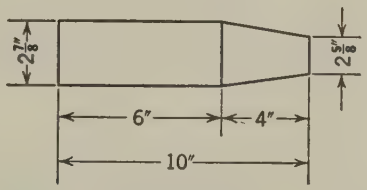


Fig. 310

SOLUTION.

Taper per inch $= \frac{1}{12} \times \frac{3}{4} = \frac{1}{16}''$

Taper in whole length of piece of work $= 10 \times \frac{1}{16} = \frac{5}{8}''$

Tail stock offset $= \frac{1}{2}$ of $\frac{5}{8} = \frac{5}{16}''$

It will be noticed that even though the tapered part is only 4″ long, the taper for the whole length of the piece is found and the offset is computed as one-half of this overall taper.

Example 3

Compute the tail stock offset for turning tapers at both ends of the piston rod in Fig. 311.

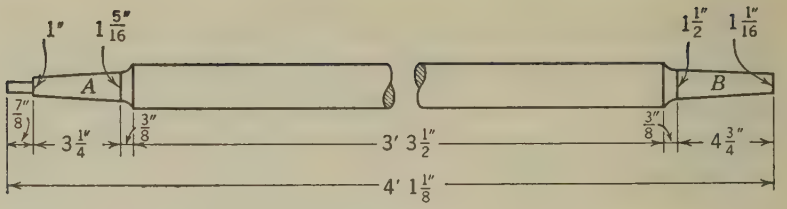


Fig. 311

SOLUTION. To turn the left end taper at A, we must compute the taper for the whole length of the rod as if it were tapered the same amount per inch as the part at A. The tail stock will then be offset half of the total taper so computed.

Taper in part A $= 1\frac{5}{16} - 1 = \frac{5}{16}''$
Taper per inch $= \frac{5}{16} \div 3\frac{1}{4} = \frac{5}{16} \times \frac{4}{13} = \frac{5}{52}''$
Total length $= 4'\ 1\frac{1}{8}'' = 49\frac{1}{8}''$
Taper in whole length of rod $= 49\frac{1}{8} \times \frac{5}{52} = 4.724''$
Tail stock offset $= \frac{1}{2}$ of $4.724 = 2.362''$ or $2\frac{23}{64}''$

To turn the taper at the right end we again find the taper in the entire length of the rod as if it were tapered the same amount per inch as the right end B. The tail stock offset will be one-half of the total taper.

Taper in part $B = 1\frac{1}{2} - 1\frac{1}{16} = \frac{7}{16}''$
Taper per inch $= \frac{7}{16} \div 4\frac{3}{4} = \frac{7}{16} \times \frac{4}{19} = \frac{7}{76}''$
Overall taper $= 49\frac{1}{8} \times \frac{7}{76} = 4.525''$
Tail stock offset $= \frac{1}{2}$ of $4.525 = 2.263''$ or $2\frac{17}{64}''$

PROBLEMS

1. Compute the tail stock offset for turning the taper on the piece of work in Fig. 312.

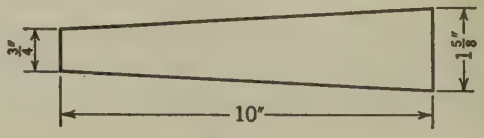


Fig. 312

2. Compute the tail stock offset for cutting the taper in Problem 5, page 349.

3. Compute the tail stock offset for cutting the taper in Problem 6, page 349.

4. Compute the tail stock offset for cutting the taper in Problem 7, page 349.

5. Compute the tail stock offset for cutting the taper in Problem 8, page 349.

6. Compute the tail stock offset for cutting the taper in Problem 9, page 350.

7. Compute the tail stock offset for cutting the taper in Problem 10, page 350.

8. Compute the tail stock offset for cutting the taper in Problem 11, page 350.

9. Compute the tail stock offset for cutting the taper in Problem 12, page 350.

10. Compute the tail stock offsets for turning the tapers at the ends of the piston rod in Fig. 313.

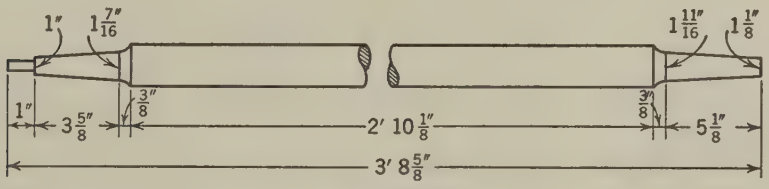


Fig. 313

228. Taper Turning by Use of the Compound Rest

This method is used where short tapers or tapers having a considerable angle are to be turned. To set the compound rest for cutting a taper, we must know the included angle, that is, the taper angle. See Fig. 314.

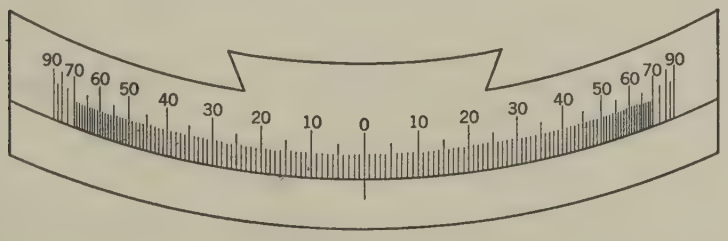


Fig. 314

Example 1

Compute the setting of the compound rest for cutting the taper on the piece in Fig. 315.

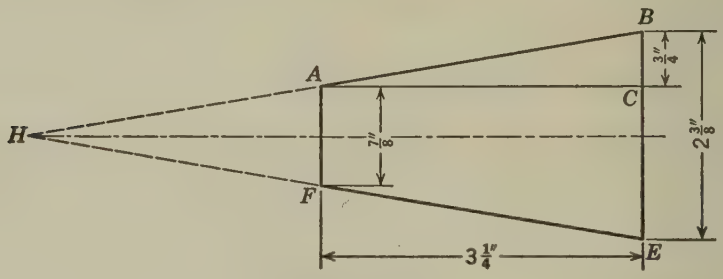


Fig. 315

SOLUTION.

$$\text{Tangent } BAC = \text{Tangent of } \tfrac{1}{2} \text{ included angle} = \frac{BC}{AC} = \frac{\frac{3}{4}}{3\frac{1}{4}}$$

$$= \tfrac{3}{4} \times \tfrac{4}{13} = \tfrac{3}{13} = .23077$$

$$\text{Angle } BAC = \tfrac{1}{2} \text{ Taper angle} = 13^\circ\, 0'$$

$$\text{Included angle} = 2 \times 13^\circ\, 0' = 26^\circ\, 0'$$

To turn this taper, the compound rest is not set to read the included angle, 26°, but to *90° minus half the included angle;* 90° − $\frac{1}{2}$ of 26° = 90° − 13° = 77° 0′.

Definition

The complement of an angle is equal to 90° minus the given angle. For example, the complement of 30° = 90° − 30° = 60°; the complement of 6° 50′ is 90° − 6° 50′ = 83° 10′.

We can therefore state the formula for setting the compound rest as follows:

Rule

Set the compound rest to the complement of one-half the included angle.

Example 2

To what angle must the compound rest be set to turn the lathe center which has an included angle of 60°? See Fig. 316.

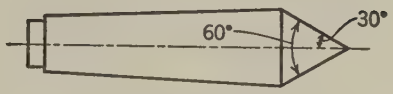


Fig. 316

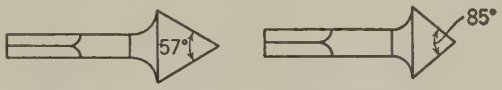
SOLUTION. Included angle = 60°

$\frac{1}{2}$ included angle = $\frac{1}{2}$ of 60° = 30°

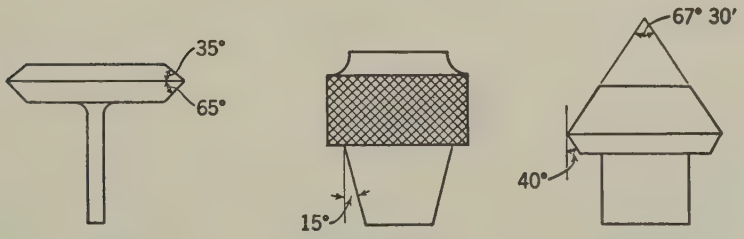
Setting of compound rest = 90° − 30° = 60°

PROBLEMS

1. To what angle must the compound rest be set to cut the tapers and bevels shown in Fig. 317?



a. Countersink *b.* Countersink



c. Valve *d.* Drill Chuck *e.* Bevel Gear

Fig. 317

2. Compute the angle for cutting the following tapers with the compound rest: Brown and Sharpe, Morse No. 5, $\frac{3}{4}''$ per foot series.

3. Figure the compound rest setting for cutting the inside bevel on the taper bushing in Fig. 318.

Fig. 318

4. Figure the compound rest setting for cutting the taper on the collet in Fig. 319.

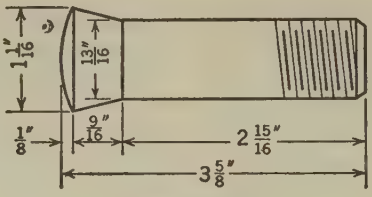


Fig. 319

5. Compute the compound rest setting for cutting the tapers at both ends of the piston rod in Fig. 320.

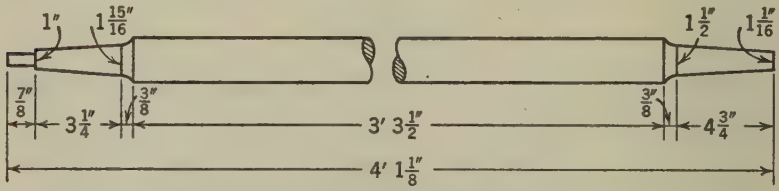


Fig. 320

6. How would you set the compound rest for cutting the following included angles: 5°; 3½°; 7° 30′; 8° 20′; 16° 50′?

229. Taper Turning by Use of the Taper Attachment

To turn a taper by means of the taper attachment, we compute the taper per foot, disregarding the length of the tapered portion and the overall length of the work. In one type of taper attachment the swivel bar which controls the taper is graduated at one end in inches per foot of taper and at the other end in degrees. The attachment can be set for any taper up to 3″ per foot, which corresponds to a taper angle of 14° 15′. See Fig. 321. In another type of attach-

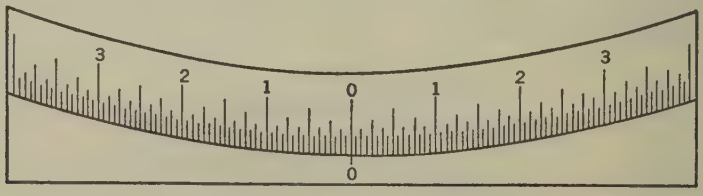


Fig. 321

ment the graduations at one end give the taper in eighths of an inch per foot, and the other end gives it in tenths of an inch per foot.

Example

Compute the setting of the taper attachment for turning the taper shown in Fig. 322.

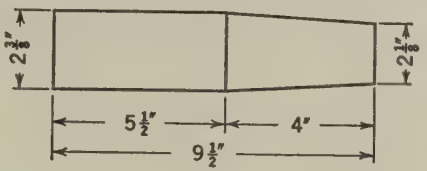


Fig. 322

SOLUTION. Taper in $4'' = 2\frac{3}{8} - 2\frac{1}{8} = \frac{1}{4}''$

Taper per inch $= \frac{1}{4} \div 4 = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}''$

Taper per foot $= 12 \times \frac{1}{16} = \frac{3}{4}''$ or $\frac{6}{8}''$

Adjust the taper attachment so that it reads the required taper per foot, namely, $\frac{6}{8}''$ per foot.

PROBLEMS

1. How would you set the taper attachment for turning a piece of work having a Brown and Sharpe taper? Morse No. 1? $\frac{3}{4}''$ per foot?

2. How should the taper attachment be set to turn tapers having the following included angles: 4° 30′; 3° 00′; 5° 30′; 7° 00′; 9° 40′?

3. Compute the setting of the taper attachment for cutting the tapers at both ends of the piston rod shown in Fig. 293.

4. Compute the setting of the taper attachment for turning the taper on the taper socket in Fig. 285.

5. Compute the setting of the taper attachment for turning the outside taper on the bushing in Fig. 286.

6. Compute the setting of the taper attachment for turning the taper on the reamer in Fig. 287.

7. Compute the setting of the taper attachment for turning the taper on the collet in Fig. 288.

8. Compute the setting of the taper attachment for turning the taper on the milling machine arbor in Fig. 289.

9. Compute the setting of the taper attachment for turning the outside taper on the drill socket in Fig. 290.

10. Compute the setting of the taper attachment for turning the taper on the drill shank in Fig. 291.

11. Compute the setting of the taper attachment for turning the taper on the mandrel in Fig. 292.

19 Speed Ratios of Pulleys and Gears

230. Gear Trains

In trains of gears, the gears that transmit power are called ***driving gears,*** whereas the others are ***followers*** or ***driven gears.***

Two meshing gears, as A and B, Fig. 323, with the same number of teeth, will revolve at the same rate of speed. Assume A to be the driving gear and B the driven. If A revolves *clockwise,* that is, in the direction of the hands of a clock, as shown by the arrow, it will cause B to revolve in the opposite direction or counter-clockwise.

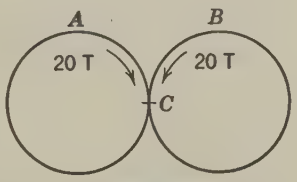


Fig. 323

Let each gear have 20 teeth. When A revolves, every tooth on it passes the point C and engages a tooth on B. When A has made one complete turn, that is, when its 20 teeth have passed the point C, they will have engaged 20 teeth on gear B, thus causing B also to make one complete turn.

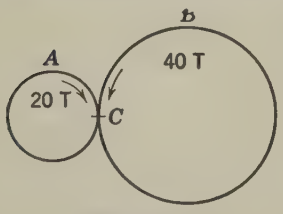


Fig. 324

If, however, A has 20 teeth and B 40 teeth, Fig. 324, in one complete turn of A its 20 teeth will pass C and will engage 20 teeth on gear B. But, since gear B has 40 teeth, gear A will have to make two complete revolutions in order

to cause B to make one complete turn. In this case the gears are said to have a speed ratio of 1 to 2; that is, one revolution of B corresponds to two revolutions of A. If A had 20 teeth and B 60 teeth, A would make three revolutions while B made one. The gears would then have a speed ratio of 1 to 3. In each case the *smaller* gear is the *faster* and the *larger* gear the *slower*.

The relations existing between the speeds of gears and their numbers of teeth is called an ***inverse ratio;*** that is, the speeds of gears vary not in the same order as the number of their teeth but in the inverse order. In other words, instead of the speed of a gear increasing with an increase in its number of teeth, it diminishes.

The rule governing the speeds of gears can be expressed as follows:

$$\frac{\text{Teeth on driven}}{\text{Teeth on driver}} = \frac{\text{Revolutions of driver}}{\text{Revolutions of driven}}$$

To find the speed of either gear, multiply the size and speed of the other gear and divide by the size of the gear whose speed is sought.

Example 1

Two meshing gears have 40 and 60 teeth respectively. If the small gear makes 120 r.p.m., how many revolutions per minute will the large gear make?

SOLUTION. $$\text{R.p.m. of large gear} = \frac{\text{Size} \times \text{R.p.m. of small gear}}{\text{Size of large gear}}$$

$$= \frac{40 \times 120}{60} = 80 \text{ r.p.m.}$$

Example 2

In Fig. 325 A has 50 teeth and B 80 teeth. If B makes 100 r.p.m., how many revolutions per minute will A make?

SOLUTION. $$\text{R.p.m. of } A = \frac{\text{Teeth} \times \text{R.p.m. of } B}{\text{Teeth of } A} = \frac{80 \times 100}{50}$$

$$= 160 \text{ r.p.m.}$$

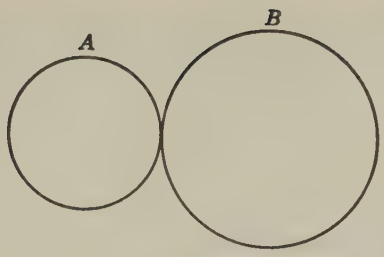


Fig. 325

To find the number of teeth on either gear, multiply the size and speed of the other gear and divide by the speed of the one whose number of teeth is sought.

Example 3

The driving gear has 24 teeth and makes 120 r.p.m. How many teeth must the driven gear have to make 80 r.p.m.?

SOLUTION. $$\text{Teeth of driven} = \frac{\text{Teeth} \times \text{R.p.m. of driver}}{\text{R.p.m. of driven}}$$

$$= \frac{24 \times 120}{80} = 36 \text{ teeth}$$

PROBLEMS

1. A gear having 120 teeth meshes with one having 24 teeth. If the large gear makes 1 revolution, how many revolutions will the small gear make?

2. In Fig. 326, gear A has 36 teeth and B has 96 teeth. If gear A makes 75 r.p.m., how many revolutions per minute will gear B make?

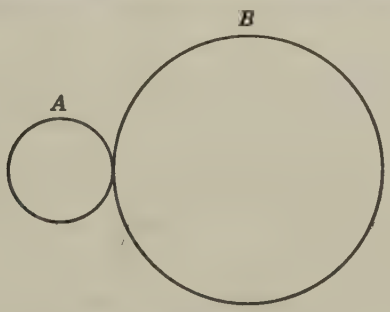


Fig. 326

3. In Fig. 327, gear A has 56 teeth and gear B has 80 teeth. If gear A makes 210 r.p.m., how many revolutions per minute will gear B make?

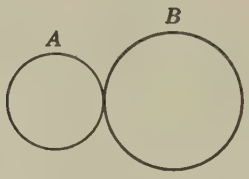


Fig. 327

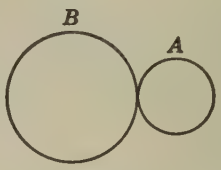


Fig. 328

4. In Fig. 328, gear A has 40 teeth and makes 180 r.p.m. If gear B is to make 120 r.p.m., how many teeth must it have?

5. A 48-tooth gear running 320 r.p.m. is required to drive another gear at 240 r.p.m. How many teeth must the driven gear have?

231. Idlers

If we desire to transmit power from shaft A to shaft B, Fig. 329, and the distance between the shafts is so great that

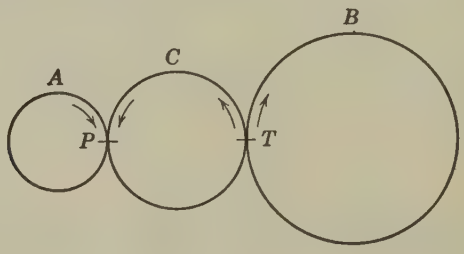


Fig. 329

it would require unusually large gears to bridge the gap, resort is had to idler gears. Gear C in Fig. 329 is such an idler.

Assume A has 20 teeth, C 40 teeth, and B 80 teeth. When A revolves once, C makes $\frac{1}{2}$ a revolution; and, while C revolves once, B makes $\frac{1}{2}$ a revolution; so that for one revolution of A the gear B makes $\frac{1}{4}$ of a revolution. That is, the speed of B is the same as if it were in direct mesh with gear A. Hence the idler has no effect on the relative speeds of the gears between which it is placed and it is omitted from all computations.

The only effect of the idler is to reverse the direction of the motion of B. Without the idler, it was seen in Fig. 323 and

in Fig. 324 that B turned counterclockwise while A turned clockwise. With the idler interposed between A and B, when A turns clockwise B also turns clockwise, as can readily be seen by a study of Fig. 329.

In case of two idlers as C and D in Fig. 330, a study of the sketch will show that the direction of rotation of B is the same as when there are no idlers. That is, idler D neutralizes the effect of idler C. In general, the effect of idlers on

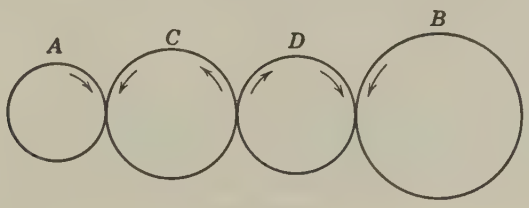


Fig. 330

the direction of motion of a follower or driven gear may be stated thus: An odd number of idlers between two gears will cause the driven gear to rotate in the same direction as the driver; an even number of idlers will cause the driven gear to rotate in a direction opposite to that of the driver.

232. To Find the Number of Teeth for a Given Speed Ratio

The method of computing the number of teeth in gears that will give a desired speed ratio is illustrated by the following example:

Example

Find two suitable gears that will give a speed ratio between driver and driven of 2 to 3.

SOLUTION. $$\frac{2}{3} = \frac{2 \times 12}{3 \times 12} = \frac{24 \text{ teeth on follower}}{36 \text{ teeth on driver}}$$

EXPLANATION. Express the desired ratio as a fraction and multiply both terms of the fraction by any convenient multiplier that will give an equivalent fraction whose numerator and de-

nominator will represent available gears. In this instance 12 was chosen as a multiplier giving the equivalent fraction $\frac{2}{3}\frac{4}{6}$. Since the speed of the driver is to the speed of the follower as 2 to 3, the driver is the larger gear and the driven is the smaller gear.

PROBLEMS

1. The speeds of two gears are in the ratio of 1 to 3. If the faster one makes 180 r.p.m., find the speed of the slower one.

2. The speed ratio of two gears is 1 to 4. The slower one makes 45 r.p.m. How many revolutions per minute does the faster one make?

3. Two gears are to have a speed ratio of 2.5 to 3. If the larger gear has 72 teeth, how many teeth must the smaller one have?

4. Find two suitable gears with a speed ratio of 3 to 4.

5. Find two suitable gears whose speed ratio shall be as 3 to $5\frac{1}{2}$.

6. In Fig. 331, A has 24 teeth, B has 36 teeth, and C has 40 teeth. If gear A makes 200 r.p.m., how many revolutions per minute will gear C make?

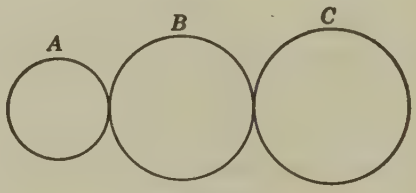


Fig. 331

7. In Fig. 332, A has 36 teeth, B has 60 teeth, C has 24 teeth, and D has 72 teeth. How many revolutions per minute will gear D make if gear A makes 175 r.p.m.?

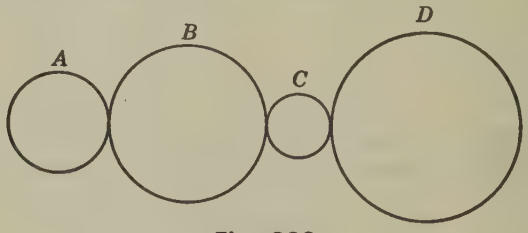


Fig. 332

233. Compound Gearing

Figure 333 shows a train of compound gears. The driver A is in mesh with B, making B a driven gear. Gear C, keyed

to the same shaft as B, is also a driver and meshes with D, so that D is a driven gear. Gears B and C, being keyed to the same shaft, revolve at the same rate of speed.

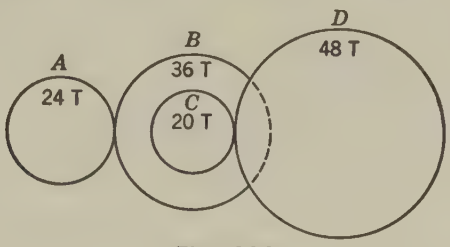


Fig. 333

The following example illustrates the method of finding the speed of a driven gear of a compound train.

Example 1

In Fig. 333, if A makes 72 r.p.m., how many revolutions per minute will gear D make?

SOLUTION. $72 \times \frac{24}{36} \times \frac{20}{48} = 20$ r.p.m.

EXPLANATION. Since B has 36 teeth and A has 24 teeth, B will make $\frac{24}{36}$ as many revolutions as A. That is, $72 \times \frac{24}{36}$ represents the speed of B. But C, being keyed to the same shaft as B, revolves at the same rate of speed; hence, when A makes 72 revolutions, C will make $72 \times \frac{24}{36}$ revolutions. Since D has 48 teeth, whereas C has 20 teeth, D will make $\frac{20}{48}$ as many revolutions as C, or $72 \times \frac{24}{36} \times \frac{20}{48} = 20$ r.p.m.

A similar analysis is applicable to the case where the number of teeth of a driven gear is wanted, or the speed or the number of teeth of a driver is desired.

In the solution of the preceding example it will be noticed that all the numbers in the numerator applied to the drivers while those in the denominator applied to the driven gears. This fact enables us to state the following rule:

Rule

To find the revolutions per minute or number of teeth of any driven gear in a train, divide the continued product of the given

factors of the driving gears by the continued product of the given factors of the driven gears.

Example 2

In Fig. 333, if A makes 72 r.p.m., how many teeth must D have if it is to make 20 r.p.m.?

SOLUTION. $$\frac{\text{Product of driver data}}{\text{Product of driven data}} = \frac{72 \times 24 \times 20}{36 \times 20}$$

$$= 48 \text{ teeth}$$

To find the number of teeth or revolutions per minute of a driving gear, divide the continued product of all that is known about the driven gears by the continued product of all that is known about the driving gears.

Example 3

In Fig. 333, how many revolutions per minute must gear A make if gear D is required to make 20 r.p.m.?

SOLUTION. $$\frac{\text{Product of driven data}}{\text{Product of driver data}} = \frac{36 \times 48 \times 20}{24 \times 20}$$

$$= 72 \text{ r.p.m.}$$

If a certain speed ratio is required between the first driver and the last driven in a compound gear train, we proceed as in the following example.

Example 4

Find four suitable gears that will transmit motion from A to D at the ratio of 3 to 5. See Fig. 334.

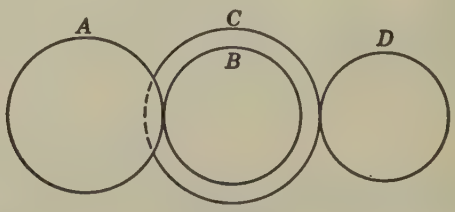


Fig. 334

SOLUTION.

$$\frac{3}{5} = \frac{2}{2\frac{1}{2}} \times \frac{1\frac{1}{2}}{2} = \frac{2 \times 16}{2\frac{1}{2} \times 16} \times \frac{1\frac{1}{2} \times 24}{2 \times 24} = \frac{\overset{B}{32} \times \overset{D}{36}}{\underset{A}{40} \times \underset{C}{48}} \begin{matrix} = \text{Followers} \\ = \text{Drivers} \end{matrix}$$

EXPLANATION. Write the desired speed ratio as a fraction, $\frac{3}{5}$. Split each term into two factors, namely, $3 = 2 \times 1\frac{1}{2}$ and $5 = 2\frac{1}{2} \times 2$, giving the two fractions, $\frac{2}{2\frac{1}{2}}$ and $\frac{1\frac{1}{2}}{2}$. Treat each of these fractions separately as in the case of a simple gear train; that is, multiply each term by some convenient multiplier which will give an equivalent fraction whose terms will express available gears. The terms of the fraction $\frac{2}{2\frac{1}{2}}$ are multiplied by 16, giving the gears 32 and 40; and the terms of the fraction $\frac{1\frac{1}{2}}{2}$ are multiplied by 24, giving the gears 36 and 48. Since speeds of gears are inversely proportional to the number of their teeth, we have the proportion:

$$\frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{Teeth of driven}}{\text{Teeth of driver}}$$

Hence the two numbers in the numerator of the result represent driven gears in the train while the two numbers in the denominator represent drivers. In mesh they would appear as in Fig. 334, where A drives B and C drives D.

234. Worm and Gear

To find the revolutions per minute of a worm wheel when we know the revolutions per minute of the worm and also whether it is single or double threaded, we proceed as in the following example.

Example 5

If the worm A is triple-threaded and makes 100 r.p.m., how many revolutions per minute will the worm gear B make if it has 50 teeth? See Fig. 335.

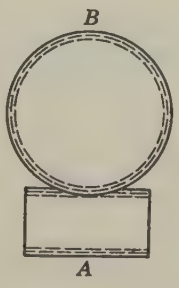


Fig. 335

SOLUTION. $\dfrac{100 \times 3}{50} = 6$ r.p.m.

EXPLANATION. The worm thread is regarded as a gear with 1 tooth if single-threaded, 2 teeth if double-threaded, etc. Hence, to find the revolutions per minute of the worm gear, we multiply the revolutions per minute and the number of threads of the worm and divide by the number of teeth on the wheel.

235. Trains of Spur, Bevel, and Worm Gearing

Problems involving speed ratios of bevel gears are solved exactly as problems with spur gears.

Example 6

If gear F in Fig. 336 makes 240 r.p.m. and has 48 teeth, E has 32 teeth, D has 20 teeth, C has 24 teeth, worm A is double-threaded,

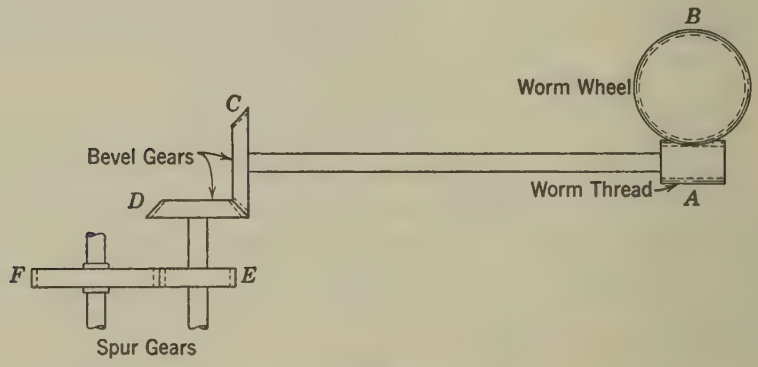


Fig. 336

and worm gear B has 40 teeth, how many revolutions per minute will gear B make?

SOLUTION. R.p.m. of gear $B = \dfrac{\text{Product of driver data}}{\text{Product of driven data}}$

$$= \frac{240 \times 48 \times 20 \times 2}{32 \times 24 \times 40}$$

$$= 15 \text{ r.p.m.}$$

236. Pulley Trains

The formulas used for gear trains are applicable also to pulley trains, except that in the case of pulleys the diameter is used instead of the number of teeth.

Example 7

Pulley A in Fig. 337 is 8″ in diameter and makes 120 r.p.m. How many revolutions per minute does pulley B make if its diameter is 12″?

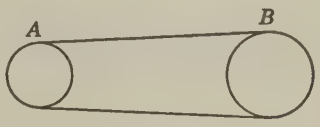


Fig. 337

SOLUTION.

$$\text{R.p.m. of } B = \frac{8 \times 120}{12} = 80$$

Example 8

In Fig. 338 pulley A is 10″ in diameter and makes 140 r.p.m. What must be the diameter of B if it is required to make 100 r.p.m.?

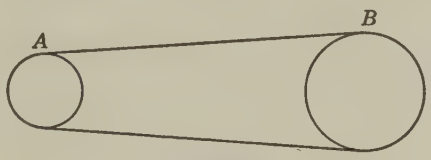


Fig. 338

SOLUTION. $$\text{Diameter of } B = \frac{10 \times 140}{100} = 14''$$

Example 9

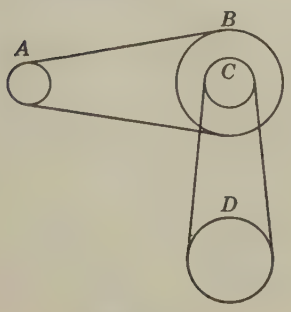


Fig. 339

In Fig. 339 A is 8″ in diameter, B is 20″, C is 10″, and D is 16″ in diameter. If A makes 200 r.p.m., how many revolutions per minute will D make?

SOLUTION.

$$\text{Speed of } D = \frac{\text{Product of all driver data}}{\text{Product of all driven data}}$$

$$= \frac{200 \times 8 \times 10}{20 \times 16} = 50 \text{ r.p.m.}$$

PROBLEMS

1. In Fig. 340, gear A has 28 teeth, B has 48 teeth, C has 24 teeth, and D has 32 teeth. If gear A makes 180 r.p.m., how many revolutions per minute will gear D make?

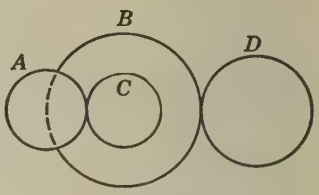


Fig. 340

2. Figure 341 shows a compound gear train. Find the speed ratio between shaft A and shaft D.

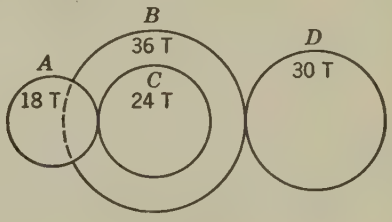


Fig. 341

3. If A in Fig. 342 makes 42 r.p.m., how many teeth must D have in order to make 105 r.p.m.?

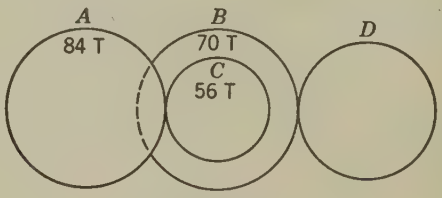


Fig. 342

4. In Fig. 343, if A makes 1 revolution, how many revolutions will D make?

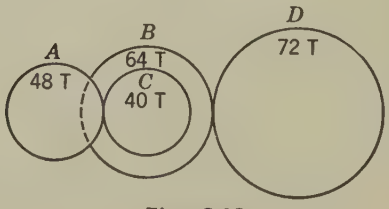


Fig. 343

5. In Problem 4, what must be the speed of A in order to cause D to make 50 r.p.m.?

6. In Fig. 344, how many teeth must gear C have if gear A makes 100 r.p.m. and gear D makes 180 r.p.m.?

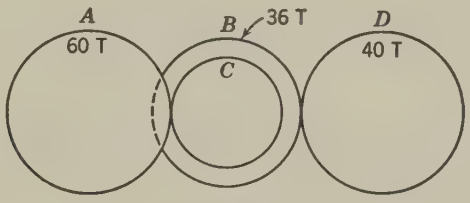


Fig. 344

7. In Fig. 345, gear A has 40 teeth; B, 36 teeth; C, 60 teeth; D, 48 teeth; and E, 72 teeth. How many revolutions per minute will E make if A makes 240 r.p.m.?

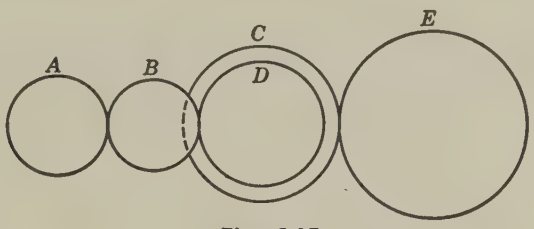


Fig. 345

8. Gear E in Fig. 346 is required to make 100 r.p.m. while A makes 75 r.p.m. How many teeth must E have if A has 24 teeth, B 64, C 32, and D 40 teeth?

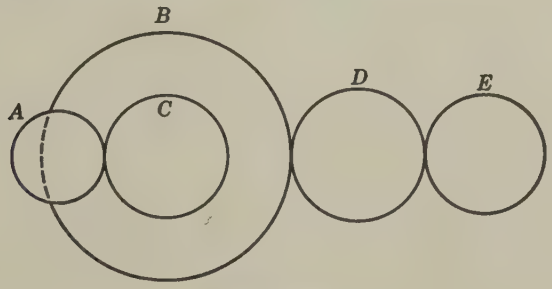


Fig. 346

9. If a speed ratio of 2 to 5 is required between A and D, in Fig. 347, what gears would you use?

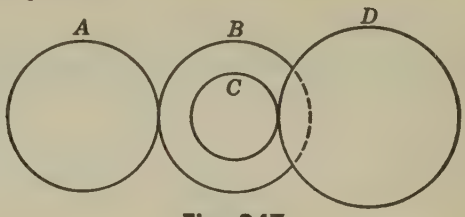


Fig. 347

10. The smaller pulley in Fig. 348 is 12″ in diameter and the larger 20″. How many revolutions per minute will the larger pulley make if the smaller pulley makes 80 r.p.m.?

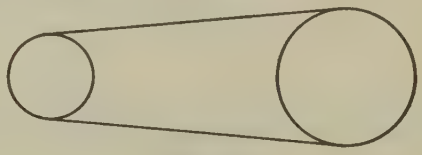


Fig. 348

11. If the large pulley in Fig. 349 is 18″ in diameter and makes 60 r.p.m., what must be the diameter of the small pulley to make 90 r.p.m.?

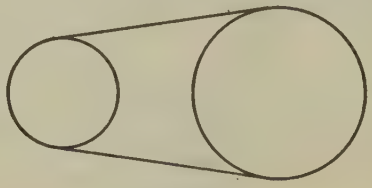


Fig. 349

12. In Fig. 350, pulley A is 14″ in diameter, B 10″, C 16″, and D 12″. If A makes 120 r.p.m., how many revolutions per minute will D make?

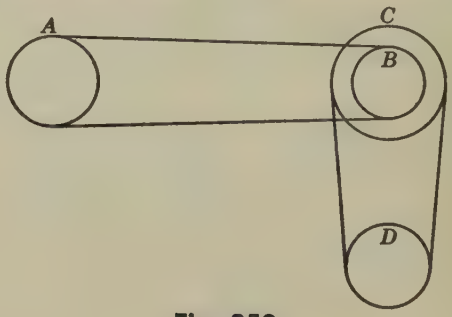


Fig. 350

13. The pulleys of the train in Fig. 351 have the following diameters: A 12″, B 20″, C 8″, D 16″, E 10″, F 12″. If A makes 75 r.p.m. how many revolutions per minute will F make?

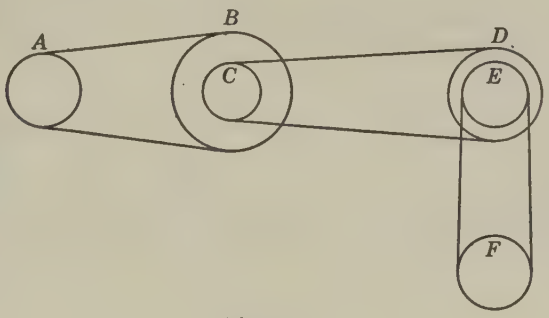


Fig. 351

14. It is required to transmit power from A to D in Fig. 352 at a speed ratio of 2 to 3. Find four suitable pulleys for this train.

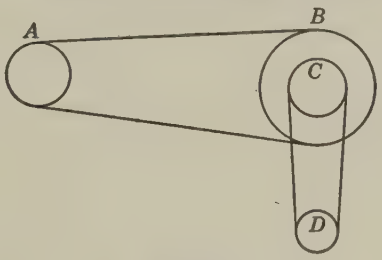


Fig. 352

15. A cone pulley having steps of 11″, 9″, and $6\frac{1}{2}$″ in diameter, and running at 140 r.p.m., drives another cone pulley having diameters of 5″, 7″, and $9\frac{1}{2}$″. What speeds can be obtained?

16. A driving pulley 36″ in diameter on a line shaft making 200 r.p.m. is belted to a 16″ pulley on a countershaft. The countershaft has a pulley 14″ in diameter belted to a 6″ pulley on an emery wheel. How many revolutions per minute will the emery wheel make?

17. A countershaft running at 240 r.p.m. has a cone pulley with steps 11″, 8″, and 5″ in diameter driving a cone on a lathe with steps 6″, 9″, and 12″ in diameter. What range of speeds can be obtained?

18. In Fig. 336, the driving gear F has 44 teeth and makes 280 r.p.m. Gear E has 28 teeth, D 36 teeth, C 24 teeth, and the worm A is triple-threaded. If the worm gear has 32 teeth, how many revolutions per minute will it make?

19. If the single-threaded worm A in Fig. 336 makes 140 r.p.m., how many revolutions per minute will the worm wheel B make if it has 48 teeth?

20. In Fig. 353, A has 60 teeth and makes 160 r.p.m. B has 28 teeth, C 24 teeth, D 20 teeth, E 36 teeth, and F 32 teeth. How many revolutions per minute will F make?

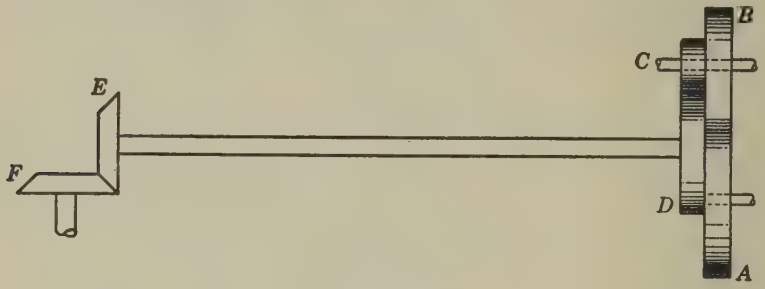


Fig. 353

21. In Fig. 353, how many teeth must F have if it is required to make 320 r.p.m. and the other conditions are as follows: A makes 120 r.p.m. and has 72 teeth, B has 24 teeth, C 40 teeth, D 36 teeth, and E 32 teeth?

20 Screw Threads

237. Pitch

The *pitch* of a screw thread is the distance between corresponding points of two adjacent threads. In Fig. 354 the space marked P indicates the pitch for the threads shown.

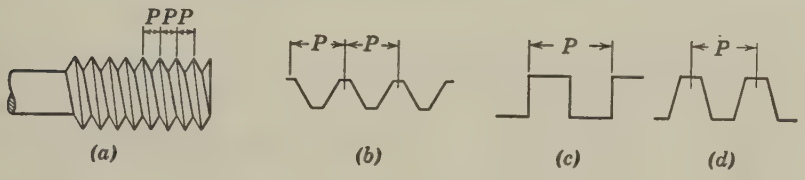


Fig. 354

If, for example, the distance between threads is $\frac{1}{8}''$, we say that the thread is $\frac{1}{8}''$-pitch. It is evident that, if the pitch is $\frac{1}{8}''$, there are eight threads to the inch. Hence a machinist will often speak of an 8-pitch thread instead of a $\frac{1}{8}''$-pitch thread. In this chapter both methods of designating pitch will be employed, as the meaning in either case is unmistakable.

From the definition of pitch it follows that:

$$\text{Pitch} = \frac{1}{\text{Number of threads per inch}}$$

and

$$\text{Number of threads per inch} = \frac{1}{\text{Pitch}}$$

Example 1

Find the pitch of a thread having 12 threads per inch.

SOLUTION. $$\text{Pitch} = \frac{1}{\text{Number of threads per inch}} = \frac{1''}{12}$$

Example 2

Find the pitch of a screw having $4\frac{1}{2}$ threads per inch.

SOLUTION. $$\text{Pitch} = \frac{1'}{\text{Number of threads per inch}}$$

$$= \frac{1}{4\frac{1}{2}} = 1 \div \frac{9}{2} = 1 \times \frac{2}{9} = \frac{2''}{9}$$

Example 3

A screw has a pitch of $\frac{5}{8}''$. Find the number of threads per inch.

SOLUTION.

$$\text{Number of threads per inch} = \frac{1}{\text{Pitch}} = \frac{1'}{\frac{5}{8}} = 1 \times \frac{8}{5}$$

$$= \frac{8}{5} = 1\frac{3}{5} \text{ threads per inch}$$

PROBLEMS

1. What is the pitch of a screw having $3\frac{1}{2}$ threads per inch?
2. A screw having 5 threads per inch has what pitch?
3. Find the pitch of a screw having $5\frac{1}{2}$ threads per inch.
4. Find the pitch of a screw having 18 threads per inch.
5. Compute the pitch of a screw with $2\frac{3}{4}$ threads per inch.
6. A $\frac{3}{4}''$-pitch screw has how many threads per inch?
7. How many threads per inch on a screw of $\frac{2}{7}''$ pitch?
8. A screw has a pitch of $\frac{3}{8}''$. Find the number of threads per inch.
9. Find the number of threads per inch in a $\frac{7}{16}''$-pitch screw.
10. The pitch of a thread is $\frac{1}{14}''$. How many threads are there in $1''$?

238. Lead

The *lead* of a screw is the distance the nut advances on the screw in one turn, or the distance the screw would advance if given one complete turn.

In a *single-threaded* screw the lead is *equal* to the pitch; that is, in one complete turn, the screw advances a distance equal to the pitch. In a *double-threaded* screw the lead is *twice* the pitch; in a *triple-threaded* screw the lead is *three times* the pitch; etc.

In a screw of $\frac{1}{8}''$-pitch the lead has the following values:

Single-threaded, lead = pitch = $\frac{1}{8}''$

Double-threaded, lead = $2 \times$ pitch = $2 \times \frac{1}{8} = \frac{1}{4}''$

Triple-threaded, lead = $3 \times$ pitch = $3 \times \frac{1}{8} = \frac{3}{8}''$

Figure 355 illustrates the relation between pitch and lead in a single, double, triple, and quadruple square thread.

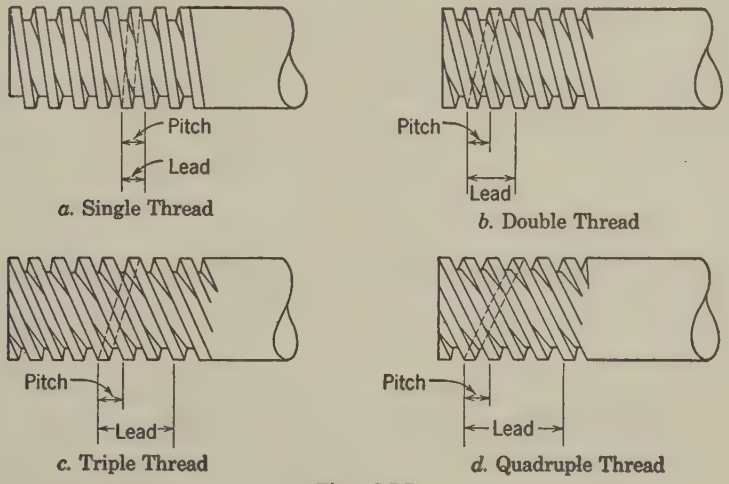


Fig. 355

PROBLEMS

1. A single-threaded screw has 12 threads per inch. What is its pitch? What is its lead?
2. What is the lead of an 8-pitch double-threaded screw?
3. What is the lead of a triple 8-pitch thread?
4. The lead of a double-threaded screw is $\frac{3}{8}''$. What is its pitch?
5. What is the lead of a $\frac{3}{16}''$-pitch quadruple thread?
6. Find the pitch of a screw with a triple thread whose lead is $\frac{3}{4}''$.
7. A single thread of $\frac{1}{16}''$-pitch has what lead?
8. What is the pitch of a double-threaded screw having a lead of $\frac{3}{32}''$?

239. Definitions Applying to Screw Threads

Figure 356 illustrates the following terms:

The *top* of the thread is called the *point* or *crest.*

The ***bottom*** of the thread is called the *root.*

The ***depth*** of the thread is the vertical distance between the top and the bottom of the thread. Twice the depth is known as the ***double depth.***

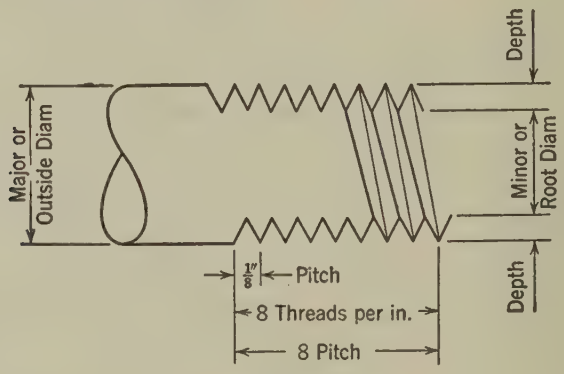


Fig. 356

The major ***diameter,*** sometimes referred to as the *outside* diameter, is the largest diameter of the thread of the screw or nut.

The ***minor*** or ***root diameter*** is the smallest diameter of the thread of the screw or nut; that is, it is the diameter at the roots or bottoms of the threads. It is evident from the sketch that the root diameter of a thread is equal to the major diameter minus the double depth.

240. Sharp V-Thread

The sharp V-thread is so named because the sides of the thread form the letter V. A horizontal line joining two successive roots will form with the sides of the thread, an equilateral triangle ABC (Fig. 357). If the pitch of the thread is 1″, the sides of the triangle will be 1″ each. The depth of the thread, h, which is a perpendicular from the crest C

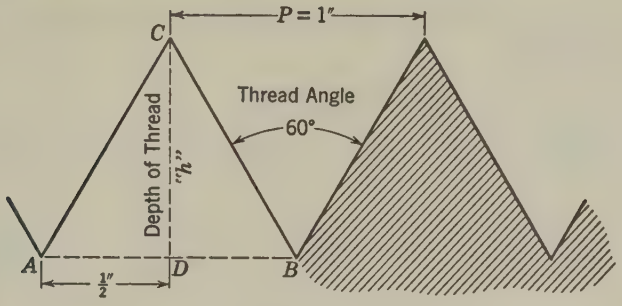


Fig. 357

to the side AB, divides the triangle into two equal right triangles. In the right triangle, ACD,

$$h = \sqrt{\overline{AC}^2 - \overline{AD}^2} = \sqrt{1^2 - (\tfrac{1}{2})^2} = \sqrt{1 - \tfrac{1}{4}} = \sqrt{\tfrac{3}{4}} = .866''$$

That is, the depth of a sharp V-thread of 1″ pitch is .866″. The depth of a ½″-pitch V-thread is equal to one-half of 0.866, or 0.433″. Hence the rule:

Rule

The depth of a sharp V-thread is found by multiplying the pitch by 0.866.

Example

Find the depth of a V-thread having 16 threads per inch.

SOLUTION. Depth = Pitch × 0.866 = $\frac{1}{16}$ × 0.866 = .0541″

PROBLEMS

Find the depth of V-threads having the following number of threads per inch.

1. 20	**2.** 18	**3.** 14	**4.** 12	**5.** 11
6. 9	**7.** 8	**8.** 7	**9.** 6	**10.** 5

241. Double Depth of Sharp V-Thread

Twice the depth of a thread is called the *double depth.* The double depth of a 1″-pitch sharp V-thread is 2 × .866

$= 1.732''$; the double depth of a $\frac{1}{4}''$-pitch V-thread is $\frac{1}{4} \times 1.732 = .433''$.

PROBLEMS

Find the double depth of V-threads with the following number of threads per inch.

1. 3	2. $3\frac{1}{4}$	3. $3\frac{1}{2}$	4. 4	5. $4\frac{1}{2}$
6. 5	7. 8	8. 12	9. 16	10. 20

242. Minor Diameter

Figure 356 shows that the minor diameter of a screw thread is equal to the major diameter minus the double depth.

Example

Find the minor diameter of a $\frac{3}{8}''$–16 V-thread. ($\frac{3}{8}''$ = major diameter; 16 = number of threads per inch.)

SOLUTION.

Major diameter $= \frac{3}{8}$	$= 0.3750''$
Double depth $= \frac{1}{16} \times 1.732$	$= 0.1082''$
Minor diameter	$= 0.2668''$

PROBLEMS

Find the minor diameter of the following V-thread screws.

1. $\frac{1}{4}''$–20	2. $\frac{7}{16}''$–14	3. $\frac{9}{16}''$–12	4. $\frac{13}{16}''$–10	5. $\frac{15}{16}''$–9
6. $1\frac{1}{4}''$–7	7. $1\frac{1}{2}''$–6	8. $1\frac{3}{4}''$–5	9. $2\frac{1}{4}''$–$4\frac{1}{2}$	10. $2\frac{7}{8}''$–4

243. Tap Drill Sizes

To tap a nut or hole is to cut a thread in it. A tap drill is used for drilling holes previous to tapping. Although theoretically the diameter of the drill is equal to the minor diameter of the bolt for which the nut is to be used, in practice the tap drills are somewhat larger than the minor diameter of the thread. See Fig. 358. The tap-drill diameter for a sharp V-thread is equal to the minor diameter plus one-quarter of the pitch.

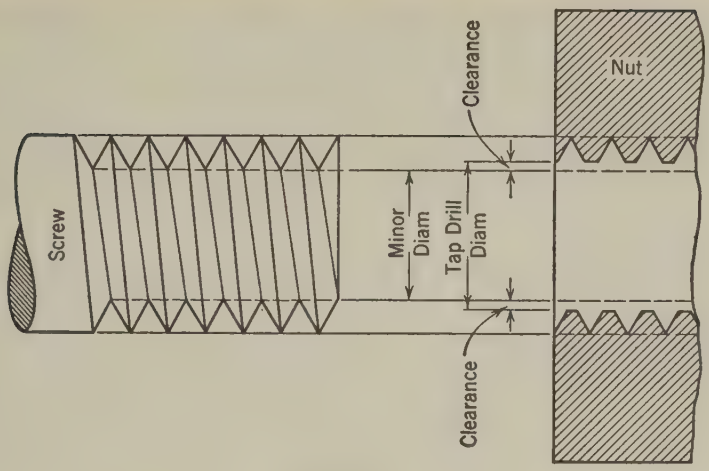


Fig. 358

Example

Find the proper size tap drill for a $\frac{7}{16}''$–14 V-thread.

SOLUTION.

Major diameter	$= \frac{7}{16}''$	$= .4375''$
Double depth	$= \frac{1}{14} \times 1.732$	$= .1237''$
Minor diameter		$= .3138''$
Clearance	$= \frac{1}{4} \times$ Pitch	
	$= \frac{1}{4} \times \frac{1}{14} = \frac{1}{56}$	$= .0179''$
Drill size		$= .3317''$

According to the table of decimal equivalents, the nearest $\frac{1}{64}''$ to .3317 is .3281, which is the decimal equivalent of $\frac{21}{64}''$. Hence a $\frac{21}{64}''$ drill is to be used.

PROBLEMS

Find the proper tap drill sizes for the following V-threads.

1. $\frac{5}{16}''$–18	2. $\frac{3}{8}''$–16	3. $\frac{1}{2}''$–12	4. $\frac{5}{8}''$–11	5. $\frac{11}{16}''$–11
6. $\frac{3}{4}''$–10	7. $\frac{7}{8}''$–9	8. $1''$–8	9. $1\frac{3}{8}''$–6	10. $2''$–$4\frac{1}{2}$

244. American Standard or American National Thread

There are two series of American Standard threads: (1) the American Standard coarse-thread series and (2) the American Standard fine-thread series.

The American Standard coarse-thread series comprises the former U. S. Standard threads supplemented in the sizes below $\frac{1}{4}''$ by part of the A.S.M.E. Standards. See Table VI.

The American Standard fine-thread series comprises the regular screw thread series of the S.A.E. supplemented in sizes below $\frac{1}{4}''$ by the A.S.M.E. fine-thread series. See Table VII.

The American Standard thread is a 60-degree thread with flattened tops and bottoms. If we start with a V-thread and divide the height or depth into eight equal parts, the American Standard is formed by eliminating $\frac{1}{8}$ of the depth at the top and $\frac{1}{8}$ of the depth at the bottom. The depth of the resulting thread is therefore equal to $\frac{3}{4}$ of the depth of the V-thread of the same pitch.

If, as in Fig. 359, the pitch is $1''$, the depth of the V-thread is $0.866''$, and the depth of the corresponding American

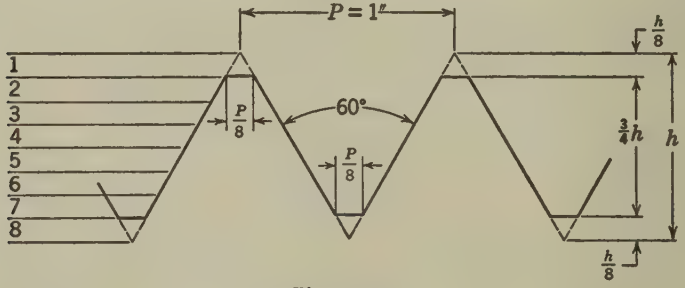


Fig. 359

Standard thread is $\frac{3}{4}$ of 0.866 or $0.6495''$. The width of the thread at top and bottom is $\frac{1}{8}$ of the pitch.

The constant 0.6495 should be memorized by the student to enable him to compute the proportions of an American Standard thread of any pitch.

Example

Find the depth of an American Standard thread having 16 threads to the inch.

SOLUTION. Depth = Pitch $\times$ 0.6495 = $\frac{1}{16} \times 0.6495$

$= 0.0406''$

PROBLEMS

Find the single depth of the following American Standard threads.

1. Seven threads per inch
2. Eight threads per inch
3. $\frac{1}{10}''$ pitch
4. $\frac{1}{12}''$ pitch
5. $5\frac{1}{2}$ threads per inch
6. Thirteen threads per inch

245. Double Depth of American Standard Thread

The double depth of an American Standard thread of $1''$ pitch is obviously equal to $2 \times .6495''$ or $1.299''$. The double depth of an American Standard thread of any pitch is equal to the *pitch multiplied by* 1.299.

Example

Find the double depth of an American Standard thread having 16 threads per inch.

SOLUTION. Double depth = Pitch $\times$ 1.299

$$= \frac{1}{16} \times 1.299 = 0.0812''$$

PROBLEMS

Find the double depth of the following American Standard threads.

1. Pitch = $\frac{1}{20}''$
2. $4\frac{1}{2}$ threads per inch
3. Pitch = $\frac{1}{11}''$
4. Nine threads per inch
5. Six threads per inch
6. Eleven threads per inch

246. Minor Diameter of American Standard Thread

The minor diameter is equal to the major diameter minus the double depth.

Example

Find the minor diameter of a $\frac{5}{8}''$–11 American Standard thread.

SOLUTION.

Major diameter = $\frac{5}{8}''$	$= 0.6250''$
Double depth = $\frac{1}{11} \times 1.299$	$= 0.1181''$
Minor diameter	$= 0.5069''$

PROBLEMS

Find the minor diameter of the following American Standard threads.

1. $\frac{1}{4}''$–20	2. $\frac{3}{8}''$–16	3. $\frac{1}{2}''$–13	4. $\frac{3}{4}''$–10	5. $1''$–8
6. $1\frac{1}{2}''$–6	7. $1\frac{3}{4}''$–5	8. $2''$–$4\frac{1}{2}$	9. $2\frac{3}{4}''$–4	10. $1\frac{1}{8}''$–7

247. Size of Tap Drill for American Standard Threads

1. *Commercial* tap drills are designed to allow an engagement of $\frac{3}{4}$ of the full depth of the screw thread. For a nut to engage the screw $\frac{3}{4}$ of the depth of the thread, the tap drill diameter must be equal to the minor diameter of the screw plus $\frac{1}{4}$ the double depth of the thread.

Example 1

Find the commercial size tap drill (that is, for $\frac{3}{4}$ engagement) for a $\frac{7}{8}''$–9 American Standard coarse-thread.

SOLUTION.

Major diameter $= \frac{7}{8}''$	$= 0.8750''$
Double depth $= \frac{1}{9} \times 1.299$	$= 0.1443''$
Minor diameter	$= 0.7307''$

$$\text{Clearance} = \frac{\text{Double depth of thread}}{4}$$

$$= \frac{.1443}{4} \qquad = 0.0361''$$

$$\text{Drill size} \qquad = 0.7668''$$

The nearest $\frac{1}{64}''$ is $\frac{49}{64}''$. Hence we use a drill $\frac{49}{64}''$ in diameter.

2. The *minimum* standard tap drill adopted by the American Standards Association is for a nut that will engage the screw thread $\frac{5}{6}$ of the depth of the thread. The tap drill size for such a nut must be equal to the minor diameter of the screw thread plus $\frac{1}{6}$ of the double depth of the thread.

Example 2

Find the tap drill size for a $\frac{7}{8}''$–9 American Standard coarse-thread for $\frac{5}{6}$ engagement.

SOLUTION. Major diameter = $\frac{7}{8}''$ = 0.8750″
Double depth = $\frac{1}{9} \times 1.299$ = 0.1443″
Minor diameter = 0.7307″

$$\text{Clearance} = \frac{\text{Double depth of thread}}{6}$$

$$= \frac{.1443}{6} = 0.0241''$$

Drill size = 0.7548″

The nearest $\frac{1}{64}''$ is $\frac{3}{4}''$. Hence we would use a drill $\frac{3}{4}''$ in diameter.

PROBLEMS

Find the commercial tap drill size and also the minimum tap drill size for each of the following American Standard threads.

1. $\frac{5}{16}''$–18	**2.** $\frac{7}{16}''$–14	**3.** $\frac{7}{8}''$–9	**4.** 1″–8	**5.** $1\frac{3}{8}''$–6
6. $1\frac{3}{4}''$–5	**7.** 4″–4	**8.** $1\frac{1}{2}''$–6	**9.** $2\frac{1}{4}''$–$4\frac{1}{2}$	**10.** $2\frac{1}{2}''$–4

248. Width of Point of Tool

It is obvious that the shape of the point of the tool for cutting an American Standard thread must be the same as the bottom of the thread. The point of the tool, therefore, is flattened an amount equal to $\frac{1}{8}$ of the pitch of the thread to be cut.

Example

Find the width of tool point for cutting a $\frac{1}{4}''$-pitch American Standard thread.

SOLUTION. Width of tool point = $\frac{1}{8} \times$ pitch = $\frac{1}{8} \times \frac{1}{4} = \frac{1}{32}''$

PROBLEMS

Find the width of tool point required to cut the following American Standard threads.

1. Pitch = $\frac{2}{9}''$	**2.** Pitch = $\frac{1}{4}''$	**3.** Pitch = $\frac{1}{16}''$
4. Pitch = $\frac{1}{7}''$	**5.** Pitch = $\frac{1}{18}''$	**6.** Pitch = $\frac{2}{7}''$

249. Square Thread

The square thread, Fig. 360, is so called because a cross-section of the thread is theoretically a square. The sketch is self-explanatory. The thickness of the thread and the width of the space are each equal to $\frac{1}{2}$ the pitch. In the nut, however, the space is slightly more than half the pitch, to allow for a sliding fit. The depth of the thread, though theoretically equal to half the pitch, is actually cut 0.005″ deeper than half the pitch.

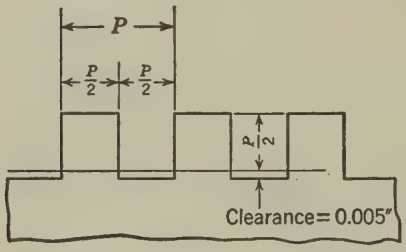


Fig. 360

The width of the tool point for cutting a square thread is equal to the width of the space, which is half the pitch.

250. Tap Drill for Square Thread

The relation between a square-threaded screw and its nut is shown in Fig. 361. It will be seen that there is a clearance

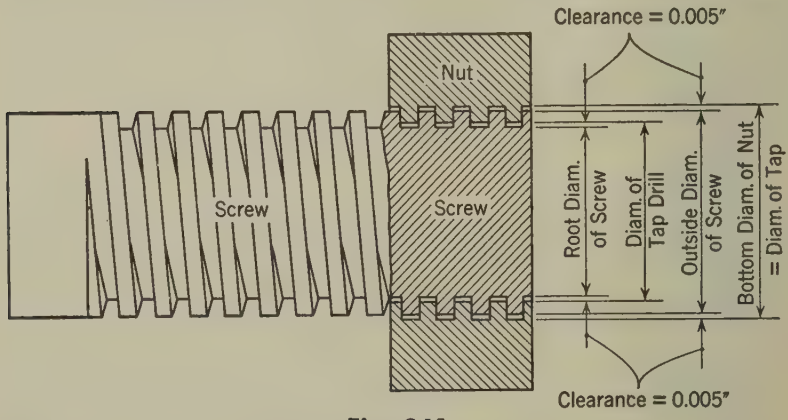


Fig. 361

of 0.005″ between the screw threads and the nut threads, both top and bottom. The tap drill therefore has to be $2 \times 0.005''$ or 0.01″ larger than the root diameter of the

screw, and the tap diameter will be $2 \times 0.005''$ or $0.01''$ larger than the outside diameter of the screw.

Example 1

Find the double depth of a $\frac{5}{8}''$–8 square thread.

SOLUTION. Double depth = Pitch + 2 × Clearance
$= \frac{1}{8} + 2 \times 0.005'' = 0.125'' + 0.01''$
$= 0.135''$

PROBLEMS

1. Find the double depth of a $\frac{3}{4}''$–6 square thread.
2. Find the double depth of a $\frac{7}{8}''$–$4\frac{1}{2}$ square thread.
3. Find the double depth of a square thread screw $1''$ in diameter with a double thread of $\frac{2}{5}''$ lead.
4. Find the double depth of a square thread screw $4''$ in diameter and 3 threads per inch.
5. Find the double depth of a $2\frac{1}{2}''$ square thread screw with 4 threads per inch.

Example 2

Find the root diameter of a $\frac{3}{8}''$–16 square thread screw.

SOLUTION.

Screw size = $\frac{3}{8}''$	$= 0.3750''$
Double depth = $\frac{1}{16} + 0.01 = 0.0625 + 0.01$	$= 0.0725''$
Root diameter	$= 0.3025''$

PROBLEMS

Find the root diameters of the square thread screws in the problems following Example 1.

Example 3

Find the size of tap drill for a $\frac{5}{8}''$–8 square thread screw.

SOLUTION. Figure 361 shows that the diameter of the tap drill is $2 \times 0.005''$ larger than the root diameter of the thread

Screw size = $\frac{5}{8}''$	$= 0.6250''$
Pitch = $\frac{1}{8}''$	$= 0.1250''$
Diameter of tap drill	$= 0.5000'' = \frac{1}{2}''$

PROBLEMS

Find the tap drill sizes for the square thread screws in the problems following Example 1.

251. The Acme 29-Degree Screw Thread

The Acme 29-degree screw thread is best understood by a study of its derivation from the square thread. In Fig.

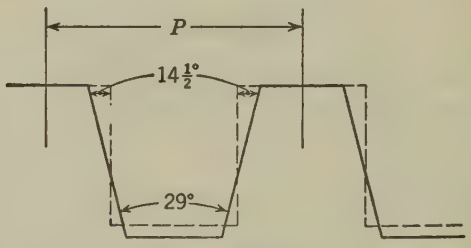


Fig. 362

362 we start with a square thread and draw lines making an angle of $14\frac{1}{2}°$ with the vertical sides of the square thread. The sides of the resulting Acme thread will therefore make an angle of 29° with each other. The depth of the Acme thread is .01″ greater than half the pitch of the thread.

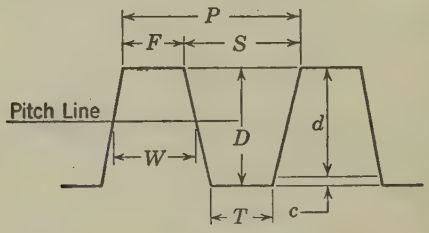


Fig. 363

The dimensions of the Acme thread for the screw, Fig. 363, are as follows:

F = Flat on top = 0.3707 × Pitch
S = Width of space = 0.6293 × Pitch

T = Bottom flat = Width of tool point for cutting thread
$= .3707 \times \text{Pitch} - 0.0052''$

D = Depth $= \dfrac{P}{2} + .010''$

Example 1

Find the root diameter, top flat, and bottom flat of an $\frac{3}{4}''$–8 Acme 29-degree screw thread.

SOLUTION. Screw size $= \frac{3}{4}''$ $= 0.7500''$

$$\text{Depth} = \frac{P}{2} + 0.010'' = \tfrac{1}{2} \times \tfrac{1}{8} + .010''$$
$$= 0.0625 + 0.010 = 0.0725''$$
$$\text{Double depth} = 2 \times 0.0725 = 0.1450''$$
$$\text{Root diameter} = 0.6050''$$

$$\text{Top flat} = 0.3707 \times \text{Pitch} = 0.3707 \times \tfrac{1}{8} = .0463''$$
$$\text{Bottom flat} = 0.3707 \times \text{Pitch} - 0.0052$$
$$= 0.3707 \times \tfrac{1}{8} - 0.0052 = 0.0463 - 0.0052$$
$$= 0.0411''$$

252. Tap Drill and Tap, Acme Thread

The relative sizes of the Acme screw and the tap drill and tap are readily seen in Fig. 364. The clearances between the

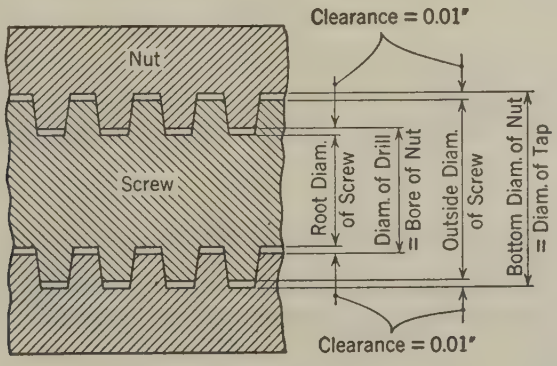


Fig. 364

screw threads and the nut threads are all 0.010″. The tap drill for the nut is therefore 2×0.010 or 0.020″ larger than

the root diameter of the screw, and the diameter of the tap is 0.020″ larger than the outside diameter of the screw. The dimensions of the threads on the tap are as follows:

$$\begin{aligned} \text{Top flat} &= 0.3707 \times \text{Pitch} - 0.0052'' \\ \text{Bottom flat} &= 0.3707 \times \text{Pitch} - 0.0052'' \\ \text{Depth} &= \frac{P}{2} + 0.020'' \end{aligned}$$

Example 2

A $\frac{3}{4}$″ bolt has an Acme 29-degree thread of $\frac{1}{8}$″ pitch. Find tap drill size, bottom and top flats of tap thread, and outside diameter of tap.

SOLUTION.

$$\begin{aligned} \text{Tap drill size} &= \text{Root diameter} + 2 \times 0.01'' \\ &= 0.605 + 0.02 \\ &= 0.625'' \\ \text{Bottom flat} &= 0.3707 \times \text{Pitch} - 0.0052 \\ &= 0.3707 \times \tfrac{1}{8} - 0.0052 \\ &= 0.0463 - 0.0052 = 0.0411'' \end{aligned}$$

Top flat = same as bottom flat = 0.0411″

$$\begin{aligned} \text{Outside diameter of tap} &= \text{Outside diameter of screw} + 2 \times 0.01 \\ &= 0.750 + 0.020 = 0.7700'' \end{aligned}$$

PROBLEMS

In each of the following five Acme 29-degree screw threads, find: (*a*) root diameter; (*b*) top flat of screw; (*c*) bottom flat of screw; (*d*) tap drill size; (*e*) bottom flat of tap; (*f*) top flat of tap; (*g*) outside diameter of tap.

1. 2″–4 **2.** $1\frac{1}{2}$″–$5\frac{1}{2}$ **3.** $2\frac{1}{8}$″–$3\frac{1}{2}$ **4.** $\frac{1}{2}$″–10 **5.** 1″–7

253. The Brown and Sharpe 29-Degree Worm Thread

For dimensions of this thread, which is similar to the 29-degree Acme thread, see Section 306 on page 471.

254. Metric Standard Screw Threads

The form of the metric thread and the dimensions of the parts are exactly like those of the American Standard thread; that is, the sides form an angle of 60° and the thread is flat-

tened $\frac{1}{8}$ top and bottom. See Fig. 365. Both the diameter and the pitch of the screw are given in millimeters. See metric conversion table on page 567.

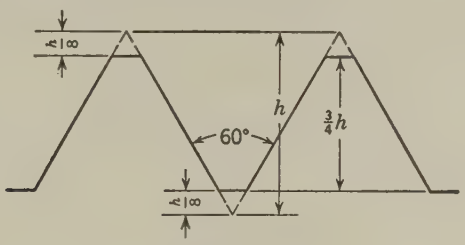


Fig. 365

Example 1

Find the double depth of a 6.5-mm. pitch thread.

SOLUTION. 6.5 mm. = 6.5 × .0394″ = .2559″
Double depth = Pitch × 1.299 = .2559 × 1.299
= .3324″

Example 2

Find the root diameter of a metric screw having an outside diameter of 10 mm. and a pitch of 1.5 mm.

SOLUTION. 10 mm. = 0.3937″; 1.5 mm. = 0.0591″.

Outside diameter = 0.3937″
Double depth = 0.0591 × 1.299 = 0.0767″
Root diameter = 0.3170″

Example 3

What size drill should be used for a metric thread 26 mm. in diameter and 3-mm. pitch?

SOLUTION. Diameter = 26 mm. = 1.0236″
Pitch = 3 mm. = 0.1181″
Diameter = 1.0236″
Double depth = 0.1181 × 1.299 = 0.1534″
Root diameter = 0.8702″
Clearance = $\frac{1}{8}$ Pitch = $\frac{1}{8}$ × .1181 = 0.0148″
Drill size = 0.8850″

The nearest 64th of an inch above this is $\frac{57}{64}$″. Hence a $\frac{57}{64}$″ drill should be used.

Example 4

Find the width of tool point for a 7-mm. pitch thread.

SOLUTION. Pitch = 7 mm. = 0.2756″
Width of tool point = $\frac{1}{8}$ Pitch = $\frac{1}{8} \times 0.2756$
= 0.0345″

NOTE. To compute the various parts of a metric thread, first convert the given metric dimensions into inches and then proceed exactly as in American Standard threads.

PROBLEMS

In each of the following metric threads find: (*a*) double depth; (*b*) root diameter; (*c*) size of drill; (*d*) width of point of tool.

	DIAMETER	PITCH
1.	3 mm.	.5 mm.
2.	5 mm.	.75 mm.
3.	6 mm.	1.0 mm.
4.	9 mm.	1.0 mm.
5.	12 mm.	1.5 mm.
6.	20 mm.	2.5 mm.
7.	22 mm.	2.5 mm.
8.	24 mm.	3.0 mm.
9.	30 mm.	3.5 mm.
10.	40 mm.	4.0 mm.

255. Whitworth Standard Threads

The Whitworth or British standard thread is a 55-degree V-thread with rounded top and bottom. If we start with a 55-degree V-thread, the Whitworth is obtained by eliminating $\frac{1}{6}$ of the height of the V both top and bottom and round-

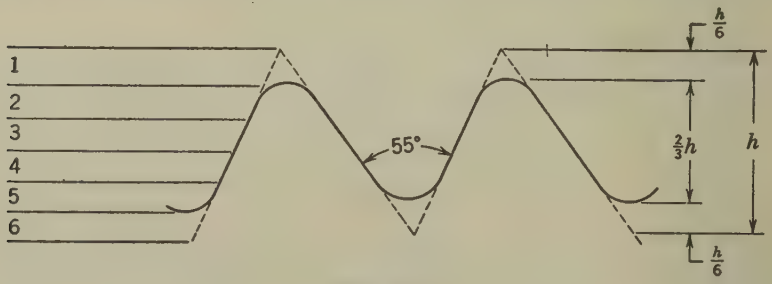


Fig. 366

ing off the thread at the point and at the root as shown in Fig. 366. The depth of the resulting thread is therefore $\frac{2}{3}$ of the depth of the corresponding V-thread.

The following are the dimensions of the Whitworth standard thread:

$$\text{Depth} = \text{Pitch} \times 0.6403$$
$$\text{Radius} = \text{Pitch} \times 0.1373$$

Example 1

Find the double depth of a Whitworth standard thread having 12 threads per inch.

SOLUTION. Double depth $= 2 \times \frac{1}{12} \times 0.6403 = 0.1067''$

PROBLEMS

Find the double depth of the following Whitworth threads.

1. Pitch $= \frac{1}{24}''$
2. Pitch $= \frac{1}{10}''$
3. 9 threads per inch
4. 6 threads per inch
5. $3\frac{1}{2}$ threads per inch

Example 2

Find the root diameter of a $\frac{1}{4}''$–20 Whitworth standard thread.

SOLUTION.

Outside diameter $= \frac{1}{4}''$	$= 0.2500''$
Double depth $= 2 \times \frac{1}{20} \times 0.6403$	$= 0.0640''$
Root diameter	$= 0.1860''$

PROBLEMS

Find the root diameter of the following Whitworth threads.

1. $\frac{3}{8}''$–16
2. $\frac{5}{8}''$–11
3. $\frac{3}{4}''$–10
4. $1\frac{7}{8}''$–$4\frac{1}{2}$
5. $3\frac{5}{8}''$–$3\frac{1}{4}$
6. $\frac{11}{16}''$–11

256. Tap Drill Size, Whitworth Thread

The diameter of the tap drill used in boring a nut blank for a Whitworth thread is larger by $\frac{1}{8}$ of the pitch than the root diameter of the thread.

Example

Find the proper tap drill size for a $\frac{7}{8}''$–9 Whitworth thread.

SOLUTION.

Outside diameter = $\frac{7}{8}''$	= 0.8750″
Double depth = $2 \times \frac{1}{9} \times 0.6403$	= 0.1423″
Root diameter	= 0.7327″
Clearance = $\frac{1}{8} \times$ Pitch = $\frac{1}{8} \times \frac{1}{9}$	= 0.0139″
Drill size	= 0.7466″

The nearest $\frac{1}{64}''$ above this decimal is $\frac{3}{4}''$. Hence a $\frac{3}{4}''$ drill should be used.

PROBLEMS

Find the proper size drill to be used for each of the following Whitworth threads.

1. $\frac{5}{16}''$–18	**2.** $\frac{1}{2}''$–12	**3.** $\frac{13}{16}''$–10	**4.** 1″–8	**5.** $1\frac{1}{4}''$–7
6. $1\frac{1}{2}''$–6	**7.** 2″–$4\frac{1}{2}$	**8.** $2\frac{1}{2}''$–4	**9.** 3″–$3\frac{1}{2}$	**10.** $5\frac{1}{4}''$–$2\frac{5}{8}$

257. Radius of Tool Point, Whitworth Thread

Example

Find the radius of tool point for cutting a Whitworth standard thread having $4\frac{1}{2}$ threads per inch.

SOLUTION. $\text{Pitch} = \dfrac{1}{4\frac{1}{2}} = \dfrac{2''}{9}$;

$$\text{Radius} = \text{Pitch} \times 0.1373 = \tfrac{2}{9} \times 0.1373$$
$$= 0.0305''$$

PROBLEMS

Find the radius of tool point for cutting the threads in each of the 10 problems of Section 256.

258. American Standard Taper Pipe Threads

A study of Fig. 367 will show the following characteristics of the American Standard taper pipe threads.

There are three varieties of thread on the pipe. At the end of the pipe, in the section marked *A*, the threads are

perfect both at top and bottom. The length of section A is found by Briggs' formula:

$$A = (0.8D + 4.8) \times P$$

where D is the actual external diameter and P is the pitch of the thread. The taper of section A is $\frac{1}{16}''$ per inch or $\frac{3}{4}''$ per foot measured on the diameter. The threads in this section are 60-degree V-threads with top and bottom slightly flattened. The depth of the threads, therefore, instead of being $0.866 \times P$, is only $0.8 \times P$.

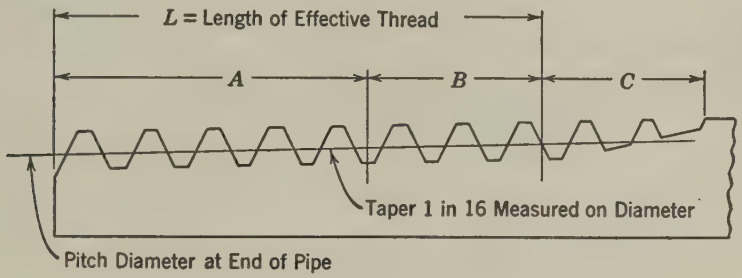


Fig. 367

In the section marked B in the diagram, there are *two* threads having the same taper at the bottom as the threads in section A, but with flat tops.

In the section marked C, there are 3 to 4 imperfect threads with flat tops and bottoms, the imperfections being caused by the chamfer of the threading die.

The value of section L, the length of effective thread, which consists of $A + B$, is found by the formula:

$$L = (0.8D + 6.8)P$$

Example

Find the depth of a perfect American Standard pipe thread of $\frac{1}{8}''$ pitch.

SOLUTION. Depth $= 0.8 \times P = 0.8 \times \frac{1}{8} = 0.1''$

PROBLEMS

Find the depth of the following perfect American Standard pipe threads.

1. $\frac{1}{27}''$ pitch
2. $\frac{1}{18}''$ pitch
3. $\frac{1}{14}''$ pitch
4. $11\frac{1}{2}$ threads per inch

259. Length of Part Having Perfect Threads, and Length of Effective Thread

Example

A pipe whose actual external diameter is 2.875″ is threaded with American Standard pipe threads of $\frac{1}{8}''$ pitch. What is the length of the part with the perfect threads, and what is the length of effective thread?

SOLUTION.
$$\begin{aligned} A &= (0.8D + 4.8) \times P \\ &= (0.8 \times 2.875 + 4.8) \times \tfrac{1}{8} \\ &= (2.3 + 4.8) \times \tfrac{1}{8} \\ &= 7.1 \times \tfrac{1}{8} = 0.89'' \\ L &= (0.8D + 6.8)P \\ &= (0.8 \times 2.875 + 6.8)P \\ &= (2.3 + 6.8)P \\ &= 9.1 \times \tfrac{1}{8} = 1.14'' \end{aligned}$$

PROBLEMS

Find the length of the part in which the threads are perfect and the effective length of thread in the following pipes having American Standard threads.

	Actual Outside Diameter	Number of Threads per Inch
1.	0.405″	27
2.	0.675″	18
3.	1.050″	14
4.	1.9″	$11\frac{1}{2}$
5.	4.0″	8

260. Thickness of Metal between Bottom of Thread and Inside of Pipe in Straight Pipe Thread

It is evident from the sketch, Fig. 368, that the thickness of the metal between the bottom of the thread and the in-

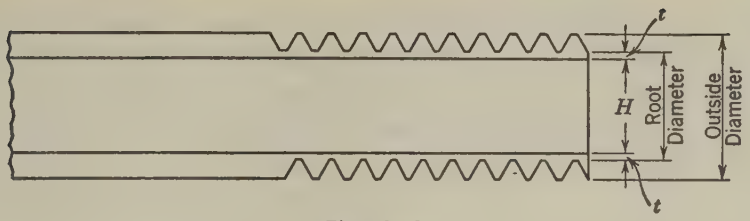


Fig. 368

side of the pipe is equal to half the difference between the root diameter of the thread and the actual inside diameter of the pipe.

Example

Find the amount of metal left between the bottom of a $\frac{1}{8}''$ thread and the inside of the pipe on a pipe having an actual inside diameter of 3.548″ and an outside diameter of 4″.

SOLUTION.

Outside diameter	= 4.000″
Double depth = $2 \times \frac{1}{8} \times .8$	= 0.200″
Root diameter	= 3.800″
Inside diameter of pipe	= 3.548″
	0.252″
Thickness of metal = $\frac{1}{2} \times 0.252$	= 0.126″

PROBLEMS

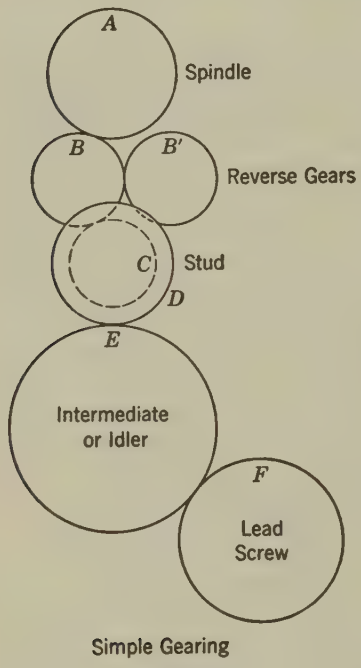
Find the thickness of the metal between the bottom of the thread and the inside of the pipe in each of the following pipes.

	Actual Inside Diameter	Actual Outside Diameter	Number of Threads per Inch
1.	0.364″	0.540″	18
2.	0.623″	0.840″	14
3.	1.048″	1.315″	$11\frac{1}{2}$
4.	3.067″	3.5″	8
5.	9.000″	9.688″	8

261. Lathe Gearing for Cutting Screw Threads

One of the most important uses to which a lathe is put is cutting screw threads. This is accomplished by closing the

split nut attached to the carriage so as to engage the lead screw. In cutting a desired number of threads per inch on a screw, it is evident that while the tool travels a distance of 1 in., the piece of work must make a number of revolutions equal to the required number of threads per inch. The speed of the tool depends on the speed of the lead screw, which in turn depends upon the pitch of the lead screw; while the speed of rotation of the work depends upon the speed of the spindle. Hence, if the lathe is so geared that the spindle and the lead screw revolve at the same rate of speed, the number of threads per inch cut on the work will be the same as the number of threads per inch on the lead screw. If, for example, the lead screw has 6 threads per inch, the piece of work will revolve 6 times while the tool travels 1 in., and the resulting screw will have 6 threads per inch.



Simple Gearing

Fig. 369

It is obvious that, if we want to cut twelve threads per inch, the piece of work will have to revolve twelve times while the tool travels 1 in., that is, while the lead screw revolves six times. But, for the work to revolve twice as fast as the lead screw, the spindle must turn twice as fast as the lead screw. This is accomplished by the change gears D and F in Fig. 369, which shows the arrangement of the gears in a simple geared lathe.

A = Spindle gear which turns the work

B and B' = Reverse gears to give direction to the lead

screw according as right or left screw is required

C = Fixed stud gear
D = Change stud gear
E = Intermediate or idler gear
F = Lead screw gear

Case I

If the ratio of spindle to stud is 1 to 1, that is, if gear A and gear C have the same number of teeth, then the pitch of the work will be the same as the lead screw pitch if gears D and F are of equal size. If a screw is required having three times as many threads per inch as the lead screw, the spindle will have to revolve three times as fast as the lead screw, and therefore a gear with $\frac{1}{3}$ as many teeth must be placed on the stud as on the lead screw.

The preceding may be put into the following simple formula:

$$\frac{\text{Lead screw}}{\text{Screw to cut}} = \frac{\text{Stud gear}}{\text{Lead screw gear}}$$

which means that the ratio of the stud gear to the lead screw gear must be the same as the ratio of the lead screw to the screw we wish to cut.

Example

What change gears are required to cut a screw with ten threads per inch if the lead screw has six threads per inch?

SOLUTION. $$\frac{\text{Lead screw}}{\text{Screw to cut}} = \frac{6}{10} = \frac{6 \times 4}{10 \times 4} = \frac{24}{40}$$

EXPLANATION. Write a fraction with the number of threads per inch on the lead screw as the numerator and the number of threads per inch on the screw to be cut as the denominator. Since the teeth of the stud gear and the lead screw gear must be in the same ratio as the terms of this fraction, we multiply both terms of the fraction by any multiplier that will give us an equivalent

fraction whose terms will represent suitable gears. In this case 4 was chosen as the multiplier, giving gears 24 and 40. If the 24-tooth gear is placed on the stud D and the 40-tooth gear on the lead screw F, the ratio of the speed of the stud (and therefore of the spindle) to the speed of the lead screw will be as 40 is to 24, or 10 to 6. Therefore the piece of work will revolve 10 times to 6 turns of the lead screw, and the result will be a screw having 10 threads per inch.

Any other multiplier might be selected which would give other gears that would produce the same result as 24 and 40. For example,

$$\frac{\text{Lead screw}}{\text{Screw to cut}} = \frac{6}{10} = \frac{6 \times 6}{10 \times 6} = \frac{36}{60}$$

Multiplying both terms of the fraction by 6 gives the gears 36 and 60. Placing the 36 on the stud and the 60 on the lead screw, the speed ratio between the spindle and the lead screw is the same as before, namely 10 to 6, and we get a screw with 10 threads per inch.

NOTE. As idler E does not affect the relative speeds of D and F, it does not enter into the computations.

PROBLEMS

Find suitable change gears to be used in the cases given below. Select gears from the following set, which is furnished with a certain lathe: **24–28–32–36–40–44–48–52–56–60–64–68–72–76–80–84–88–92–96–100–104–108–112–116–120.**

1. To cut 8 threads per inch with a 6-pitch lead screw.
2. To cut 14 threads per inch with an 8-pitch lead screw.
3. To cut 12 threads per inch with an 8-pitch lead screw.
4. To cut a thread of $\frac{1}{13}''$ pitch with a lead screw whose pitch is $\frac{1}{6}''$.
5. To cut 10 threads per inch with a 4-pitch lead screw.
6. Determine the gears for stud and lead screw when 18 threads per inch are to be cut with a lead screw whose pitch is $\frac{1}{8}''$.
7. A lathe having an 8-pitch lead screw is found to have a 32 gear on the stud and an 88 gear on the lead screw. How many threads per inch will it cut on a piece of work?
8. A lathe whose lead screw has 6 threads per inch has a lead screw gear of 36 teeth. What gear must be placed on the spindle to cut a thread of $\frac{1}{4}''$ pitch?

9. It is desired to cut 20 threads per inch on a screw. If the pitch of the lead screw is $\frac{1}{8}''$ and the stud gear has 32 teeth, what gear must be placed on the lead screw?

10. It is found that a lathe having a stud gear of 36 teeth and a lead screw gear of 84 teeth cuts a screw with 7 threads per inch. What is the pitch of the lead screw?

Case II

If spindle gear A and stud gear C have not the same number of teeth, that is, if the ratio of spindle to stud is not 1 to 1, we proceed as follows in computing the change gears:

1. As in Case I write a fraction expressing the ratio of the number of threads on the lead screw to the number of threads to be cut.
2. Multiply this ratio by the ratio of the number of teeth on stud gear to the number of teeth on spindle gear.
3. Find suitable gears by multiplying the result of step 2 by any convenient multiplier.

Example

What change gears are required to cut 16 threads per inch with a lead screw having 6 threads per inch when the spindle gear has 20 teeth and the stud gear has 40 teeth?

SOLUTION.

$$\frac{\text{Lead screw}}{\text{Screw to cut}} = \frac{6}{16}$$

$$\frac{\text{Fixed stud gear}}{\text{Spindle gear}} = \frac{40}{20}$$

$$\text{Change gears} = \frac{6}{16} \times \frac{40}{20} = \frac{3}{4}$$

$$\frac{3}{4} \times \frac{12}{12} = \frac{36}{48}$$

Stud gear = 36 teeth

Lead screw gear = 48 teeth

PROBLEMS

1. If the ratio of spindle to stud is 1 to 3 and the lead screw has 8 threads per inch, what change gears are required to cut 12 threads per inch?

2. Compute the necessary change gears for cutting 20 threads per inch with a lead screw having 8 threads per inch when the spindle gear has 18 teeth and the stud gear has 36 teeth.

3. Find the change gears for cutting a $\frac{1}{6}''$ pitch screw with a $\frac{1}{8}''$ pitch lead screw when the spindle gear has 16 teeth and the stud gear has 32 teeth.

4. If the spindle gear has 16 teeth and the stud gear 48 teeth, what change gears are required for cutting a screw with 8 threads to the inch if the lead screw has 12 threads to the inch?

5. What gear must be placed on the stud of a lathe if the ratio of spindle to stud is 1 to 2, the lead screw pitch is $\frac{1}{6}''$, an 80-tooth gear is on the lead screw, and a pitch of $\frac{1}{10}''$ is wanted on a screw?

262. Fractional Threads

To cut a fractional number of threads per inch, the same method is followed as for whole threads.

Example

Find the change gears necessary for cutting $11\frac{1}{2}$ threads per inch with a lead screw having 6 threads per inch.

SOLUTION. $$\frac{\text{Lead screw}}{\text{Screw to cut}} = \frac{6}{11\frac{1}{2}} = \frac{6 \times 8}{11\frac{1}{2} \times 8} = \frac{48}{92}$$

Stud gear = 48 teeth

Lead screw gear = 92 teeth

PROBLEMS

1. Find the change gears for cutting $4\frac{1}{2}$ threads per inch with an 8-pitch lead screw and a spindle to stud ratio of 1 to 1.

2. With a spindle to stud ratio of 1 to 2, find the change gears for cutting a screw with $3\frac{1}{2}$ threads per inch, the lead screw having 6 threads per inch.

3. It is desired to cut $5\frac{1}{2}$ threads per inch on a bolt. With a spindle to stud ratio of 1 to 1 and a lead screw having 6 threads per inch, what change gears are necessary?

4. What change gears would be required in Problem 3 if the spindle to stud ratio were 1 to 2?

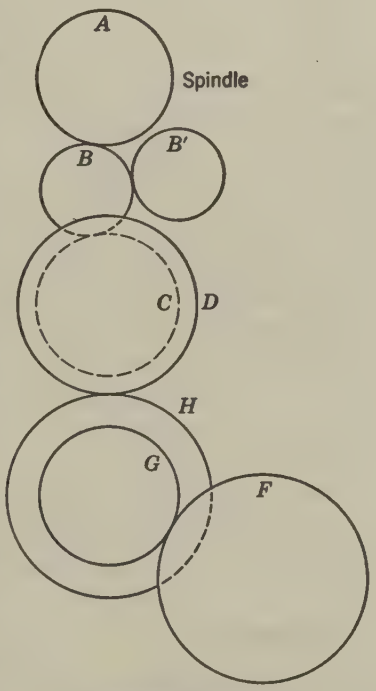
5. What change gears would be required in Problem 3 if the spindle to stud ratio were 1 to 3?

In Problems 6, 7, 8, 9, and 10, assume spindle to stud ratio 1 to 1 and compute the change gears required.

6. Required to cut $2\frac{3}{8}$ threads per inch with a 6-pitch lead screw.
7. Required to cut $2\frac{3}{4}$ threads per inch with a 6-pitch lead screw.
8. Required to cut $2\frac{1}{4}$ threads per inch with an 8-pitch lead screw.
9. Required to cut $2\frac{5}{8}$ threads per inch with an 8-pitch lead screw.
10. Required to cut $4\frac{1}{2}$ threads per inch with a 6-pitch lead screw.

263. Use of Compound Gearing

Sometimes it is impossible to obtain the required speed ratio between spindle and lead screw for cutting a desired thread by simple gearing with the gears available. Again, certain cases may require gears with an unusually large number of teeth. To obviate these possibilities, recourse is had to *compound* gearing. In this system the idler E of Fig. 369 is replaced by two gears G and H as shown in Fig. 370. One of these compound gears meshes with the change gear D while the other meshes with the lead screw gear F. The following examples will illustrate the method of computing compound gearing.



Compound Gearing

Fig. 370

Example 1

Find the compound gearing required to cut a screw having 35 threads per inch with a lead screw having 6 threads per inch, the ratio of spindle to stud being 1 to 1.

SOLUTION. $$\frac{\text{Lead screw}}{\text{Screw to cut}} = \frac{6}{35} = \frac{2}{5} \times \frac{3}{7} = \frac{2 \times 16}{5 \times 16} \times \frac{3 \times 8}{7 \times 8}$$

$$= \frac{32}{80} \times \frac{24}{56}$$

EXPLANATION. 1. Express in the form of a fraction the ratio of the number of threads per inch in the lead screw to the number of threads per inch in the screw to be cut. This gives the fraction $\frac{6}{35}$.

2. Separate each term of this fraction into two factors; $6 = 2 \times 3$ and $35 = 5 \times 7$. This gives the two fractions $\frac{2}{5}$ and $\frac{3}{7}$.

3. Treat each of the fractions exactly as in simple gearing; that is, multiply both numerator and denominator by any convenient number that will give an equivalent fraction whose terms will represent suitable gears. In this case the terms of the fraction $\frac{2}{5}$ are multiplied by 16, giving the fraction $\frac{32}{80}$; and the terms of the fraction $\frac{3}{7}$ are multiplied by 8, giving the fraction $\frac{24}{56}$.

4. The numerators 32 and 24 represent driving gears, and the denominators 80 and 56 represent driven gears. Either of the drivers may be placed on the stud and either of the driven may be placed on the lead screw.

If any of these four gears are not among those furnished with the lathe, use, instead of 16 and 8, such other multipliers as will give available gears.

If the ratio of fixed stud gear to spindle is not 1 to 1, multiply the ratio of lead screw to screw to be cut by the ratio of stud to spindle and then proceed as in Example 1 of compound gearing.

Example 2

Find the compound gearing required to cut 5 threads per inch on a lathe having a lead screw with 6 threads per inch, the ratio of fixed stud gear to spindle gear being 2 to 1.

SOLUTION. $$\frac{\text{Lead screw}}{\text{Screw to cut}} = \frac{6}{5}$$

$$\frac{\text{Fixed stud gear}}{\text{Spindle gear}} = \frac{2}{1}$$

$$\frac{6}{5} \times \frac{2}{1} = \frac{12}{5}$$

$$\frac{12}{5} = \frac{3}{2} \times \frac{4}{2\frac{1}{2}} = \frac{3 \times 12}{2 \times 12} \times \frac{4 \times 12}{2\frac{1}{2} \times 12} = \frac{36}{24} \times \frac{48}{30}$$

The driving gears are 36 and 48.
The driven gears are 24 and 30.

PROBLEMS

NOTE. *Unless otherwise stated, the ratio of spindle to stud is 1 to 1.*

1. Calculate by compounding the change gears necessary to cut 20 threads per inch in a lathe with a lead of 6.

2. Find by compounding the change gears required for cutting $3\frac{1}{2}$ threads per inch in a lathe with a screw constant of 6.

3. Find by compounding the change gears for cutting a $\frac{1}{24}''$ pitch thread on a lathe with a lead of 6.

4. On a lathe having 8 threads per inch on the lead screw, a 72 gear is used on the lead screw and a 36 gear on the stud. If the compound gears are 48 and 24, how many threads per inch will be cut?

NOTE. *In giving the sizes of the compounding gears, the first mentioned is the driver; the second, the driven gear.*

5. Find, by compounding, suitable gears to be used in cutting 23 threads per inch with a lead screw having 6 threads per inch.

6. What gears should be used to cut 16 threads per inch on a lathe having 8 threads per inch on the lead screw if the compounding gears are 30 and 60?

7. With a lead screw having 6 threads per inch, a 20-tooth stud gear, and an 80-tooth lead screw gear, what compounding gears could be used to cut 48 threads per inch?

8. It is desired to cut a screw with $5\frac{1}{2}$ threads per inch. If the lead screw has 6 threads per inch and the ratio of spindle to stud is 1 to 2, find the necessary change gears.

9. What change gears are required to cut $3\frac{3}{4}$ threads per inch on a lathe having a lead screw of $\frac{1}{6}''$ pitch, if the compound gears are 24 and 48?

10. What change gears are required to cut a double-threaded screw of $\frac{3}{4}''$ lead if the lead screw has 6 threads per inch?

11. What change gears are required to cut $2\frac{2}{3}$ threads per inch on a lathe with a lead of 6, and 24 and 48 compound gears?

12. What change gears are required to cut $2\frac{7}{8}$ threads per inch on a lathe with a lead of 6?

13. We are required to cut a triple-threaded screw of $\frac{3}{8}''$ pitch. What change gears are required if the lead screw has 8 threads per inch?

14. We are required to cut a quadruple-threaded screw with a lead of $1\frac{1}{4}''$. Compute the compound gearing required if the lead screw has 6 threads per inch.

264. Cutting Metric Threads

To cut a metric thread on a lathe whose lead screw pitch is given in inches, we proceed as follows:

Example

Find the change gears for cutting a thread of 2.5 mm. pitch on a lathe whose lead screw pitch is $\frac{1}{6}''$.

SOLUTION. First find the number of threads per inch in the screw to be cut. $1'' = 25.4$ mm.

$$\text{Number of threads per inch} = \frac{25.4}{2.5} = 10\frac{4}{25}$$

$$\text{Number of threads per inch in the lead screw} = \frac{1}{\frac{1}{6}} = 6$$

$$\frac{\text{Lead screw}}{\text{Screw to cut}} = \frac{6}{10\frac{4}{25}} = \frac{6}{\frac{254}{25}} = 6 \times \frac{25}{254} = \frac{150}{254}$$

$$\frac{150}{254} = \frac{2}{2} \times \frac{75}{127} = \frac{2 \times 24}{2 \times 24} \times \frac{75 \times 1}{127 \times 1} = \frac{48}{48} \times \frac{75}{127}$$

Gears 48 and 75 are drivers.
Gears 48 and 127 are driven.

NOTE. A special 127-tooth gear is generally used in cutting metric threads on a lathe whose lead screw pitch is given in inches.

PROBLEMS

1. Compute the gears required to cut a thread of 3 mm. pitch with a lead screw of 6.

2. What change gears are necessary by compounding to cut a $3\frac{1}{2}$-mm. pitch thread with a lead screw of 6?

3. It is required to cut a thread of 4 mm. pitch with a lead screw of 8. Find the compound gearing necessary.

4. Calculate by compounding the change gears necessary for cutting a thread of 7.5 mm. pitch with a lead screw of 6.

5. What change gears are required for cutting a thread of 10 mm. pitch with a lead screw of 6?

21 Cutting Speed and Feed

265. Cutting Speed, Surface Speed, or Rim Speed

In the interest of efficiency and economy the machinist should run his machines at various speeds, depending on the nature of the metal that is being worked, the kind of cut (that is, whether it is a roughing or a finishing cut), the feed and depth of cut, the material and shape of the cutting tool, and the strength and design of the machine. Machinery handbooks contain tables giving the proper speeds at which various metals should be worked on the different machines. Likewise, in the operation of grindstones, emery wheels, and flywheels, it is necessary to know the speed at which they may be run to obtain the best results. The proper speed for grindstones and emery wheels is determined by the strength of the stone and the nature of the grinding to be done.

Problems involving cutting speed or rim speed may be grouped in the following three classes:

1. *Given the diameter of the piece of work or wheel and the revolutions per minute, to find the cutting speed or rim speed in feet per minute.*

2. *Given the diameter and the speed in feet per minute, to find the number of revolutions per minute.*

3. *Given the speed in feet per minute and the revolutions per minute, to find the diameter.*

266. Cutting Speed on the Lathe

The *cutting speed* is the rate at which a tool passes over a piece of work. It is always expressed in *feet per minute*. For the lathe, cutting speed may be defined as the number of linear feet on the surface of the work that passes the point of the tool in one minute. In every complete turn of a piece of work in the lathe, a distance equal to the circumference of the work passes the point of the tool. Hence the cutting speed in feet per minute is found by multiplying the circumference of the work expressed in feet by the number of revolutions per minute.

Example 1

Find the cutting speed of a piece of cast iron $\frac{7}{8}''$ in diameter turned in a lathe at 180 r.p.m.

SOLUTION. Cutting speed = Circumference $\times$ R.p.m.

$$= \tfrac{7}{8} \times \tfrac{22}{7} \times \tfrac{1}{12} \times 180$$

$$= 41\tfrac{1}{4} \text{ ft. per minute}$$

EXPLANATION. Multiplying the diameter $\frac{7}{8}$ by $\frac{22}{7}$ gives the circumference in inches. Dividing by 12 gives the circumference in feet. That is, in each turn of the piece of work, $(\frac{7}{8} \times \frac{22}{7} \times \frac{1}{12})$ feet will pass the point of the tool. And, since the piece revolves 180 times in 1 minute, multiplying $(\frac{7}{8} \times \frac{22}{7} \times \frac{1}{12})$ by 180 gives the number of feet on the surface of the work that will pass the tool in 1 minute.

267. Surface Speed or Rim Speed

By the *surface speed* or *rim speed* of an emery wheel or flywheel is meant the number of feet that a point on the rim or circumference of the wheel travels in 1 minute. Every time that a wheel makes one complete turn, a point on its rim travels a distance equal to its circumference. Hence the number of feet that a point on the rim will travel in a minute is equal to the number of turns that the wheel makes in 1 minute multiplied by its circumference expressed in feet.

Example 2

Find the surface speed of an emery wheel 8″ in diameter making 240 r.p.m.

SOLUTION. $8 \times \frac{22}{7} \times \frac{1}{12} \times 240 = 503$ ft. per minute

EXPLANATION. In each turn of the wheel, a point on its rim travels a distance equal to its circumference, that is, $8 \times \frac{22}{7}$. Divide this by 12 to express the circumference in feet. Since the wheel turns 240 times in 1 minute, a point on the rim will travel in 1 minute, 240 times the length of one circumference; hence, multiply the circumference by 240.

PROBLEMS

1. A piece $1\frac{1}{4}$″ in diameter is turned in the lathe at 165 r.p.m. Find the cutting speed in feet per minute.

2. Calculate the cutting speed in turning a brass rod $1\frac{3}{8}$″ in diameter at 240 r.p.m.

3. The diameter of a piece that is being turned is $2\frac{1}{4}$″. If the piece makes 110 r.p.m., what is the cutting speed?

4. Find the cutting speed when a piece $1\frac{1}{8}$″ in diameter is turned at 120 r.p.m.

5. A 3″ rod is turned in a lathe at 75 r.p.m. What is the cutting speed?

6. Find the surface speed of an emery wheel 12″ in diameter making 210 r.p.m.

7. An 8′ flywheel makes 200 r.p.m. Find the rim speed.

8. A 12″ emery wheel is driven by a motor which makes 2000 r.p.m. A 3″ pulley on the motor drives an 8″ pulley on the emery wheel spindle. Find the surface speed of the emery wheel in feet per minute.

9. A bar of cold rolled machine steel $3\frac{1}{2}$″ in diameter is turned in the lathe at 32 r.p.m. What is the cutting speed?

10. A brass rod $\frac{7}{8}$″ in diameter is turned in the lathe at 320 r.p.m. What is the cutting speed?

268. To Find the Revolutions per Minute When the Diameter and Cutting Speed Are Known

Example 1

At how many revolutions per minute should a lathe be run to give a cutting speed of 36 ft. per minute when turning a rod $2\frac{1}{2}$″ in diameter?

SOLUTION. Circumference $= \frac{5}{2} \times \frac{22}{7} \times \frac{1}{12} = \frac{55}{84}$ ft.

R.p.m. = Cutting speed ÷ Circumference

$= 36 \div \frac{55}{84} = 36 \times \frac{84}{55} = 55$

EXPLANATION. Since in every turn of the piece of work a distance equal to its circumference passes the cutting tool, for 36 ft. of the surface of the work to pass the tool, the piece must make as many times one turn as the length of the circumference is contained in 36 ft. Hence the rule:

R.p.m. = Cutting speed ÷ Circumference

Example 2

A 14″ emery wheel, Fig. 371, is required to have a surface speed of 5500 ft. per minute. The wheel is driven by a 4″ pulley on the same spindle belted to a 10″ pulley. What must be the revolutions per minute of the 10″ pulley?

SOLUTION. R.p.m. of emery wheel

$$= \frac{\text{Rim speed}}{\text{Circumference}}$$

Circumference of wheel

$= 14 \times \frac{22}{7} \times \frac{1}{12} = \frac{22}{6}$ feet

R.p.m. of wheel $= 5500 \div \frac{22}{6}$

$= 5500 \times \frac{6}{22} = 1500$

R.p.m. of 4″ pulley = 1500

R.p.m. of 10″ pulley

$$= \frac{4 \times 1500}{10} = 600$$

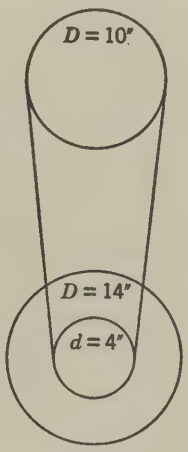


Fig. 371

EXPLANATION. The circumference of the emery wheel is found by multiplying its diameter by $\frac{22}{7}$. We then divide by 12 to express the circumference in feet. This gives $\frac{22}{6}$ feet. We next find the revolutions per minute of the wheel by dividing the rim speed by the circumference, which gives 1500 r.p.m. But since the 4″ pulley is on the same spindle as the emery wheel, it will make the same number of revolutions per minute. As the 10″

pulley is belted to the 4″ pulley, the 10″ pulley will make $\frac{4}{10}$ as many revolutions per minute as the 4″ pulley, or $\frac{4}{10} \times 1500 = 600$ r.p.m.

PROBLEMS

1. What should be the maximum number of revolutions per minute of a 6′ flywheel if any point on its rim should travel not more than a mile per minute?

2. At how many revolutions per minute should a 6″ gear blank be turned if the cutting speed is to be 42 ft. per minute?

3. A certain grindstone may run with a rim speed of 1200 ft. per minute. What should be the revolutions per minute if the diameter of the stone is 30″?

4. At how many revolutions per minute may a 4′ grindstone be run if it can stand a surface speed of 700 ft. per minute?

5. The cutting speed of a piece of work in a lathe is 250 ft. per minute. If the diameter is $2\frac{1}{2}$″, what must be its revolutions per minute?

6. How many revolutions per minute must a 24″ circular saw make to give a cutting speed of 3600 ft. per minute?

7. How many revolutions per minute should a steel shaft $1\frac{3}{4}$″ in diameter make if it is to be turned in the lathe with a cutting speed of 30 ft. per minute?

8. A crank pin of annealed tool steel $3\frac{3}{4}$″ in diameter is turned in the lathe with a cutting speed of 20 ft. per minute. How many revolutions per minute must it make?

9. The surface of a cast iron pulley 12″ in diameter is turned in the lathe with a cutting speed of 40 ft. per minute. How many revolutions per minute must it make?

10. The cutting speed on a cast iron cylinder $4\frac{1}{2}$″ in diameter that is bored in a lathe is 42 ft. per minute. How many revolutions per minute does it make?

269. To Find the Diameter When the Rim Speed and the Revolutions per Minute Are Known

Example

An emery wheel which is required to have a rim speed of a mile a minute is to be placed on a spindle which makes 1600 r.p.m. What must be the diameter of the emery wheel?

SOLUTION. $\text{Circumference} = \dfrac{\text{Rim speed}}{\text{r.p.m.}} = \dfrac{5280}{1600} \times 12 = 39.6''$

$$\text{Diameter} = \text{Circumference} \div \tfrac{22}{7}$$

$$= 39.6 \div \tfrac{22}{7} = 39.6 \times \tfrac{7}{22}$$

$$= 12.6''$$

EXPLANATION. In a mile there are 5280 ft. Since in 1600 turns a point on the rim travels 5280 ft., in one turn it will travel $\frac{1}{1600}$ of 5280 ft. But in one turn a point on the rim travels a distance equal to the circumference; hence, dividing the rim speed by the turns made in one minute gives the length of the circumference in feet. We multiply by 12 to change it to inches. To find the diameter, we divide the circumference by $\frac{22}{7}$, giving 12.6″. A 12″ wheel would be used.

PROBLEMS

1. An emery wheel having an allowable surface speed of 4500 ft. per minute makes 1500 r.p.m. What is its diameter?

2. A flywheel making 330 r.p.m. has a rim speed of a mile per minute. What is its diameter?

3. What must be the diameter of a circular saw to give a cutting speed of 3000 ft. per minute if it revolves at the rate of 400 r.p.m.?

4. An emery wheel which is required to have a surface speed of 5000 ft. per minute is driven by a 2000 r.p.m. motor. What must be the diameter of the emery wheel if the motor pulley is 3″ in diameter and the pulley on the emery wheel spindle is 8″ in diameter?

5. If the motor in Problem 4 is used to drive a 12″ emery wheel, what must be the size of the pulley on the emery wheel spindle if a speed of 4500 ft. per minute is required?

6. What is the diameter of a rod that makes 48 r.p.m. and has a cutting speed of 36 ft. per minute?

7. What is the diameter of a crank which makes 24 r.p.m. and has a cutting speed of 20 ft. per minute?

8. A brass rod is turned in the lathe at 400 r.p.m. with a cutting speed of 75 ft. per minute. What is the diameter?

9. The bearings at the ends of a crank shaft are turned in a lathe at 18 r.p.m. with a cutting speed of 30 ft. per minute. What is the diameter?

10. The rim speed of a pulley making 50 r.p.m. is 150 ft. per minute. What is the diameter of the pulley?

270. Drill Press and Milling Machine

The cutting speed of a drill is the number of feet traveled in 1 minute by the outer corners of the cutting edges.

Example 1

A 2″ hole is drilled in a piece of work with the drill revolving at the rate of 240 r.p.m. What is the cutting speed?

SOLUTION. Cutting speed = Circumference of drill × R.p.m.

$$= 2 \times \tfrac{22}{7} \times \tfrac{1}{12} \times 240$$

$$= 126 \text{ ft. per minute}$$

EXPLANATION. In each complete revolution of the drill the outer corners of the cutting edges travel a distance equal to the circumference of the drill. Since the drill makes 240 r.p.m., the outer corners of the cutting edges will travel 240 circumferences in 1 minute. Multiplying the diameter by $\frac{22}{7}$ gives $2 \times \frac{22}{7}$. Dividing by 12 gives the circumference in feet. Multiplying by 240 gives the speed in feet per minute.

In the case of milling machines, the cutting speed is found by multiplying the circumference of the cutter by the revolutions per minute.

Example 2

Find the cutting speed of a 5″ milling cutter making 30 r.p.m.

SOLUTION. Cutting speed = Circumference × R.p.m.

$$= 5 \times \tfrac{22}{7} \times \tfrac{1}{12} \times 30$$

$$= 39\tfrac{2}{7} \text{ ft. per minute}$$

EXPLANATION. Multiply the diameter by $\frac{22}{7}$ to get the circumference in inches. Dividing by 12 gives the circumference in feet. Multiplying by 30 (the revolutions per minute) gives the cutting speed in feet per minute.

PROBLEMS

1. A $1\frac{3}{4}''$ hole is bored with a drill running at 80 r.p.m. Find the cutting speed.

2. What is the cutting speed of a drill boring a $\frac{5}{8}''$ hole at 220 r.p.m.?

3. Find the cutting speed of a $2\frac{3}{4}''$ milling cutter making 75 r.p.m.

4. Calculate the cutting speed of a $3\frac{1}{2}''$ milling cutter making 120 r.p.m.

5. A milling cutter $2\frac{1}{4}''$ in diameter makes 50 r.p.m. What is the cutting speed?

6. What is the cutting speed of a $1\frac{7}{8}''$ drill making 60 r.p.m.?

7. Compute the cutting speed of a $1\frac{1}{2}''$ drill making 72 r.p.m.

8. Find the cutting speed of a $5\frac{1}{2}''$ milling cutter making 40 r.p.m.

9. A 2″ milling cutter makes 100 r.p.m. Calculate the cutting speed.

10. What is the cutting speed of a $\frac{3}{4}''$ drill making 128 r.p.m.?

To find the revolutions per minute of a drill when the diameter and the cutting speed are known proceed as in the following example:

Example

How many revolutions per minute must a $1\frac{1}{2}''$ drill make in order to have a cutting speed of 30 ft. per minute?

SOLUTION. R.p.m. = Cutting speed ÷ Circumference

$$\text{Circumference} = 1\tfrac{1}{2} \times \tfrac{22}{7} \times \tfrac{1}{12} = \tfrac{11}{28} \text{ ft.}$$

$$\text{R.p.m.} = 30 \div \tfrac{11}{28} = 30 \times \tfrac{28}{11} = 76\tfrac{4}{11}$$

EXPLANATION. In each turn of the drill, the outer corners of the cutting edges travel a distance equal to one circumference. We find the circumference in feet by multiplying the diameter by $\frac{22}{7}$ and dividing by 12. This gives $\frac{3}{2} \times \frac{22}{7} \times \frac{1}{12} = \frac{11}{28}$ ft. That is, while the drill makes one revolution, the outer corners of the cutting edges travel $\frac{11}{28}$ ft. If the corners are to travel 30 ft. in a minute, the drill must revolve as many times as $\frac{11}{28}$ is contained in 30. Hence, dividing 30 by $\frac{11}{28}$, we get $30 \times \frac{28}{11} = 76\frac{4}{11}$ r.p.m.

To find the revolutions per minute of the milling cutter for a desired cutting speed when the diameter of the cutter is known, proceed exactly as in the case of finding the revolutions per minute of a drill when the diameter and the cutting speed are given; that is, divide the cutting speed by the circumference of the cutter.

PROBLEMS

1. A $\frac{7}{8}''$ hole is bored in the drill press with a cutting speed of 32 ft. per minute. Find the revolutions per minute of the drill.

2. A $1\frac{1}{4}''$ drill is boring a hole in a piece of iron with a cutting speed of 35 ft. per minute. How many revolutions per minute must it make?

3. How many revolutions per minute must a $7\frac{1}{2}''$ milling cutter make to have a cutting speed of 50 ft. per minute?

4. An 8″ milling cutter has a cutting speed of 38 ft. per minute. Find the revolutions per minute.

5. At what speed must a $\frac{3}{4}''$ drill be run to have a cutting speed of 25 ft. per minute?

6. How fast must a 9″ milling cutter be run to have a cutting speed of 48 ft. per minute?

7. What must be the revolutions per minute of a $2\frac{3}{4}''$ milling cutter to have a speed of 120 ft. per minute?

8. How many revolutions per minute does a $1\frac{3}{16}''$ drill make if its cutting speed is 33 ft. per minute?

9. A $2\frac{1}{4}''$ drill has a cutting speed of 50 ft. per minute. At how many revolutions per minute is the drill running?

10. A $6\frac{1}{2}''$ milling cutter has a cutting speed of 44 ft. per minute. Compute the revolutions per minute.

To find the size of cutter to use with a given number of revolutions per minute and cutting speed proceed as in the following example:

Example

What must be the diameter of a milling cutter to have a cutting speed of 24 ft. per minute when it is run at 15 r.p.m.?

SOLUTION. Circumference = Cutting speed ÷ R.p.m.

$$= \frac{24}{15} \times 12 = 19.2''$$

Diameter = Circumference ÷ $\frac{22}{7}$

$$= 19.2 \times \frac{7}{22} = 6.1''$$

Use a 6″ cutter.

PROBLEMS

1. What size milling cutter should be used at 40 r.p.m. to give a cutting speed of 40 ft. per minute?

2. A milling cutter having a cutting speed of 60 ft. per minute is making 120 r.p.m. What is its diameter?

3. Find the size of the milling cutter running at 46 r.p.m. with a cutting speed of 46 ft. per minute.

4. What is the diameter of a milling cutter having a cutting speed of 28 ft. per minute and making 52 r.p.m.?

5. What must be the diameter of a milling cutter if it is to have a cutting speed of 90 ft. per minute and run at 45 r.p.m.?

271. Cutting Speed on the Planer

In figuring the *actual or net cutting speed* on the planer, we must take into account the time occupied by the idle return stroke. If a planer has a forward stroke of 16 ft. per minute and a return stroke of 32 ft. per minute, the return stroke will take half as much time as the forward stroke. Assuming the cut to be 16 ft. long, the forward stroke will take 1 minute and the return, $\frac{1}{2}$ minute. The complete stroke, cut and return, will take $1\frac{1}{2}$ minutes. But since it takes $1\frac{1}{2}$ minutes to cut 16 ft., it will cut in 1 minute a distance equal to $16 \div 1\frac{1}{2} = 16 \times \frac{2}{3} = 10\frac{2}{3}$ ft., which is called the net or actual cutting speed of the planer. That is, the tool in the forward stroke moves at the rate of 16 ft. per minute while in contact with the work, but, owing to the time lost by the idle return stroke, the actual or net result is only an average of $10\frac{2}{3}$ ft. cut for every minute that the planer is operating.

Example

What is the net or actual cutting speed of a planer making 10 strokes per minute, each 2′ 6″ long, if the reverse is 3 to 1? Find the theoretical cutting speed.

SOLUTION. $2' 6'' = 2\frac{1}{2}'$

Distance traveled in one direction in one minute

$$= 10 \times 2\tfrac{1}{2} = 25' = \text{Net cutting speed}$$

Since the reverse is 3 to 1, the forward stroke will take up $\frac{3}{4}$ of each minute, while the return takes $\frac{1}{4}$. That is, it takes the planer $\frac{3}{4}$ of a minute to travel the 25′. Hence in 1 minute the theoretical cutting speed would be

$$25 \div \tfrac{3}{4} = 25 \times \tfrac{4}{3} = 33\tfrac{1}{3} \text{ ft.}$$

PROBLEMS

1. A planer has a forward stroke of 18 ft. per minute and a return twice as fast. Find the actual and the theoretical cutting speeds.

2. A planer with a 4 to 1 return makes 12 strokes a minute, each stroke 27″ long. Find the net and the theoretical cutting speeds.

3. Find the actual and the theoretical cutting speeds of a planer making 14 strokes per minute, the length of stroke being 3′ 6″ and the return stroke 3 to 1.

4. What are the net and the theoretical cutting speeds of a planer having a forward stroke of 24 ft. per minute and a return of 2 to 1?

5. With a return of 2 to 1 and a stroke of 4′ 3″, what are the net and the theoretical cutting speeds of a planer making 8 strokes a minute?

6. A planer with a 3 to 1 return makes 15 strokes per minute. Find the actual and the theoretical cutting speeds if the stroke is 28″ long.

272. Time Required for a Job on the Planer

Example

Find the time required for one complete cut on a casting 1′ 3″ × 3′ 6″ if the cutting speed is 20 ft. per minute, the feed $\frac{1}{4}$″ per stroke, and the return 2 to 1. Allow a clearance of 3″ at each end.

NOTE. A feed of $\frac{1}{4}$″ per stroke means that for each forward stroke there is a sidewise motion of the tool of $\frac{1}{4}$″.

SOLUTION.

$$\text{Total length of forward stroke} = 3' 6'' + \text{Clearance}$$
$$= 3' 6'' + 6'' = 4' 0''$$

Since the cutting speed is 20 ft. per minute while the length of the forward stroke is only 4 ft., the forward stroke will take only $\frac{4}{20}$ or $\frac{1}{5}$ of a minute. The return stroke, being twice as fast, will take $\frac{1}{2}$ of $\frac{1}{5}$ or $\frac{1}{10}$ of a minute.

The time required for one complete stroke $= \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$ of a minute. Since the feed is $\frac{1}{4}$″, four strokes will be required for every inch of width of the casting.

Total number of strokes $= 15 \times 4 = 60$.

Since every complete stroke takes $\frac{3}{10}$ of a minute, 60 strokes will take $60 \times \frac{3}{10} = 18$ minutes.

PROBLEMS

1. Find the time required for taking a complete cut on a plate $2' \times 3'$, if the cutting speed is 30 ft. per minute, the return is 4 to 1, and the feed $\frac{1}{8}''$. The clearance at each end is $3''$.

2. A planer has a forward movement of 24 ft. a minute and a return of 3 to 1. How long will it take for a complete cut on a casting $1'\ 8'' \times 5'\ 6''$ with a feed of $\frac{3}{16}''$? Allow for clearance.

3. How long will it take to finish one face of a casting $2'\ 6'' \times 4'\ 0''$ if the planer has a forward stroke of 20 ft. per minute, a return of 3 to 1, a roughing feed of $\frac{1}{16}''$, and a finishing feed of $\frac{1}{8}''$? Allow $3''$ at each end for the tool to clear.

4. With a forward stroke of 36 ft. per minute, a return of 2 to 1, and a feed of $\frac{3}{32}''$, how long will it take for one complete cut on a face $15'' \times 3'\ 4''$? Allow $3''$ at each end for tool clearance.

5. Find the time required for finishing a casting $2'\ 6'' \times 6'\ 0''$ on a planer having a forward movement of 40 ft. per minute and a return of 2 to 1, if a feed of $\frac{1}{8}''$ is used for roughing and $\frac{1}{4}''$ for finishing. Clearance for tool is $3''$ at each end.

273. Number of Strokes per Minute

To find the number of strokes per minute for a given length of stroke and given cutting and return speeds proceed as in the following example:

Example

Find the number of strokes per minute a planer must make if the forward or cutting speed is 28 ft. per minute, the return 2 to 1, and the length of stroke 8′ including clearance.

SOLUTION.

Time required for forward stroke $= \frac{8}{28} = \frac{2}{7}$ minute

Time required for return stroke $= \frac{1}{2}$ of $\frac{2}{7} = \frac{1}{7}$ minute

Time required for complete stroke $= \frac{1}{7} + \frac{2}{7} = \frac{3}{7}$ minute

Since each complete stroke takes $\frac{3}{7}$ of a minute, in 1 minute the planer will make $1 \div \frac{3}{7}$ or $1 \times \frac{7}{3} = 2\frac{1}{3}$ strokes.

PROBLEMS

NOTE. *In each of the following problems the given length of stroke includes the clearance for the tool at both ends.*

1. Find the number of strokes per minute of a planer having a forward stroke of 24 ft. per minute, a return of 2 to 1, and a length of stroke of 6′.

2. How many strokes per minute must a planer make if its forward movement is 30 ft. per minute, its return 2 to 1, and length of stroke 4′?

3. A planer has a cutting stroke of 20 ft. per minute, a return of 3 to 1, and a length of stroke of 5′ 6″. How many strokes will it make in a minute?

4. Compute the number of strokes per minute for a cut 6′ 6″ long if the cutting speed of the planer is 24 ft. per minute and the return 2 to 1.

5. The forward movement of a planer is 32 ft. per minute and the return is 2 to 1. Compute the number of strokes per minute required for a stroke 5′ long.

274. Cutting Speed of Shapers

The cutting speed of a *geared shaper* is figured in exactly the same way as the cutting speed of a planer.

The method of computing the cutting speed of a *crank shaper* is illustrated in the following example:

Example

Find the cutting speed of a shaper making 30 r.p.m. with a reverse of 2 to 1 if the length of stroke is 14″.

SOLUTION.

Length of stroke $= 14'' = 1\frac{1}{6}'$
Total length of metal cut in one minute $= 30 \times 1\frac{1}{6} = 35'$
Cutting speed $= 35 \times \frac{3}{2} = 52.5$ ft. per minute

EXPLANATION. Since there is a stroke for each revolution, there will be 30 strokes per minute. Hence the actual total length of metal cut per minute is 30 times the length of stroke, or $30 \times 1\frac{1}{6} = 35'$. That is, 35′ is the actual net cutting speed. But since the return is twice as fast as the forward stroke, the return takes $\frac{1}{3}$ of the minute while the forward stroke takes $\frac{2}{3}$ of the minute. That is, 35′ of metal is cut in $\frac{2}{3}$ of a minute. Hence for a whole minute the cutting speed of the shaper is $35 \div \frac{2}{3} = 35 \times \frac{3}{2} = 52.5$ ft. per minute.

PROBLEMS

1. A crank shaper with a 2 to 1 return makes 28 strokes of 1′ length in a minute. Find the cutting speed.

2. Find the cutting speed of a crank shaper making 25 r.p.m. with a reverse of 3 to 1, if the length of stroke is 10″.

3. With a return of 2 to 1 and a speed of 27 r.p.m., what is the cutting speed of a crank shaper if the length of stroke is 15″?

4. What is the cutting speed of a crank shaper making 32 r.p.m. with a 2 to 1 return if the length of stroke is 14″?

5. A crank shaper having a 3 to 1 return makes 28 r.p.m. and the length of cut is 9″. What is the cutting speed?

275. Cutting Feed of a Lathe

The *cutting feed of a lathe* is the distance the tool travels sidewise along the work while the piece of work makes one revolution. If, for example, the tool moves $\frac{1}{25}$″ while the piece turns around once, the feed is $\frac{1}{25}$″. Sometimes the feed is stated as so many revolutions of the piece for every inch of travel of the tool. A feed of 25 means that the work makes 25 revolutions while the tool travels 1 in.

Example 1

A piece of work revolves 60 times while the tool travels $\frac{3}{4}$″. Find the feed.

SOLUTION 1. Feed $= 60 \div \frac{3}{4} = 60 \times \frac{4}{3} = 80$

EXPLANATION. Since the piece revolves 60 times while the tool travels $\frac{3}{4}$″, for the tool to move a whole inch, the piece will revolve $60 \times \frac{4}{3}$ or 80 times.

SOLUTION 2. Feed $= \frac{1}{60}$ of $\frac{3}{4} = \frac{1}{80}$″

EXPLANATION. Since in 60 turns of the piece of work the tool moves $\frac{3}{4}$″, in one turn of the work, the tool will travel $\frac{1}{60}$ of $\frac{3}{4}$ or $\frac{1}{80}$″.

Example 2

With a feed of 28, how many revolutions are required to take a cut over a piece of work 8″ long?

SOLUTION. Total revolutions required $= 8 \times 28 = 224$

EXPLANATION. Since the piece must revolve 28 times for each inch of travel of the tool, for 8″ it must revolve 8×28 or 224 times.

Example 3

Find the time required to take one cut over a piece of work 14″ long and $1\frac{1}{2}$″ in diameter if the cutting speed is 32 ft. per minute and the feed 20.

SOLUTION.

Circumference of work $= \frac{22}{7} \times 1\frac{1}{2} = \frac{33}{7}$″

Cutting speed in inches $= 32 \times 12 = 384$″

R.p.m. = Cutting speed ÷ Circumference

$= 384 \div \frac{33}{7} = 384 \times \frac{7}{33} = \frac{896}{11}$

Total revolutions required for one cut

$= 14 \times 20 = 280$

Time required for one cut = Total revolutions ÷ R.p.m.

$= 280 \div \frac{896}{11} = 280 \times \frac{11}{896}$

$= 3.4$ minutes

PROBLEMS

1. Find the feed of a lathe if the tool travels $3\frac{1}{4}$″ while the work makes 120 r.p.m.

2. With a feed of 30, how many revolutions are required to take a cut over a piece of work $6\frac{1}{2}$″ long?

3. A shaft 28″ long is being turned in a lathe at a speed of 48 r.p.m. and a feed of 25. Find the time required for one cut.

4. Find the time required for one cut over a piece of work $1\frac{3}{4}$″ in diameter and 20″ long which is being turned in a lathe with a cutting speed of 36 and a feed of 18.

5. With a feed of 32 and a cutting speed of 24 ft. per minute, how long will it take for a cut over a piece $2\frac{1}{2}$″ in diameter and 8″ long?

6. A piece of tool steel $1\frac{7}{8}$″ in diameter and 15″ long is being turned in the lathe. If the cutting speed is 24 ft. per minute and the feed $\frac{1}{20}$″, find the time required for one cut.

7. With a feed of 22 and a cutting speed of 27 ft. per minute, how many inches of a piece of 2″ steel will be turned in 10 minutes?

8. How long will it take for one cut over a $3\frac{1}{8}''$ rod $18''$ long with a cutting speed of 35 ft. per minute and a feed of $\frac{1}{25}''$?

9. A lathe tool moves $3\frac{1}{4}''$ along the work in 1 minute while the lathe makes 250 r.p.m. What is the feed?

276. Drill Press Feed

The feed of a drill press is the distance the drill penetrates the work in one revolution. If the drill revolves 100 times while it penetrates a distance of 1 in., the feed is $\frac{1}{100}''$ per revolution.

Example

Find the time required to drill through a piece of work $3''$ thick with a drill making 240 r.p.m. if the feed per revolution is $0.007''$.

SOLUTION. Penetration in 1 minute $= 240 \times 0.007$

$= 1.680''$

Time to drill through $3''$ $= 3 \div 1.68$

$= 1.788$ minutes

$= 1$ minute 47 seconds

EXPLANATION. Since the drill penetrates $0.007''$ for each turn, in a minute it will advance 240 times $0.007''$ or $1.68''$. To drill through $3''$, it will take as many times 1 minute as $1.68''$ is contained in $3''$, or $3 \div 1.68 = 1.788$ minutes.

PROBLEMS

1. With a speed of 250 r.p.m. and a feed of $0.018''$ per revolution, how long will it take to drill through a piece of cast iron $4\frac{1}{4}''$ thick?

2. A $1\frac{1}{8}''$ drill making 280 r.p.m. is fed $0.016''$ per revolution. Find the time required to drill through a piece $3\frac{7}{16}''$ thick.

3. How long will it take to drill 24 holes in a plate $1\frac{1}{8}''$ thick with a drill making 225 r.p.m. and a feed of $0.015''$ per revolution? Allow $\frac{1}{2}$ minute for adjusting the plate after each hole is drilled.

4. With a feed of $0.025''$ and a speed of 280 r.p.m., how long will it take to drill through a piece $3\frac{7}{8}''$ thick?

5. At a speed of 300 r.p.m. and a feed of $0.02''$, how long will it take to drill through a piece $4\frac{1}{2}''$ thick?

277. Milling Machine Feed

The feed of the milling machine is the distance traveled by the table while the cutter makes one revolution. If a milling cutter revolves 80 times while the table moves 1″, the feed is $\frac{1}{80}$″.

Example 1

Find the feed per minute of a milling cutter if the feed is 0.02″ per revolution and the cutter makes 120 r.p.m.

SOLUTION. Feed per minute = $120 \times 0.02 = 2.4$″

EXPLANATION. Since the cutter makes 120 revolutions in a minute, it will cut 120×0.02″ or 2.4″ in a minute.

Example 2

How long will it take for a cutter to traverse a piece of work 10″ long if it makes 32 r.p.m. and the feed is $\frac{1}{40}$″?

SOLUTION. Total revolutions required = $10 \times 40 = 400$

Time required = $400 \div 32$

= 12.5 minutes

EXPLANATION. With a feed of $\frac{1}{40}$″, 40 revolutions of the cutter are required for each inch traversed. Since the cutter makes 32 revolutions in 1 minute, to make 400 revolutions it will take $400 \div 32$ or 12.5 minutes.

Example 3

A 2″ milling cutter makes 210 r.p.m. If the table feed is 7″ per minute, find the cutting speed. Find the feed per tooth if the cutter has 14 teeth.

SOLUTION.

Cutting speed = Circumference × R.p.m.

= $2 \times \frac{22}{7} \times \frac{1}{12} \times 210 = 110$ ft. per minute

Feed per revolution = $7 \div 210 = \frac{1}{210} = 0.0333$″

Feed per tooth = $0.0333 \div 14 = 0.0024$″

EXPLANATION. The cutting speed is found by multiplying the circumference by the revolutions per minute. Since the table moves 7″ in the time that the cutter makes 210 revolutions, in one revolution of the cutter the table will move $\frac{1}{210}$ of 7″ or $\frac{1}{30}$″. Since there are 14 teeth in the cutter, the feed per tooth will be $\frac{1}{14}$ of the feed per revolution or $\frac{1}{14}$ of 0.0333″ = 0.0024″.

PROBLEMS

1. A milling cutter revolves 75 times while the table moves 1″. Find the feed per revolution.

2. A milling cutter making 150 r.p.m. has a feed of 0.02″. What is the feed per minute?

3. What is the feed per revolution of a milling cutter which makes 80 r.p.m. with a table feed of $6\frac{1}{2}$″? What is the feed per tooth if the cutter has 10 teeth?

4. A $\frac{3}{4}$″ end mill makes 210 r.p.m. and the table feed is $8\frac{1}{2}$″ per minute. Find the cutting speed. Find the feed per tooth if the end mill has 8 teeth.

5. A milling cutter makes 100 r.p.m. with a table feed of $5\frac{3}{8}$″. What is the feed per revolution? What is the feed per tooth if the cutter has 12 teeth?

6. A $\frac{3}{4}$″ cutter with 10 teeth makes 200 r.p.m. with a table feed of $8\frac{3}{4}$″. Find the cutting speed and the feed per tooth.

7. A milling cutter $2\frac{1}{4}$″ in diameter has 12 teeth. If the cutting speed is 75 ft. per minute, find the feed per minute when every tooth is allowed to cut a chip 0.005″ thick.

8. A milling cutter 3″ in diameter has 15 teeth. If it makes 48 r.p.m. and the table feed is $2\frac{3}{4}$″ per minute, find the thickness of the chip cut by each tooth.

9. A $\frac{7}{8}$″ end mill with 8 teeth has a surface speed of 35 ft. per minute. What is the feed per minute if each tooth cuts a chip 0.004″ thick?

22 Gears

278. Spur Gears

Two round steel disks like those shown in Fig. 372 will drive each other if pressed close together. This is called a friction drive and may be used where little power is to be transmitted. It is quite evident, however, that if any considerable power is required, the two cylindrical disks will slip on each other. To prevent this slippage and make a

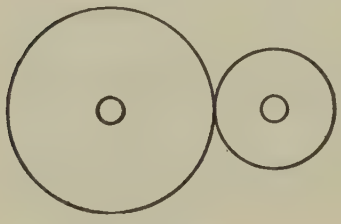


Fig. 372

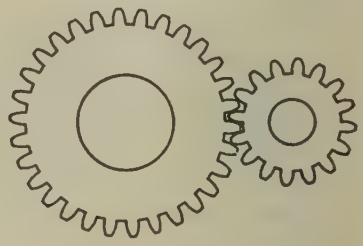


Fig. 373

positive drive, teeth are provided on each disk and corresponding grooves for the teeth to enter. Such cylindrical disks or wheels are called ***gear wheels*** or ***gears***. A pair of gears is shown in Fig. 373.

Pitch Circle

The circle representing the disk on which the teeth are built is called the ***pitch circle***. It is not visible at all on the actual gear, yet it remains the most important element in the design of the gear. See Fig. 374 and Fig. 375.

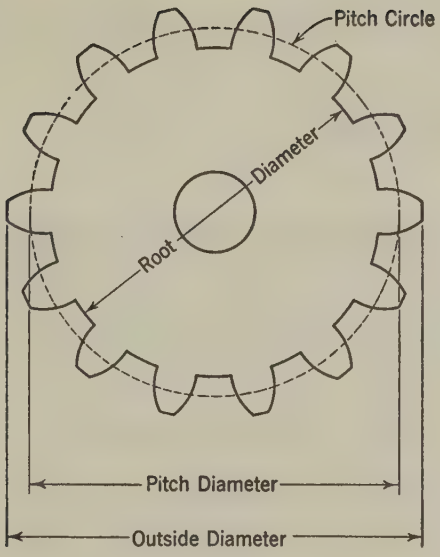
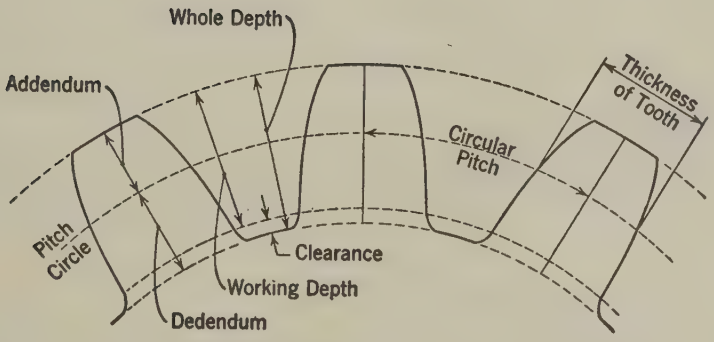


Fig. 374



Gear Tooth Parts

Fig. 375

Pitch Diameter

The diameter of the pitch circle is called the ***pitch diameter*** of the gear.

Gear Tooth Parts

See Fig. 375.

Addendum, Dedendum, and Clearance

The projection of the gear tooth above the pitch circle is called the ***addendum.*** The part of the tooth below the pitch circle is called the ***dedendum.*** Its depth is equal to that of the addendum plus an additional depth called the ***clearance.***

Whole Depth and Working Depth

The sum of the addendum and the dedendum constitutes the ***whole depth.*** This is also called the ***cutting depth,*** as it is the depth of cut required in cutting the gear from the blank. The whole depth minus the clearance is called the ***working depth,*** that is, the depth to which the teeth of the mating gear enter. The part of the working depth below the pitch circle is equal to the depth of the addendum. Hence the working depth is equal to twice the depth of the addendum. The whole depth of a gear tooth is therefore equal to two addenda plus the clearance.

Outside Diameter

The ***outside diameter,*** or the diameter of the blank, is evidently equal to the pitch diameter plus twice the addendum. See Fig. 374.

The Root Diameter

The ***root diameter*** or the diameter between the bottoms of the teeth is equal to the outside diameter minus twice the whole depth.

Circular Pitch

The distance, P_c, between the centers of two adjoining teeth is called the ***circular pitch.*** It is measured on the circumference of the pitch circle and is equal to that circumference divided by the

number of teeth. If we call the diameter of the pitch circle D and the number of teeth N, then the circumference of the pitch circle $= \pi \times D$; and the circular pitch $= \dfrac{\pi \times D}{N}$; or, $P_c = \dfrac{\pi \times D}{N}$.

Also, since $N \times P_c =$ the pitch circumference, we have

$$\text{Pitch diameter} = \frac{\text{Pitch circumference}}{\pi}; \text{ or, } D = \frac{N \times P_c}{\pi}$$

Also, since $\dfrac{\text{Pitch circumference}}{\text{Circular pitch}} = \text{Number of teeth}$, we have

$$\text{Number of teeth, } N = \frac{\pi \times D}{P_c}$$

The ***thickness of tooth*** is half the circular pitch.

Example 1

Find the circular pitch of a gear which has 48 teeth and a pitch diameter of 6″.

SOLUTION. $P_c = \dfrac{\text{Pitch circumference}}{\text{Number of teeth}} = \dfrac{\pi \times D}{N}$

$$= \frac{3.1416 \times 6}{48} = 0.3927''$$

Example 2

Find the pitch diameter of a gear which has 45 teeth and a circular pitch of 0.3142″.

SOLUTION. $\text{Pitch diameter} = \dfrac{\text{Circumference of pitch circle}}{\pi}$

$$D = \frac{N \times P_c}{\pi} = \frac{45 \times 0.3142}{3.1416} = 4.5''$$

Example 3

Find the number of teeth in a gear which has a pitch diameter of $12\frac{1}{2}$″ and a circular pitch of 0.7854″.

SOLUTION. $\text{Number of teeth} = \dfrac{\text{Circumference of pitch circle}}{P_c}$

$$N = \frac{\pi \times D}{P_c} = \frac{3.1416 \times 12.5}{0.7854} = 50 \text{ teeth}$$

Example 4

Find the thickness of tooth of the gear in Example 3.

SOLUTION. Thickness of tooth $= \frac{P_c}{2} = \frac{0.7854}{2} = 0.3927''$

PROBLEMS

1. Find the circular pitch of a gear which has 60 teeth and a pitch diameter of $7\frac{1}{2}''$.
2. What is the circular pitch of a 70-tooth gear if the pitch diameter is $3\frac{1}{2}''$?
3. A gear which has a pitch diameter of 22″ has 55 teeth. Find the circular pitch.
4. Find the circular pitch of a 70-tooth gear with a 40″ pitch diameter.
5. Find the circular pitch of a 36-tooth gear with a 4″ pitch diameter.
6. A gear has 65 teeth and 0.6283″ circular pitch. Find the pitch diameter.
7. A gear has 31 teeth and a circular pitch of 1.5708″. Find the pitch diameter.
8. Find the pitch diameter of an 82-tooth gear whose circular pitch is 0.1963″.
9. A gear has 30 teeth and 0.3927″ circular pitch. Find the pitch diameter.
10. Find the pitch diameter of a 45-tooth gear whose circular pitch is 2.0944″.
11. How many teeth has a gear whose circular pitch is 3.1416″ and whose pitch diameter is 82″?
12. Find the number of teeth in a gear with a pitch diameter of 48″ and a circular pitch of 2.5133″.
13. Find the number of teeth on a gear with a pitch diameter of 4″ and a circular pitch of 0.6283″.
14. Find the number of teeth on a gear with a pitch diameter of $5\frac{1}{2}''$ and a circular pitch of 0.2244″.
15. Find the number of teeth on a gear with a pitch diameter of $2\frac{1}{4}''$ and a circular pitch of 0.2618″.
16. Find the thickness of tooth in the gears in Problems 11 to 15.

279. Diametral Pitch

In modern gear design the circular pitch is used very little. Another pitch, called the ***diametral pitch,*** is commonly used because of its convenience in computation.

When we speak of the pitch of a gear, we mean diametral pitch unless circular pitch is specified.

The diametral pitch is the number of teeth per inch of pitch diameter.

Diametral pitch is not a distance, as pitch is commonly understood. For instance, if a gear has a pitch diameter of 3″ and 24 teeth, it has $\frac{24}{3}$ or 8 teeth for every inch of its pitch diameter; that is, its diametral pitch is 8. It is called an 8-pitch gear. It will mesh only with other 8-pitch gears. A 60-tooth, 8-pitch gear has a pitch diameter of $\frac{60}{8}$ or $7\frac{1}{2}$″; a 40-tooth, 8-pitch gear has a pitch diameter of $\frac{40}{8}$ or 5″.

Since the diametral pitch means the number of teeth per inch of pitch diameter, the number of teeth on a gear is evidently equal to the pitch diameter times the pitch. A 6-pitch gear with a pitch diameter of $5\frac{1}{2}$″ will have $5\frac{1}{2} \times 6$ or 33 teeth. This relation between pitch, number of teeth, and pitch diameter simplifies gear computation to a great extent.

Using the following letters, D for pitch diameter,
P for diametral pitch,
N for number of teeth,

we have, by definition:

$$\text{Diametral pitch} = \frac{\text{Number of teeth}}{\text{Pitch diameter}}; \quad \text{or} \quad P = \frac{N}{D}$$

Also, Number of teeth

$$= \text{Pitch diameter} \times \text{Pitch}; \quad \text{or} \quad N = D \times P.$$

$$\text{Also, Pitch diameter} = \frac{\text{Number of teeth}}{\text{Diametral pitch}}; \quad \text{or} \quad D = \frac{N}{P}.$$

Example 1

Find the diametral pitch of a gear which has 72 teeth and a pitch diameter of $4\frac{1}{2}$″.

SOLUTION. $P = \frac{N}{D} = \frac{72}{4\frac{1}{2}} = 16$. The gear is a 16-pitch gear.

Example 2

What is the pitch diameter of a 12-pitch gear with 45 teeth?

SOLUTION. $D = \dfrac{N}{P} = \dfrac{45}{12} = 3\frac{3}{4}$. The pitch diameter is $3\frac{3}{4}''$.

Example 3

How many teeth must a 5-pitch gear have if its pitch diameter is 6.2″?

SOLUTION. $N = D \times P = 6.2 \times 5 = 31$ teeth. The gear has 31 teeth.

PROBLEMS

1. Find the diametral pitch of a 68-tooth gear whose pitch diameter is 17″.

2. Find the diametral pitch of a 40-tooth gear whose pitch diameter is 16″.

3. Find the diametral pitch of a 30-tooth gear whose pitch diameter is $2\frac{1}{2}''$.

4. Find the diametral pitch of a 65-tooth gear whose pitch diameter is $3\frac{1}{4}''$.

5. Find the diametral pitch of a 124-tooth gear whose pitch diameter is $7\frac{3}{4}''$.

6. How many teeth are there on a 12-pitch gear whose pitch diameter is $5\frac{1}{2}''$?

7. How many teeth are there on a 9-pitch gear whose pitch diameter is 11″?

8. How many teeth are there on a 20-pitch gear whose pitch diameter is 9.1″?

9. How many teeth are there on a 4-pitch gear whose pitch diameter is $16\frac{3}{4}''$?

10. How many teeth are there on a 16-pitch gear whose pitch diameter is $3\frac{7}{8}''$?

11. Find the pitch diameter of a 70-tooth, 5-pitch gear.

12. Find the pitch diameter of a 48-tooth, 9-pitch gear.

13. Find the pitch diameter of an 18-tooth, 8-pitch gear.

14. Find the pitch diameter of a 45-tooth, $2\frac{1}{2}$-pitch gear.

15. Find the pitch diameter of a 27-tooth, $1\frac{1}{2}$-pitch gear.

280. Proportions of Gear Teeth

In standard gears *the addendum is the reciprocal of the pitch;* that is, addendum $= \frac{1}{\text{Pitch}}$. In a 10-pitch gear the addendum is $\frac{1}{10}''$; in a 4-pitch gear the addendum is $\frac{1}{4}''$. The dedendum is equal in depth to the addendum plus the clearance. The *clearance* is usually stated as $\frac{1}{20}$ of the circular pitch.

281. Relation between Circular Pitch and Diametral Pitch

It is often necessary to convert circular pitch into diametral pitch, and vice versa. The circular pitch, being part of the circumference of the pitch circle, is found by dividing that circumference by the number of teeth in the gear.

Let D = Pitch diameter
N = Number of teeth
P = Pitch
P_c = Circular pitch

$$\text{Pitch circumference} = \pi \times \text{Pitch diameter} = \pi \times D$$

$$\text{Circular pitch} = \frac{\text{Pitch circumference}}{\text{Number of teeth}}$$

that is,

$$P_c = \frac{\pi \times D}{N}$$

But

$$N = D \times P$$

Writing this in place of N, we get

$$P_c = \frac{\pi \times D}{P \times D}$$

from which, by cancellation, we get

Formula for converting diametral pitch into circular pitch:

$$P_c = \frac{\pi}{P}$$

To convert circular pitch into diametral pitch we proceed as follows:

$$\text{Diametral pitch} = \frac{\text{Number of teeth}}{\text{Pitch diameter}}$$

that is,

$$P = \frac{N}{D}$$

But

$$N = \frac{\pi \times D}{P_c}$$

Putting this value in place of N in the formula, we get

$$P = \frac{\pi \times D}{P_c \times D}$$

which, by cancellation of D, becomes the formula for converting circular pitch into diametral pitch:

$$P = \frac{\pi}{P_c}$$

Example 1

Find the circular pitch of a 12-pitch gear.

SOLUTION. $P_c = \dfrac{\pi}{P} = \dfrac{3.1416}{12} = 0.2618''$

Example 2

Find the diametral pitch of a gear whose circular pitch is 0.6283″.

SOLUTION. $P = \dfrac{\pi}{P_c} = \dfrac{3.1416}{0.6283} = 5$

PROBLEMS

Find the circular pitch of the following gears.

1. A 6-pitch gear
2. A $2\frac{1}{2}$-pitch gear
3. A 10-pitch gear
4. A $1\frac{1}{4}$-pitch gear
5. An 18-pitch gear
6. A 20-pitch gear
7. A $1\frac{1}{2}$-pitch gear
8. A 3-pitch gear
9. An 8-pitch gear
10. A 14-pitch gear

Find the diametral pitch of the following gears.

11. A gear whose circular pitch is 1.2566″
12. A gear whose circular pitch is 0.1571″
13. A gear whose circular pitch is 0.3491″
14. A gear whose circular pitch is 1.7952″
15. A gear whose circular pitch is 1.5708″
16. A gear whose circular pitch is $\frac{3}{4}$″
17. A gear whose circular pitch is $1\frac{1}{2}$″
18. A gear whose circular pitch is 0.3927″
19. A gear whose circular pitch is 0.1963″
20. A gear whose circular pitch is 2.5133″

282. Clearance

The *clearance*, which is usually stated as $\frac{1}{20}$ of the circular pitch, can also be expressed in terms of the diametral pitch.

$$\text{Clearance} = \tfrac{1}{20} \text{ of } P_c = \tfrac{1}{20} \text{ of } \frac{\pi}{P}$$

$$= \tfrac{1}{20} \times \frac{3.1416}{P} = \frac{0.157}{P}$$

Example 1

Find the clearance of a gear whose circular pitch is 0.3491″.

SOLUTION. $\text{Clearance} = \dfrac{P_c}{20} = \dfrac{0.3491}{20} = 0.0175''$

Example 2

Find the clearance of a 12-pitch gear.

SOLUTION. $\text{Clearance} = \dfrac{0.157}{P} = \dfrac{0.157}{12} = 0.0131''$

PROBLEMS

Find the clearance in each of the following gears.

1. A 16-pitch gear
2. A 12-pitch gear
3. A $2\frac{1}{2}$-pitch gear
4. An 8-pitch gear
5. A $1\frac{3}{4}$-pitch gear
6. A gear whose circular pitch is 1.5708″
7. A gear whose circular pitch is 0.2244″

8. A gear whose circular pitch is 0.6283″
9. A gear whose circular pitch is 0.2618″
10. A gear whose circular pitch is 0.75″

283. Depth of Tooth

The whole depth of tooth = Addendum + Dedendum

$$= \frac{1}{P} + \frac{1}{P} + \frac{0.157}{P}$$

$$= \frac{2.157}{P}$$

Since Dedendum = Addendum + Clearance

Whole depth = Addendum + Addendum + Clearance

Example 1

Find the whole depth of tooth of an 8-pitch gear.

SOLUTION. Whole depth $= \frac{2.157}{P} = \frac{2.157}{8} = 0.2696''$

Example 2

Find the whole depth of tooth of a gear whose circular pitch is 0.6283″.

SOLUTION. $P = \frac{\pi}{P_c} = \frac{3.1416}{0.6283} = 5$

Whole depth $= \frac{2.157}{P} = \frac{2.157}{5} = 0.4314''$

PROBLEMS

Find the whole depth of tooth in each of the following gears.

1. A 16-pitch gear
2. A 12-pitch gear
3. A $2\frac{1}{2}$-pitch gear
4. An 8-pitch gear
5. A $1\frac{3}{4}$-pitch gear
6. A gear whose circular pitch is 1.5708″
7. A gear whose circular pitch is 0.2244″
8. A gear whose circular pitch is 0.6283″
9. A gear whose circular pitch is 0.2618″
10. A gear whose circular pitch is 0.75″

284. Outside Diameter

The outside diameter of a gear

$$= \text{Pitch diameter} + 2 \times \text{Addendum}$$

$$= D + \frac{1}{P} + \frac{1}{P}$$

$$= D + \frac{2}{P}$$

Example

Find the outside diameter of an 8-pitch gear whose diameter is $5\frac{1}{2}''$.

SOLUTION.

$$\text{Outside diameter} = D + \frac{2}{P} = 5\tfrac{1}{2} + \tfrac{2}{8} = 5\tfrac{1}{2} + \tfrac{1}{4} = 5\tfrac{3}{4}''$$

PROBLEMS

Find the outside diameters of the following gears.

1. A 16-pitch gear with a pitch diameter of $8\frac{1}{4}''$
2. A $2\frac{1}{2}$-pitch gear whose pitch diameter is $24''$
3. A 16-pitch gear whose pitch diameter is $1.875''$
4. A 10-pitch gear whose pitch diameter is $3.1''$
5. A 20-pitch gear whose pitch diameter is $9\frac{1}{4}''$
6. A 4-pitch gear whose pitch diameter is $12''$
7. A 12-pitch gear with a pitch diameter of $3\frac{1}{4}''$
8. A 9-pitch gear whose pitch diameter is $5.333''$
9. A 14-pitch gear whose pitch diameter is $11\frac{1}{2}''$
10. A 6-pitch gear whose pitch diameter is $3.667''$

285. Pitch Diameter

To find the pitch diameter when the outside diameter and the pitch are given, the method is as follows:

$$\text{Pitch diameter} = \text{Outside diameter} - 2 \times \text{Addendum}$$

Using the symbol OD for outside diameter, we have the formula,

$$D = OD - \frac{2}{P}$$

The formula for finding the pitch diameter when the outside diameter and the number of teeth are given is as follows:

$$D = \frac{N \times OD}{N + 2}$$

This formula states that we must multiply the outside diameter by the number of teeth and divide that product by the number of teeth plus 2.

Example 1

Find the pitch diameter of an 8-pitch gear whose outside diameter is $6\frac{1}{4}''$.

SOLUTION. $D = OD - \frac{2}{P} = 6\frac{1}{4} - \frac{2}{8} = 6\frac{1}{4} - \frac{1}{4} = 6''$

Example 2

Find the pitch diameter of a 66-tooth gear whose oustide diameter is $8\frac{1}{2}''$.

SOLUTION. $D = \frac{N \times OD}{N + 2} = \frac{66 \times 8\frac{1}{2}}{66 + 2} = \frac{561}{68} = 8\frac{1}{4}''$

PROBLEMS

Find the pitch diameter of the following gears.

1. A gear which has 58 teeth and an outside diameter of $7\frac{1}{2}''$
2. A 40-tooth gear whose outside diameter is $16.8''$
3. A gear which has 114 teeth and an outside diameter of $9.667''$
4. A 31-tooth gear whose outside diameter is $3.3''$
5. A 57-tooth gear whose outside diameter is $14\frac{3}{4}''$
6. An 18-pitch gear whose outside diameter is $5\frac{1}{2}''$
7. A 14-pitch gear whose outside diameter is $3.286''$
8. A $2\frac{1}{2}$-pitch gear whose outside diameter is $40.8''$
9. A 6-pitch gear whose outside diameter is $15''$
10. A 12-pitch gear whose outside diameter is $7.417''$

286. Use of Formulas

The student will notice that frequently there are several formulas for finding a required item. He must select the

formula which contains the information given. For instance, to find the pitch diameter of a gear whose outside diameter is $7\frac{3}{4}''$ and which has 60 teeth, we select the formula which gives D in terms of OD and N, namely $D = \dfrac{OD \times N}{N + 2}$. When the circular pitch is given, it will often be found convenient to convert it into diametral pitch and continue the calculation in terms of diametral pitch.

Example

Find the depth of cut and size of blank for a 6-pitch gear of 80 teeth.

SOLUTION. Depth of cut is the same as the whole depth.

$$\text{Depth of cut} = \frac{2.157}{P} = \frac{2.157}{6} = 0.3595''$$

Size of blank is the same as the outside diameter.

$$\text{Size of blank} = \frac{N + 2}{P} = \frac{80 + 2}{6} = \frac{82}{6} = 13.667''$$

PROBLEMS

Compute the missing dimensions of the gears in the following table.

	Pitch	Pitch Diameter	Number of Teeth	Circular Pitch	Addendum	Clearance	Dedendum	Working Depth	Whole Depth	Outside Diameter
1.	10	$2\frac{1}{2}''$								
2.	16		25							
3.	12									$4\frac{2}{3}''$
4.			32							$8\frac{1}{2}''$
5.		3.417″	41							
6.		3.8″		0.1571″						
7.			55	1.2566″						
8.				0.3927″						$9\frac{1}{2}''$
9.	$1\frac{1}{2}$		90							
10.	14		75							

FORMULAS FOR GEAR COMPUTATION

To Find	Symbol	Having	Formula
Pitch	P	Circular pitch	$P = \frac{\pi}{P_c}$
Circular pitch	P_c	Pitch	$P_c = \frac{\pi}{P}$
	P_c	Pitch diameter and number of teeth	$P_c = \frac{\pi \times D}{N}$
Pitch	P	Pitch diameter and number of teeth	$P = \frac{N}{D}$
	P	Outside di meter and number of teeth	$P = \frac{N + 2}{OD}$
Pitch diameter	D	Number of teeth and pitch	$D = \frac{N}{P}$
	D	Number of teeth and circular pitch	$D = \frac{N \times P_c}{\pi}$
	D	Outside diameter and number of teeth	$D = \frac{OD \times N}{N + 2}$
	D	Outside diameter and pitch	$D = OD - \frac{2}{P}$
Outside diameter	OD	Number of teeth and pitch	$OD = \frac{N + 2}{P}$
	OD	Pitch diameter and pitch	$OD = D + \frac{2}{P}$
	OD	Number of teeth and circular pitch	$OD = \frac{(N + 2) \times P_c}{\pi}$
	OD	Pitch diameter and circular pitch	$OD = D + \frac{2P_c}{\pi}$
Number of teeth	N	Pitch diameter and pitch	$N = D \times P$
	N	Pitch diameter and circular pitch	$N = \frac{D \times \pi}{P_c}$
Thickness of tooth	T	Circular pitch	$T = \frac{P_c}{2}$
Addendum	Add	Pitch	$Add = \frac{1}{P}$
	Add	Circular pitch	$Add = \frac{P_c}{\pi}$
Dedendum	Ded	Pitch	$Ded = \frac{1.157}{P}$
	Ded	Circular pitch	$Ded = .3683P_c$
Clearance	Cl	Circular pitch	$Cl = \frac{P_c}{20}$
	Cl	Pitch	$Cl = \frac{0.157}{P}$
Whole depth	WD	Pitch	$WD = \frac{2.157}{P}$
	WD	Circular pitch	$WD = 0.6866 \times P_c$
Working depth	wd	Pitch	$wd = \frac{2}{P}$
	wd	Circular pitch	$wd = \frac{2 \times P_c}{\pi}$

287. Racks

The proportions of the teeth in a rack are the same as in a gear. The linear pitch of the rack is the same as the circular pitch of the gears that mesh with the rack. See Fig. 376, which shows the tooth proportions for a rack to mesh with a 6-pitch gear.

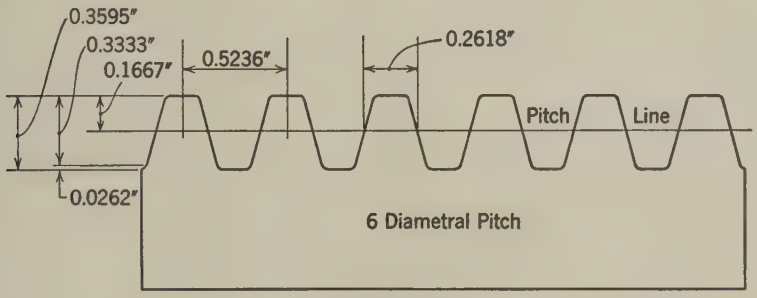


Fig. 376

Example 1

Find the linear pitch of a 5-pitch rack.

SOLUTION. Linear pitch = Circular pitch;

$$P_c = \frac{\pi}{P} = \frac{3.1416}{5} = 0.6283''$$

Example 2

Find the depth of cut for a 12-pitch rack.

SOLUTION.

$$\text{Depth of cut} = \text{Whole depth} = \frac{2.157}{P} = \frac{2.157}{12} = 0.1798''$$

PROBLEMS

1. Find whole depth of tooth of a 4-pitch rack.
2. Find the linear pitch of a 3-pitch rack.
3. A 6-pitch rack is driven by a 27-tooth pinion. Find the distance traveled for each turn of the pinion.
4. A rack is required to have a linear pitch of $\frac{3}{4}''$. What must be the pitch diameter of a 20-tooth pinion to mesh with the rack?
5. Find the pitch diameter of a 30-tooth pinion to mesh with a rack whose linear pitch is $1\frac{1}{8}''$.

288. Center-to-Center Distance of Gears

Since the pitch circles of gears in mesh are supposed to be in contact, it is evident that the distance from the center of one gear to the center of the other gear is equal to the sum of the pitch radii. In other words, the center-to-center distance of a pair of gears in mesh is equal to half the sum of their pitch diameters, Fig. 377(*a*).

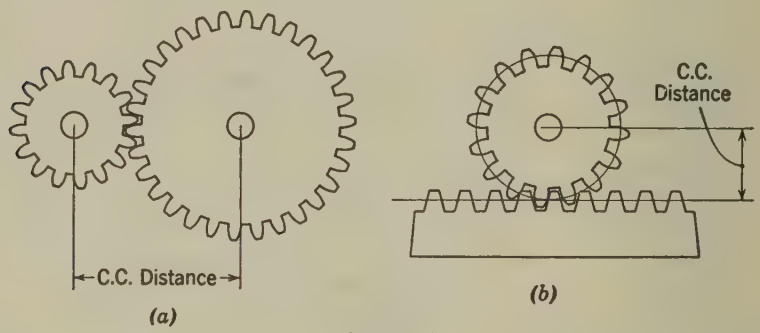


Fig. 377

In a rack and pinion the distance from the center of the pinion to the pitch line of the rack is equal to half the pitch diameter of the pinion, Fig. 377(*b*).

Example 1

Two gears in mesh have pitch diameters of 8″ and $3\frac{1}{4}$″ respectively. Find their center-to-center distance.

SOLUTION. Center-to-center distance $= \dfrac{8 + 3\frac{1}{4}}{2} = \dfrac{11\frac{1}{4}}{2} = 5\frac{5}{8}''$

Example 2

Find the center-to-center distance of a pair of 8-pitch gears, having 42 teeth and 75 teeth respectively.

SOLUTION. 75-tooth gear, $D = \dfrac{N}{P} = \dfrac{75}{8} = 9\frac{3}{8}''$

42-tooth gear, $D = \dfrac{N}{P} = \dfrac{42}{8} = 5\frac{1}{4}''$

Center-to-center distance $= \dfrac{9\frac{3}{8} + 5\frac{1}{4}}{2} = \dfrac{14\frac{5}{8}}{2} = 7\frac{5}{16}''$

PROBLEMS

1. Find the center-to-center distance of a pair of gears whose pitch diameters are $7\frac{1}{2}''$ and $5\frac{1}{4}''$.

2. A pair of 5-pitch gears have 45 teeth and 62 teeth respectively. Find the distance center to center.

3. Find the distance center to center of a 32-tooth gear and a 100-tooth gear of 12-pitch.

4. A pair of gears of $2\frac{1}{2}$-pitch have 30 teeth and 45 teeth respectively. Find the distance center to center.

5. A 35-tooth, 14-pitch pinion meshes with a rack. Find the distance from center of pinion to pitch line of rack.

6. Find the distance from the center of an 8-pitch, 21-tooth pinion to the pitch line of a rack which is driven by this pinion.

7. A pair of gears of 45 teeth and 75 teeth respectively have a center-to-center distance of $12''$. Find their pitch diameters.

8. Find the pitch diameters of a pair of gears of 49 teeth and 72 teeth if the distance center to center is $4.321''$.

9. Gear A has 40 teeth and an outside diameter of $7''$; gear B has 18 teeth and an outside diameter of $3.333''$. What should be the distance center to center when properly adjusted?

10. Find the center-to-center distance of the following pair of gears: gear A has 90 teeth and an outside diameter of $7\frac{2}{3}''$; gear B has 30 teeth and an outside diameter of $2\frac{2}{3}''$.

289. Selection of Cutters

Because the shape of the tooth changes with the number of teeth on a gear, cutters are made in sets, usually eight to the set for each pitch. The following table gives the cutter number to select for different numbers of teeth. Theoretically a different shaped cutter is required for each number of teeth, but for ordinary work the approximation is close enough if we use the cutters within the limits as listed in the table.

TABLE OF CUTTERS

Use No. 8 gear cutter for cutting gears with 12 or 13 teeth.
Use No. 7 gear cutter for cutting gears from 14 to 16 teeth.
Use No. 6 gear cutter for cutting gears from 17 to 20 teeth.
Use No. 5 gear cutter for cutting gears from 21 to 25 teeth.
Use No. 4 gear cutter for cutting gears from 26 to 34 teeth.
Use No. 3 gear cutter for cutting gears from 35 to 54 teeth.
Use No. 2 gear cutter for cutting gears from 55 to 134 teeth.
Use No. 1 gear cutter for cutting gears from 135 teeth to a rack.

PROBLEMS

Select cutters for the gears in the problems on page 445.

290. Bevel Gears

Bevel gears differ from spur gears in that the teeth are cut on the surface of a cone instead of a cylinder. Figure 378 shows two frustums of cones in contact. Such frustums of

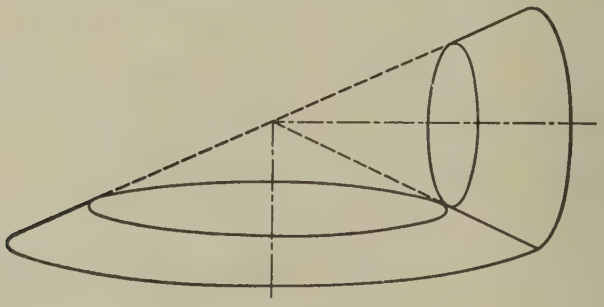


Fig. 378

cones, if pressed firmly together, will drive each other. To secure a positive drive, teeth are cut on the surface of these cones, producing a pair of bevel gears, as shown in Fig. 379.

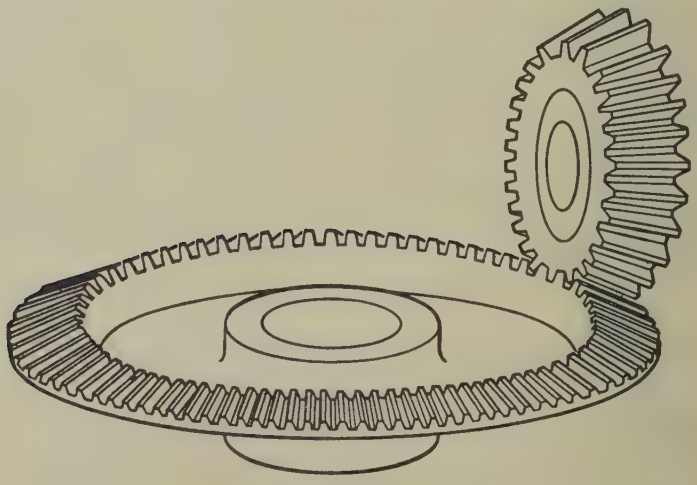


Fig. 379

291. Definitions Applying to Bevel Gears. Pitch Cones

The cones which form the basis of the bevel gears are called the *pitch cones* and their diameters are the *pitch diameters* of the bevel gears. The pitch cones are not visible at all on the finished gears, but they are the most important elements in the design.

The ***addendum*** is the projection of the tooth above the pitch cone surface.

The depth of the tooth below the pitch cone surface is called the ***dedendum.*** The depth of the dedendum is equal to the depth of the addendum plus the *clearance.*

The terms *addendum, dedendum, clearance, pitch diameter, diametral pitch, number of teeth, circular pitch, thickness of tooth,* and *width of space at pitch line* have the same meaning as in spur gears and are computed in exactly the same way. For further definitions in reference to bevel gears it will be necessary to refer to Fig. 380(*a*) and (*b*).

Figure 380(*a*) shows the pitch cones of a pair of bevel gears in mesh, the axes being at right angles to each other. Figure 380(*b*) shows a cross-section through the bevel gears built on these cones. We will call the large gear the *gear* and the small one, the *pinion.*

AB = Pitch diameter of the gear
BC = Pitch diameter of the pinion
FR = Outside diameter of the gear
HM = Outside diameter of the pinion
$OA = OB = OC$ = Pitch cone radius
Angle EOB = Pitch cone angle of the gear
Angle DOB = Pitch cone angle of the pinion
Angle AOF = Angle COH = Addendum angle (same for both gear and pinion)
Angle AOK = Angle LOC = Dedendum angle (same for both gear and pinion)
AT = Angular addendum of gear
CP = Angular addendum of pinion
Angle EOF = Turning angle of gear

Angle DOH = Turning angle of pinion
Angle EOK = Cutting angle of gear
Angle DOL = Cutting angle of pinion

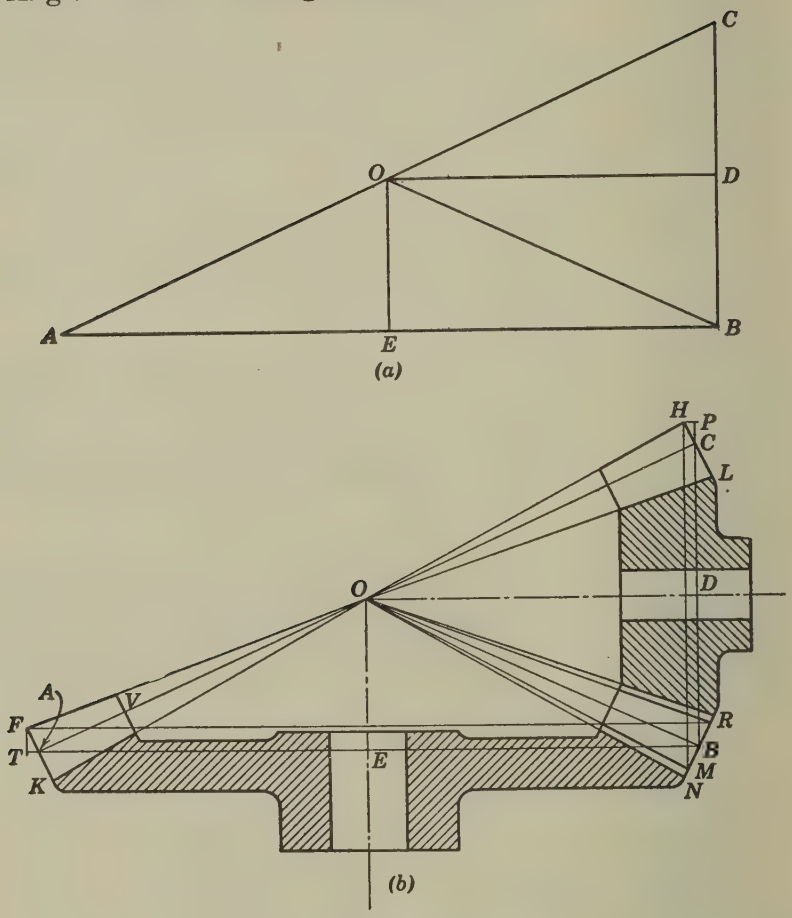


Fig. 380

The following relations are evident from the construction of the figure:

Turning angle = Pitch cone angle + Addendum angle
Cutting angle = Pitch cone angle − Dedendum angle
Outside diameter = Pitch diameter
+ Twice the angular addendum

The following example will serve to illustrate the principles of bevel gear design.

Example

Compute the dimensions of a pair of 4-pitch bevel gears with shafts at right angles, the gear to have 60 teeth and the pinion 18 teeth.

SOLUTION. PITCH DIAMETERS

$$\text{Pitch diameter of gear} = \frac{N}{P} = \frac{60}{4} = 15''$$

$$\text{Pitch diameter of pinion} = \frac{N}{P} = \frac{18}{4} = 4\tfrac{1}{2}''$$

PITCH CONE ANGLES

Construct the pitch cones by drawing $AB = 15''$ at right angles to $BC = 4\frac{1}{2}''$. Connect A and C, and from the center O of AC, draw OB and the perpendiculars OE and OD (Fig. 381).

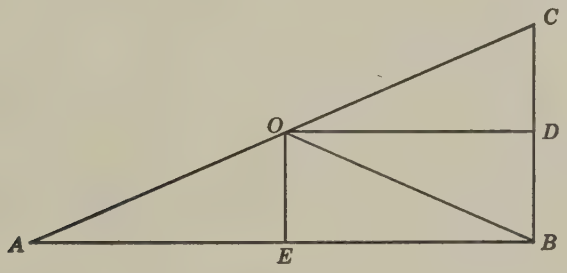


Fig. 381

Angle BOE is the pitch cone angle of the gear;
Angle BOD is the pitch cone angle of the pinion.

$$EB = \tfrac{1}{2} \text{ of } AB = \tfrac{1}{2} \text{ of } 15'' = 7\tfrac{1}{2}''$$

$$BD = \tfrac{1}{2} \text{ of } BC = \tfrac{1}{2} \text{ of } 4\tfrac{1}{2}'' = 2\tfrac{1}{4}''$$

$$\text{Tangent } BOE = \frac{BE}{OE} = \frac{7\frac{1}{2}}{2\frac{1}{4}} = 3.33333$$

Therefore angle $BOE = 73° 18' =$ pitch cone angle of gear.

$$\text{Tangent } BOD = \frac{BD}{OD} = \frac{2\frac{1}{4}}{7\frac{1}{2}} = 0.30000$$

Therefore angle $BOD = 16° 42' =$ pitch cone angle of pinion.

Proof

Sum of pitch cone angles $= 73° 18' + 16° 42' = 90°$

PROBLEMS

Compute the pitch diameters and the pitch cone angles of the following pairs of bevel gears at right angles. Draw the pitch cone diagram in each case.

1. Gear 60 teeth, Pinion 25 teeth, Pitch 4.
2. Gear 44 teeth, Pinion 24 teeth, Pitch 8.
3. Gear 72 teeth, Pinion 36 teeth, Pitch 6.
4. Gear 35 teeth, Pinion 30 teeth, Pitch 5.
5. Gear 50 teeth, Pinion 25 teeth, Pitch 10.
6. Gear 54 teeth, Pinion 30 teeth, Pitch 3.
7. Gear 100 teeth, Pinion 27 teeth, Pitch 5.
8. Gear 62 teeth, Pinion 40 teeth, Pitch 8.
9. Gear 45 teeth, Pinion 36 teeth, Pitch 4.
10. Gear 75 teeth, Pinion 24 teeth, Pitch 6.

292. Tooth Parts

Continuing the example of the preceding section, and referring to Fig. 380:

$$\text{Addendum} = \frac{1}{P} = \frac{1}{4} = 0.25'' = AF \text{ in Fig. } 380(b)$$

$$\text{Clearance} = \frac{0.157}{P} = \frac{0.157}{4} = 0.0393''$$

$$\text{Dedendum} = \text{Depth under pitch cone}$$

$$= \text{Addendum} + \text{Clearance}$$

$$= \frac{1}{P} + \frac{0.157}{P} = \frac{1.157}{P} = \frac{1.157}{4} = 0.2893''$$

Whole depth of tooth at pitch circle

$$= 0.25 + 0.25 + 0.0393 = 0.5393''$$

$= FK$ in Fig. 380(b)

$$\text{Circular pitch at pitch line} = \frac{\pi}{P} = \frac{3.1416}{4} = 0.7854''$$

$$\text{Thickness of tooth} = \text{Space between teeth} = \frac{0.7854}{2} = 0.3927''$$

PROBLEMS

Compute the addendum, clearance, dedendum, whole depth, depth under pitch cone, circular pitch, thickness of tooth, and space between teeth in each of the ten pairs of bevel gears given in Section 291.

293. Pitch Cone Radius

In Fig. 381 the pitch cone radius

$$OA = \frac{AE}{\sin \text{ angle } EOA}$$

$$OA = \frac{7.5}{\sin 73^\circ 18'} = \frac{7.5}{0.95782} = 7.83''$$

294. Addendum Angle and Turning Angle

Since the line FK in Fig. 380(b) is perpendicular to the pitch cone radius, then $\frac{FA}{AO}$ = the tangent of the addendum angle AOF; that is,

$$\text{Tangent of addendum angle} = \frac{\text{Addendum}}{\text{Pitch cone radius}} = \frac{0.25}{7.83} = 0.03193$$

$$\text{Addendum angle } AOF = 1^\circ 50'$$

The turning angle is equal to the pitch cone angle plus the addendum angle.

$$\text{Turning angle of the gear} = 73^\circ 18' + 1^\circ 50' = 75^\circ 08'$$

$$\text{Turning angle of the pinion} = 16^\circ 42' + 1^\circ 50' = 18^\circ 32'$$

295. Dedendum Angle and Cutting Angle

Tangent of dedendum angle AOK

$$= \frac{KA}{AO} = \frac{\text{Depth under pitch cone}}{\text{Pitch cone radius}}$$

$$= \frac{0.2893}{7.83} = 0.03695$$

Dedendum angle $AOK = 2° 07'$

Cutting angle = Pitch cone angle minus dedendum angle

Cutting angle of gear $= 73° 18' - 2° 07' = 71° 11'$

Cutting angle of pinion $= 16° 42' - 2° 07' = 14° 35'$

PROBLEMS

Compute the pitch cone radius, addendum angle and turning angle and the dedendum angle and cutting angle for the ten pairs of bevel gears given in Section 291.

296. Outside Diameters

The outside diameter = Pitch diameter + twice the angular addendum.

Outside Diameter of Gear

In triangle ATF, Fig. 380(b), AF is the addendum, 0.25″; angle TAF is equal to the pitch cone angle of the gear, $EOA = 73° 18'$.

Angular addendum AT

$$= AF \times \cos TAF = 0.25 \times \cos 73° 18'$$

$$= 0.25 \times 0.28736 = 0.072''$$

Outside diameter of gear

$$= 15 + 2 \times 0.072 = 15 + 0.144 = 15.144''$$

Outside Diameter of Pinion

In triangle PCH, CH is the addendum, 0.25″; angle PCH = pitch cone angle of the pinion DOC, which is 16° 42′.

Angular addendum PC

$$= CH \times \cos PCH = 0.25 \times \cos 16° 42'$$

$$= 0.25 \times 0.95782 = 0.239''$$

Outside diameter of pinion

$$= 4\tfrac{1}{2} + 2 \times 0.239 = 4.5 + 0.478 = 4.978''$$

PROBLEMS

Compute the outside diameters of the ten pairs of bevel gears given in Section 291.

297. Selecting Cutters for Bevel Gears

Bevel gears may be cut on the milling machine with spur gear cutters of the same pitch. The cutters, however, are not selected for the number of teeth on the bevel gear but for an imaginary number of teeth which is computed as follows: In Fig. 382, showing the pitch cones of the bevel gears under consideration, the line XY is drawn perpendicular to the pitch cone radius OB; that is, the line XY is a prolongation of the edge of tooth RM in 380(b). The center lines of the gear and the pinion are prolonged to meet this line.

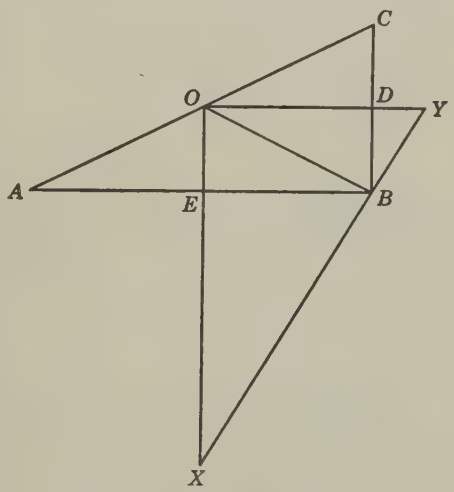


Fig. 382

BY = $BO \times$ tangent angle BOY
= Pitch cone radius $\times$ tangent of pitch cone angle of pinion
= $7.83 \times \tan 16° 42'$
= $7.83 \times 0.30001 = 2.349''$
BX = $BO \times$ tangent angle BOX
= Pitch cone radius $\times$ tangent of pitch cone angle of gear
= $7.83 \times \tan 73° 18'$
= $7.83 \times 3.33317 = 26.099''$

BY is the pitch radius of the imaginary pinion for which to select the cutter.

Diameter = $2 \times 2.349 = 4.698''$
Number of teeth = Pitch diameter $\times$ Pitch = 4.698×4
= 18.792 teeth

The pinion is to be cut with a 19-tooth cutter, that is, a No. 6 cutter.

BX is the pitch radius of the imaginary gear for which to select the cutter.

Diameter = $2 \times 26.1 = 52.2''$
Number of teeth = Pitch diameter $\times$ Pitch = 52.2×4
= 208.8 teeth

The gear is to be cut with a 209-tooth cutter, that is, a No. 1 cutter.

PROBLEMS

Select the cutters for the ten pairs of bevel gears given in Section 291.

298. Dimensions of Teeth at Small End

The *width* or *face* of the teeth, AV in 380(*b*), is usually made about one-quarter of the pitch cone radius. The dimensions of the teeth at the small end are proportional to their dimensions at the large end. The ratio between the small end dimensions and the large end dimensions

$$= \frac{\text{Pitch cone radius at small end}}{\text{Pitch cone radius at large end}}$$

The pitch cone radius being 7.83″, let us assume a width of face of 2″, making the pitch cone radius at the small end of the teeth $OV = 7.83 - 2.00 = 5.83''$. Then the ratio between the small pitch cone radius and the large pitch cone radius $= \frac{5.83}{7.83} = 0.745$. We can now find the tooth proportions at the small end by multiplying the large end dimensions by the factor 0.745.

Addendum at small end $= 0.745 \times 0.25 = 0.1862''$
Clearance at small end $= 0.745 \times 0.0393 = 0.0293''$
Dedendum at small end $= 0.745 \times 0.2893 = 0.2155''$
Whole depth at small end $= 0.745 \times 0.5393 = 0.4018''$
Circular pitch at small end $= 0.745 \times 0.7854 = 0.5851''$
Thickness of tooth at small end = space at small end
$= 0.745 \times 0.3927 = 0.2926''$

PROBLEMS

Compute the addendum, clearance, dedendum, whole depth, circular pitch, thickness of tooth, and space at the small ends of the teeth of the ten pairs of bevel gears given in Section 291. In each case assume a tooth face of approximately $\frac{1}{4}$ of the pitch cone radius but not more than 0.3 of the pitch cone radius.

299. Miter Gears

When the gear and the pinion are both the same size and their axes are at right angles, they are called ***miter gears.*** The computation in this case is greatly simplified, all elements being the same for both gears.

The pitch cone angle is 45°.

$$\text{Pitch cone radius} = \frac{\text{Pitch radius}}{\sin 45°} = \frac{\text{Pitch diameter}}{2 \times \sin 45°}$$

$$= \frac{\text{Pitch diameter}}{2 \times 0.70711} = \frac{\text{Pitch diameter}}{1.41422}$$

$$= 0.707 \times \text{Pitch diameter}$$

$$\text{Face angle} = 45° + \text{Addendum angle}$$

$$\text{Cutting angle} = 45° - \text{Dedendum angle}$$

$$\text{Angular addendum} = \text{Addendum} \times \cos 45°$$
$$= 0.70711 \times \text{Addendum}$$

$$\text{Number of teeth for which to select cutter} = \frac{\text{Number of teeth}}{\cos 45°}$$
$$= \frac{\text{Number of teeth}}{0.70711}$$
$$= 1.414 \times \text{Number of teeth}$$

TABLE OF RULES FOR BEVEL GEARS

To Find	Rule
Pitch diameter	Pitch diameter $= \frac{N}{P}$
Pitch cone angles	Tan pitch cone angle, gear $= \frac{\text{Pitch diameter, gear}}{\text{Pitch diameter, pinion}}$ Tan pitch cone angle, pinion $= \frac{\text{Pitch diameter, pinion}}{\text{Pitch diameter, gear}}$
Pitch cone radius	Pitch cone radius $= \frac{\frac{1}{2}\text{ Pitch diameter of gear}}{\text{Sine pitch cone angle, gear}}$
Tooth parts, large end. Addendum	Addendum $= \frac{1}{P}$
Clearance	Clearance $= \frac{0.157}{P}$
Dedendum or depth below pitch cone	Depth below pitch cone $= \frac{1.157}{P}$
Whole depth	Whole depth $= \frac{2.157}{P}$
Circular pitch at pitch line	Circular pitch $= \frac{\pi}{P}$
Thickness of tooth or space at pitch line	Thickness $=$ Space $= \frac{P_c}{2}$
Tooth dimensions at small end	Multiply large end dimensions by following ratio: $\frac{\text{Pitch cone radius at small end}}{\text{Pitch cone radius at large end}}$
Addendum angle	Tan addendum angle $= \frac{\text{Addendum}}{\text{Pitch cone radius}}$
Dedendum angle	Tan dedendum angle $= \frac{\text{Dedendum}}{\text{Pitch cone radius}}$
Turning angle	Face angle $=$ Pitch cone angle $+$ Addendum angle
Cutting angle	Cutting angle $=$ Pitch cone angle $-$ Dedendum angle
Angular addendum	Gear, angular addendum $=$ Addendum $\times$ Cosine of pitch cone angle of gear Pinion, angular addendum $=$ Addendum $\times$ Cosine of pitch cone angle of pinion
Outside diameter	Outside diameter $=$ Pitch diameter $+ 2$ times the angular addendum
Number of teeth for which to select cutter	Number of teeth for which to select the cutter $= \frac{\text{Actual number of teeth}}{\text{Cosine of pitch cone angle}}$

PROBLEMS

The elements required for the design, cutting, and measuring of bevel gears are the following:

Number of teeth.
Pitch.
Pitch diameter.
Pitch cone radius.
Pitch cone angle.
Addendum angle and face angle.
Dedendum angle and cutting angle.
Tooth parts: addendum, clearance, dedendum, whole depth, circular pitch, thickness of tooth, width of space at the small and large ends.
Angular addendum, gear and pinion.
Outside diameter.
Number of teeth for which to select cutter.

Compute all the above elements for the following pairs of bevel gears:

	Gear	Pinion	Pitch
1.	62 teeth	21 teeth	10
2.	90 teeth	20 teeth	$2\frac{1}{2}$
3.	75 teeth	18 teeth	8
4.	120 teeth	15 teeth	6
5.	72 teeth	63 teeth	9
6.	35 teeth	35 teeth	5
7.	62 teeth	45 teeth	5
8.	71 teeth	50 teeth	2
9.	36 teeth	36 teeth	3
10.	75 teeth	35 teeth	12

300. Spiral Gears

Spiral gears differ from spur gears in that the teeth are not parallel to the axis of the gear. The teeth of spiral gears are cut at an angle so that they wind around the pitch cylinder like a screw thread. The teeth may be cut with spur gear cutters and they have the same cross-section as the teeth of spur gears. Except for the shape of the threads a spiral gear is similar to a multiple screw thread of long lead. A complete turn of the helix is seldom seen on a spiral gear, however, because the gear is too short.

The cross-section of the teeth as seen at the end of a spiral gear is not the true cross-section. It is distorted because of

the fact that the teeth are at an angle to the axis. To see the actual shape of the teeth we must take the normal cross-section, that is, a cross-section at right angles to the direction of the teeth. See Fig. 383.

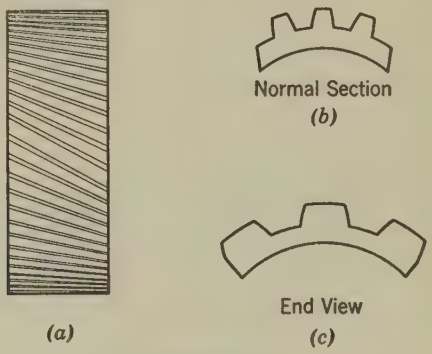


Fig. 383

NOTE. The teeth of a spiral gear do not really form a spiral but a helix, so that "helical gears" is the proper term to use. However, "spiral gears" is used in this book because that is the name by which such gears are commonly called in shop practice.

The word "pitch," as used in spiral gear calculations, means the normal diametral pitch, that is, the pitch in reference to a plane at right angles to the direction of the teeth. It is the pitch of the cutter used in cutting the gear.

301. Spiral Angle or Helix Angle

Figure 384 shows how the spiral angle is computed. Draw a right triangle in which the altitude is the pitch circumference and the base, the lead. Then the angle ACB is

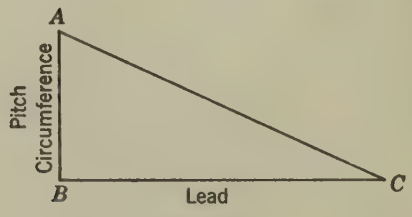


Fig. 384

the spiral angle. $\frac{AB}{BC}$ = the tangent of this angle; that is,

$$\text{Tangent of spiral angle} = \frac{\text{Pitch circumference}}{\text{Lead}}$$

302. Pitch Diameter of Spiral Gears

In spur gears the pitch diameter is found by dividing the number of teeth by the pitch; $D = \frac{N}{P}$. In spiral gears the pitch diameter is found by dividing the *product of the number of teeth and the secant of the spiral angle by the pitch.* If we call the spiral angle A, then the pitch diameter $D = \frac{N \times \text{Secant } A}{P}$.

303. Spiral Gears with Parallel Shafts

Spiral gears may have their shafts parallel or at any angle. The limits of this book will not permit a full discussion of the various possibilities, but we will make a study of the two simplest as well as most important conditions, namely, with shafts parallel and with shafts at right angles. The following example will serve to illustrate the case of spiral gears with parallel shafts.

Example

Required a pair of spiral gears of 3-pitch, with parallel shafts and a speed ratio of 3 to 1. The distance center to center of gears is $10\frac{1}{2}''$.

SOLUTION.

Let N = number of teeth on small gear.

Then $3N$ = number of teeth on large gear (speed ratio 3:1)

$$\text{Pitch diam. small gear} = \frac{N \sec A}{\text{Pitch}} = \frac{N \sec A}{3}$$

$$\text{Pitch diam. large gear} = \frac{3N \sec A}{\text{Pitch}} = \frac{3N \sec A}{3}$$

But the sum of the pitch diameters is equal to 2 × the distance center to center of gears = $2 \times 10\frac{1}{2}'' = 21''$.

$$\therefore \frac{N \sec A}{3} + \frac{3N \sec A}{3} = \frac{4N \sec A}{3} = 21$$

$$N = \frac{3 \times 21}{4 \sec A} = \frac{63}{4 \sec A} = \frac{15.75}{\sec A}$$

The spiral angle on gears with parallel shafts is usually made about 20°. It is desirable to avoid any angle larger than this in order to reduce the end thrust. Assuming a spiral angle of 20° in order to get a trial value for N, we get

$$N = \frac{15.75}{\sec 20°} = \frac{15.75}{1.0642} = 14.8$$

the number of teeth on the small gear. Since the number of teeth must be a whole number, let us assume the small gear to have 15 teeth. To maintain the required speed ratio, the large gear must have 3 × 15, or 45 teeth.

We must remember that the value of 20° for the spiral angle was only a trial value. *To get the spiral angle*, let us go back to the equation, $N = \frac{15.75}{\sec A}$. Solving for sec A, we get

$$\sec A = \frac{15.75}{N} = \frac{15.75}{15} = 1.0500$$

From the table of secants we find Angle $A = 17° 45'$ = the spiral angle.

Pitch Diameters

$$\text{Small gear, } D = \frac{15 \times \sec 17° 45'}{3} = \frac{15 \times 1.0500}{3} = 5.25''$$

$$\text{Large gear, } D = \frac{45 \times \sec 17° 45'}{3} = \frac{45 \times 1.0500}{3} = 15.75''$$

$$\text{Center-to-center distance} = \frac{\text{Sum of pitch diameters}}{2}$$

$$= \frac{5.25 + 15.75}{2} = 10.5''$$

Outside Diameters

Small gear,

$$OD = D + \frac{2}{P} = 5.25 + \frac{2}{3} = 5.25 + 0.667 = 5.917''$$

Large gear,

$$OD = D + \frac{2}{P} = 15.75 + \frac{2}{3} = 15.75 + 0.667 = 16.417''$$

Leads

From Fig. 384, $\tan A = \dfrac{\text{Pitch circumference}}{\text{Lead}}$

$$\text{Lead} = \frac{\text{Pitch circumference}}{\tan A}$$

Small gear,

$$\text{Lead} = \frac{\text{Pitch circumference}}{\tan 17^\circ 45'} = \frac{3.1416 \times 5.25}{0.32010} = 51.6''$$

Large gear,

$$\text{Lead} = \frac{\text{Pitch circumference}}{\tan 17^\circ 45'} = \frac{3.1416 \times 15.75}{0.32010} = 154.8''$$

Selection of Cutters

Spur gear cutters of 3-pitch may be used for cutting these gears on the milling machine. The cutters, however, are not selected for the actual number of teeth but for an imaginary number of teeth computed as follows:

$$\text{Number of teeth for which to select cutter} = \frac{N}{(\cos A)^3}$$

that is, the actual number of teeth on the gear divided by the cube of the cosine of the spiral angle.

$$\text{Small gear, Number of teeth} = \frac{N}{(\cos A)^3} = \frac{15}{(\cos 17^\circ 45')^3}$$

$$= \frac{15}{(0.95240)^3} = \frac{15}{0.864} = 17.4 \text{ teeth}$$

Use No. 6 cutter.

$$\text{Large gear, Number of teeth} = \frac{N}{(\cos A)^3} = \frac{45}{(\cos 17° 45')^3}$$

$$= \frac{45}{(0.95240)^3} = \frac{45}{0.864} = 52.1 \text{ teeth}$$

Use No. 3 cutter.

Summary of dimensions for a pair of spiral gears of 3-pitch, with parallel shafts, a 3 to 1 speed ratio, and a center-to-center distance of 10.5″.

Required	Gear	Pinion
Number of teeth	45	15
Spiral angle	17° 45′ (right)	17° 45′ (left)
Pitch diameter	15.75″	5.25″
Outside diameter	16.417″	5.917″
Lead	154.8″	51.6″
Cutter number to use	No. 3	No. 6

PROBLEMS

Compute the elements of the preceding table for the following pairs of spiral gears with parallel shafts.

	Pitch	Speed Ratio	Center-to-Center Distance
1.	6	2 to 1	$7\frac{1}{8}$″
2.	10	1 to 1	5″
3.	5	$2\frac{1}{2}$ to 1	$8\frac{1}{4}$″
4.	4	1.6 to 1	$11\frac{1}{2}$″
5.	8	$1\frac{1}{2}$ to 1	$4\frac{1}{2}$″
6.	10	4 to 1	$5\frac{1}{2}$″
7.	8	12 to 5	6.0625″
8.	6	3 to 1	$7\frac{1}{2}$″
9.	2	$3\frac{1}{2}$ to 2	$23\frac{1}{4}$″
10.	$1\frac{1}{2}$	2 to 1	30″

304. Spiral Gears with Shafts at Right Angles

When the shafts are at right angles, the gears will have spiral angles which are complements of each other. If we call the spiral angle of one of the gears A, then the spiral angle of the other gear will be $90° - A$. The secant of 90° minus an angle is equal to the cosecant of the angle. In the computation we will therefore use secant A for one gear and cosecant A for the other gear.

The most effective angle for the teeth of spiral gears with shafts at right angles is 45°, and we must try to get the spiral angle as near 45° as possible.

Example

Required, a pair of 4-pitch spiral gears with axes at right angles, a speed ratio of 3 to 2, and a center-to-center distance of 9″.

SOLUTION. Let N = Number of teeth on small gear.

Then $\frac{3}{2} \times N = 1.5N$ = Number of teeth on large gear.

The sum of the pitch diameters $= 2 \times 9'' = 18''$

$$\text{Pitch diameter of small gear} = \frac{N \times \sec A}{4}$$

$$\text{Pitch diameter of large gear} = \frac{1.5N \times \operatorname{cosec} A}{4}$$

$$\text{Then} \quad \frac{N \sec A}{4} + \frac{1.5N \operatorname{cosec} A}{4} = 18''$$

$$N \sec A + 1.5N \operatorname{cosec} A = 4 \times 18 = 72 \qquad (1)$$

For the spiral angle A we will assume a trial value of 45°.

$$N \sec 45° + 1.5N \operatorname{cosec} 45° = 72$$

$$N \times 1.4142 + 1.5N \times 1.4142 = 72$$

$$2.5N \times 1.4142 = 72$$

$$N \times 3.5355 = 72$$

$$N = \frac{72}{3.5355} = 20.365$$

The nearest number of teeth on the small gear that will permit a speed ratio of 3 to 2 is 20. We will therefore use 20 teeth on the small gear. The large gear must have $\frac{3}{2} \times 20$ or 30 teeth. Putting these values into equation (1), we get

$$20 \times \sec A + 30 \times \operatorname{cosec} A = 72$$

The actual value of angle A can be found only by trial, that is, by assuming various values of A near 45 until we find one that

will satisfy the equation. Using the trial value, $A = 45°$, we get

$$20 \times 1.4142 + 30 \times 1.4142 = 70.71$$

which is less than 72 and shows that 45° is not the correct angle. Let us try 43°.

$$20 \times 1.3673 + 30 \times 1.4663 = 71.335$$

This is nearer to 72 and shows that we are in the right direction. Let us try 41°.

$$20 \times 1.3250 + 30 \times 1.5242 = 72.226$$

This is more than 72 and shows that 41° is smaller than the correct value. We will try 41° 30′.

$$20 \times 1.3352 + 30 \times 1.5092 = 71.980$$

This is very close to the actual value. We will try 41° 27′.

$$20 \times 1.3342 + 30 \times 1.5106 = 72.002$$

which differs from the required value by only 0.002. Further trial would give the exact value but this is close enough for ordinary work. We therefore use 41° 27′ for the spiral angle of the small gear and 90° − 41° 27′ or 48° 33′ for the large gear.

The remainder of the computation is similar to the computation for gears with parallel shafts.

Pitch Diameters

$$\text{Small gear, } D = \frac{N \times \sec 41° 27'}{4} = \frac{20 \times 1.3342}{4} = 6.6710''$$

$$\text{Large gear, } D = \frac{N \times \sec 48° 33'}{4} = \frac{30 \times 1.5106}{4} = 11.3295''$$

$$\text{Sum of pitch diameters} = 6.6710 + 11.3295$$

$$= 18.0005''$$

$$\text{Exact distance center to center} = \frac{18.0005}{2} = 9.00025''$$

Outside Diameters

Small gear, $OD = D + \frac{2}{P} = 6.6710 + \frac{2}{4} = 7.1710''$

Large gear, $OD = D + \frac{2}{P} = 11.3295 + \frac{2}{4} = 11.8295''$

Leads

$$\text{Small gear, Lead} = \frac{\text{Pitch circumference}}{\tan \text{spiral angle}} = \frac{3.1416 \times 6.6710}{\tan 41^\circ 27'}$$

$$= \frac{3.1416 \times 6.6710}{0.88317} = 23.73''$$

$$\text{Large gear, Lead} = \frac{\text{Pitch circumference}}{\tan \text{spiral angle}} = \frac{3.1416 \times 11.3295}{\tan 48^\circ 33'}$$

$$= \frac{3.1416 \times 11.3295}{1.13228} = 31.43''$$

Selection of Cutters

Small gear,

$$\text{Number of teeth for cutter} = \frac{N}{(\cos A)^3} = \frac{20}{(\cos 41^\circ 27')^3}$$

$$= \frac{20}{(0.74953)^3} = \frac{20}{0.4211}$$

$$= 47.6 \text{ teeth}$$

Use No. 3 cutter.

Large gear,

$$\text{Number of teeth for cutter} = \frac{N}{(\cos A)^3} = \frac{30}{(\cos 48^\circ 33')^3}$$

$$= \frac{30}{(0.66197)^3} = \frac{30}{0.2901}$$

$$= 103.5 \text{ teeth}$$

Use No. 2 cutter.

Summary of dimensions for a pair of 4-pitch spiral gears with axes at right angles, a speed ratio of 3 to 2, and a center-to-center distance of 9″.

Required	Gear	Pinion
Number of teeth	30	20
Spiral angle	48° 33′ (right)	41° 27′ (right)
Pitch diameter	11.3295″	6.6710″
Outside diameter	11.8295″	7.1710″
Lead	31.43″	23.73″
Cutter number to use	No. 2	No. 3

PROBLEMS

Ten problems were given at the end of the section on spiral gears with parallel shafts. Compute the dimensions of the same ten pairs of gears *when the shafts are at right angles.*

305. Worm Gearing

A modification of right angle spiral gearing is employed in cases where the speed ratio is very large. On the small gear the angle becomes so large that the spiral is able to make several complete turns around the pitch cylinder, so

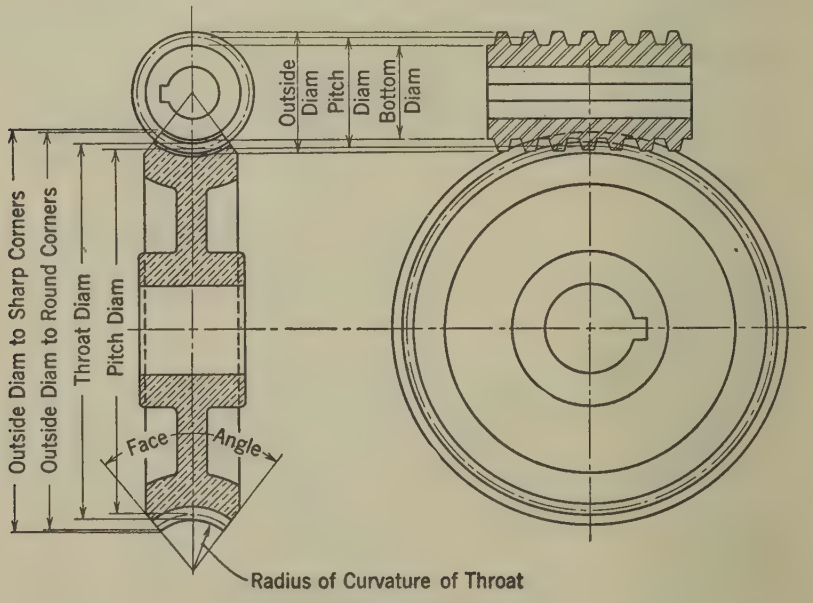


Fig. 385

that the gear really becomes a screw. This screw is called a worm, and the gear with which it meshes is called a worm-wheel. Figure 385 shows two views of a worm and gear in mesh with all the important dimensions indicated.

306. The Worm

Since the worm is really a screw, we speak of worm threads instead of teeth. The terms *pitch* and *lead* have the same meaning as in screw threads. Worms, like screws, may be single-threaded, double-threaded, triple-threaded, etc. A double-threaded worm (sometimes called a worm with two threads) with a pitch of $\frac{3}{8}''$ will have a lead of $2 \times \frac{3}{8}''$ or $\frac{3}{4}''$.

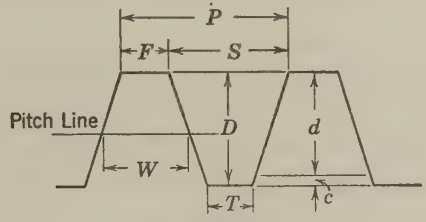


Fig. 386

In cross-section the threads appear like the Acme 29-degree screw threads, but they are deeper. See Fig. 386. The following formulas give the dimensions of the worm and worm threads:

Formulas for Worm

Addendum = 0.3183 × Pitch

Bottom flat (width of point of thread cutting tool)
= 0.31 × Pitch

Top flat = 0.335 × Pitch

Whole depth = 0.6866 × Pitch

Pitch diameter = Outside diameter − 2 × Addendum

Root diameter = Outside diameter − 2 × Whole depth

$$\text{Tangent of spiral angle} = \frac{\text{Pitch circumference}}{\text{Lead}}$$

Minimum length of worm

$$= \sqrt{8 \times \text{Addendum} \times \text{Pitch diameter of wormwheel}}$$

307. The Wormwheel

The teeth of the wormwheel are cut to fit the worm and their *angle is the complement of the spiral angle of the worm.*

The *circular pitch* of the wormwheel is equal to the *linear pitch* of the worm. The pitch circumference of the wormwheel therefore equals the linear pitch of the worm times the number of teeth. To find the pitch diameter we divide this product by 3.1416.

The face of the wormwheel is not a cylinder like that of a spur gear or a spiral gear but is hollowed out to fit the worm, thus securing larger bearing surface and greater efficiency.

The outside diameter at the center is called the ***throat diameter.***

The radius of the face curve is called the ***throat radius.***

In cutting the wormwheel, two operations are necessary, *gashing* and *hobbing*. In the gashing operation the teeth are roughly cut out by a gear cutter of proper pitch and of the same diameter as the worm and set at the gashing angle. The gear is then hobbed, that is, cut by a revolving cutter which is practically the same as the worm with the threads broken by flutes to form teeth. This hob, in cutting, either drives the wormwheel or is geared to revolve with it. The following formulas give the dimensions of the wormwheel:

Formulas for Wormwheel

$$\text{Pitch diameter} = \frac{N \times \text{Circular pitch}}{3.1416}$$

$$\text{Throat diameter} = D + 2 \times \text{Addendum}$$

$$\text{Throat radius} = \tfrac{1}{2}\ \text{Outside diameter of worm} - 2 \times \text{Addendum}$$

$$\text{Gashing angle} = \text{Complement of spiral angle of worm}$$

$$\text{Diameter of wormwheel to sharp corners} = 2 \times \text{Throat radius}\ (1 - \cos \tfrac{1}{2}\ \text{face angle}) + \text{Throat diameter}$$

Usually it is not necessary to compute this dimension, as the sharp corners are frequently trimmed off. It is usually sufficient to scale it from a carefully made drawing.

Example

Compute the dimensions of a worm and wormwheel to meet the following conditions: double-threaded worm with outside diameter of $3\frac{1}{4}''$ and lead of $\frac{7}{8}''$, speed ratio of 30 to 1.

SOLUTION. (1) *Worm dimensions.*

$$\text{Pitch} = \frac{\text{Lead}}{2} = \frac{0.875}{2} = 0.4375''$$

$$\text{Addendum} = 0.3183 \times \text{Pitch}$$
$$= 0.3183 \times 0.4375 = 0.1393''$$

Whole depth of tooth
$$= 0.6866 \times \text{Pitch} = 0.6866 \times 0.4375 = 0.3004''$$

$$\text{Bottom flat} = \text{Width of tool point for cutting thread}$$
$$= 0.31 \times \text{Pitch}$$
$$= 0.31 \times 0.4375 = 0.1356''$$

$$\text{Top flat} = 0.335 \times \text{Pitch}$$
$$= 0.335 \times 0.4375 = 0.1466''$$

$$\text{Pitch diameter} = \text{Outside diameter} - 2 \times \text{Addendum}$$
$$= 3.25 - 2 \times 0.1393$$
$$= 3.25 - 0.2786 = 2.9714''$$

$$\text{Root diameter} = \text{Outside diameter} - 2 \times \text{Whole depth}$$
$$= 3.25 - 2 \times 0.3004$$
$$= 3.25 - 0.6008 = 2.6492''$$

Spiral angle of worm:

$$\text{Tangent of spiral angle} = \frac{\text{Pitch circumference}}{\text{Lead}}$$
$$= \frac{3.1416 \times 2.9714}{0.875} = 10.6685$$

$$\text{Spiral angle} = 84^\circ 39'$$

Minimum length of worm: To get the minimum length of worm, we must first compute the pitch diameter of the wormwheel.

(2) *Wormwheel dimensions.*

Number of teeth

$$= 30 \times 2 = 60 \text{ (Speed ratio 30:1 and double thread)}$$

$$\text{Pitch diameter} = \frac{N \times \text{Pitch of worm}}{3.1416}$$

$$= \frac{2 \times 30 \times 0.4375}{3.1416} = 8.3556''$$

$$\text{Throat diameter} = \text{Pitch diameter} + 2 \times \text{Addendum}$$

$$= 8.3556 + 2 \times 0.1393$$

$$= 8.3556 + 0.2786 = 8.6342''$$

Throat radius

$$= \tfrac{1}{2} \text{ Outside diameter of worm} - 2 \times \text{Addendum}$$

$$= \tfrac{1}{2} \times 3.25 - 2 \times 0.1393 = 1.6250 - 0.2786$$

$$= 1.3464''$$

Gashing angle of wormwheel

$$= \text{Complement of spiral angle of worm}$$

$$= 90° - 84° 39' = 5° 21'$$

Diameter of wormwheel to sharp corners

$$= 2 \times \text{Throat radius } (1 - \cos \tfrac{1}{2} \text{ face angle}) + \text{Throat diameter (Assume average face angle of } 75°)$$

$$= 2 \times 1.3464(1 - \cos 37° 30') + 8.6342$$

$$= 2.6928(1 - 0.79335) + 8.6342$$

$$= 2.6928 \times 0.20665 + 8.6342$$

$$= 0.5563 + 8.6342 = 9.1905''$$

Minimum length of worm

$$= \sqrt{8 \times \text{Addendum} \times \text{Pitch diameter of gear}}$$

$$= \sqrt{8 \times .1393 \times 8.3556} = \sqrt{9.3115} = 3.05''$$

Distance center to center

$$= \frac{\text{Pitch diameter of worm} + \text{Pitch diameter of gear}}{2}$$

$$= \frac{2.9714 + 8.3556}{2} = \frac{11.3270}{2} = 5.6635''$$

(3) *Summary of dimensions.*

Worm.	Pitch	$= 0.4375''$
	Addendum	$= 0.1393''$
	Whole depth of tooth	$= 0.3004''$
	Bottom flat	$= 0.1356''$
	Top flat	$= 0.1466''$
	Pitch diameter	$= 2.9714''$
	Root diameter	$= 2.6492''$
	Spiral angle	$= 84° 39'$
	Minimum length of worm	$= 3.05''$
Wormwheel.	Number of teeth	$= 60$
	Pitch diameter	$= 8.3556''$
	Throat diameter	$= 8.6342''$
	Throat radius	$= 1.3464''$
	Gashing angle	$= 5° 21'$
	Diameter to sharp corners	$= 9.1505''$

Center-to-center distance $= 5.6635''$.

PROBLEMS

Compute the dimensions of worms and wormwheels to meet the following conditions.

	Outside Diameter of Worm	Thread of Worm	Lead of Worm	Speed Ratio
1.	$2\frac{1}{2}''$	Single	$\frac{3}{8}''$	35 to 1
2.	$1\frac{3}{4}''$	Single	$\frac{5}{16}''$	40 to 1
3.	$3''$	Single	$\frac{1}{2}''$	50 to 1
4.	$6''$	Double	$1\frac{1}{2}''$	30 to 1
5.	$4''$	Triple	$1\frac{1}{2}''$	28 to 1
6.	$1\frac{1}{2}''$	Single	$\frac{1}{4}''$	60 to 1
7.	$7\frac{1}{2}''$	Double	$1''$	45 to 1
8.	$1\frac{1}{2}''$	Double	$\frac{3}{16}''$	50 to 1
9.	$2\frac{3}{4}''$	Triple	$\frac{3}{8}''$	20 to 1
10.	$10''$	Quadruple	$2''$	25 to 1

23 Milling Machine Work

308. Indexing

The adjustment of the milling machine for turning a piece of work through part of a revolution is called ***indexing.*** Any arrangement or device for accomplishing this object is an indexing arrangement. For example, if we are making a

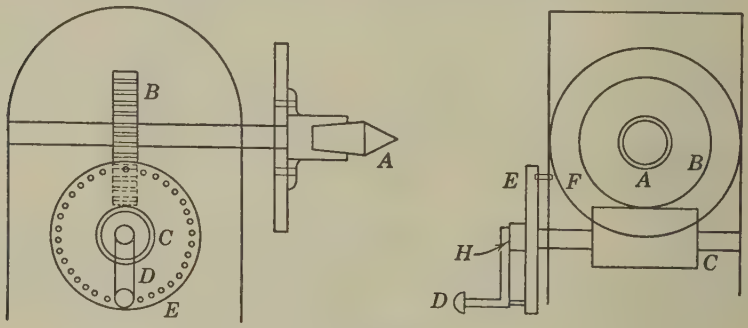


Fig. 387

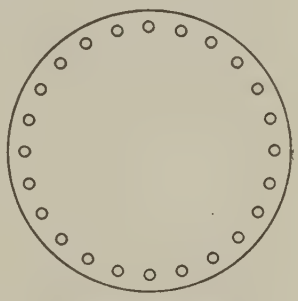
fluted reamer with ten flutes, it is necessary to turn the reamer one-tenth of a revolution after cutting each groove. In other words, we index for ten divisions. If we are cutting a gear with seventy-five teeth, we must turn the gear one seventy-fifth of a revolution after cutting each tooth; that is, we index for seventy-five divisions.

On the milling machine indexing is done by means of an attachment called the ***universal dividing head*** or the ***index-***

ing head, which is part of the standard equipment of milling machines. The dividing head is sometimes called the ***spiral head,*** since it is also used for the purpose of holding and turning the work while spiral grooves are being milled, as in twist drills and spiral gears. The essential features of its operation are seen by a study of the diagrams in Fig. 387.

309. Direct Indexing

Direct indexing, as its name implies, means indexing directly from the plate attached to the spindle. The worm and worm-gears are thrown out of mesh, and the indexing is done by means of the ***index plate*** shown in Fig. 388. This consists of a steel disk about 8″ in diameter and drilled with 24 holes equally spaced on a circle concentric with the spindle to which the plate is attached. Sometimes 30- and 36-hole circles are used. The index plate is held by a pin through one of the holes. To turn the spindle (and the piece of work which is attached to it), we release the pin and turn the plate until we have turned the work the required amount, then insert the pin again. For instance, to cut 8 flutes on a reamer, we must turn the work $\frac{1}{8}$ of a revolution after each cut. Since there are 24 holes in the index circle, we must turn the index plate $\frac{1}{8}$ of 24 or 3 holes. That is, 3 holes on a 24-hole circle will give a rotation of $\frac{1}{8}$ of a turn. To index for 6 divisions, we must make $\frac{1}{6}$ of a turn. This is done by taking $\frac{1}{6}$ of 24 or 4 holes. Four holes on the 24-hole circle give $\frac{4}{24}$ or $\frac{1}{6}$ of a turn.



24 Hole Index Plate

Fig. 388

It is evident that, by the method of direct indexing, we are limited to those numbers which are contained as factors in 24. We can index for the following numbers: 2, 3, 4, 6, 8, 12, 24. We cannot index for 10 divisions or 20 divisions or any other number which is not a factor of 24.

PROBLEMS

1. What direct indexing is required for milling a square end on a round shaft?

2. State the direct indexing for cutting three flutes, evenly spaced on a drill.

3. Give the direct indexing for cutting a milling cutter with 6 teeth; with 8 teeth.

4. State the direct indexing for cutting a pinion with 12 teeth; with 24 teeth.

310. Simple Indexing

In simple indexing, the spindle is rotated by means of the worm, C, and the worm-gear, B, shown in Fig. 387. In most modern machines the worm is single-threaded and the worm-gear has 40 teeth. One revolution of the worm, which is operated by crank D, passes one tooth of the worm-gear and turns the spindle through $\frac{1}{40}$ of a revolution. An index plate, E, is mounted on the worm-gear shaft and prevented from turning by means of the stop pin at F. To make exactly one revolution of the worm, the spring pin H in the crank handle is withdrawn from the hole it occupies in the index plate, the crank is given a complete turn, and the pin is allowed to spring back into the same hole. We have thus turned the worm through one revolution and the worm-gear and spindle through $\frac{1}{40}$ of a revolution. In other words, we have indexed for 40 divisions. Two complete turns of the worm will rotate the spindle $\frac{2}{40}$ or $\frac{1}{20}$ of a turn. By means of whole turns of the crank we can index for all numbers from 1 to 40 which are factors of 40.

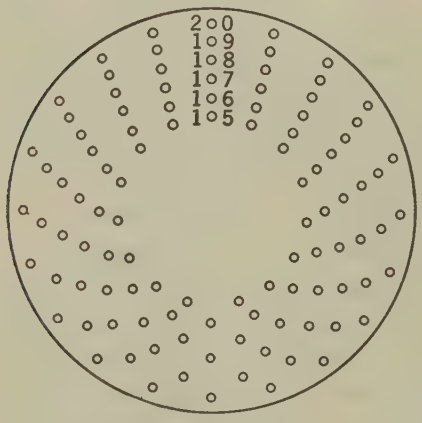


Fig. 389

The number of divisions for which we can index by this method is greatly increased by the use of the index plate *E*. Figure 389 shows one of these index plates. It has six concentric circles of equally spaced holes. These circles contain respectively 15, 16, 17, 18, 19, and 20 holes. The crank spring pin can be adjusted to any of the circles.

Suppose the crank pin is adjusted to the 20-hole circle, and we withdraw the pin and move it to the next hole. We have then turned the worm $\frac{1}{20}$ of a revolution and the spindle $\frac{1}{20}$ of $\frac{1}{40}$ or $\frac{1}{800}$ of a turn. That is, we have indexed for 800 divisions. If we move the pin to the fifth hole, we rotate the worm $\frac{5}{20}$ or $\frac{1}{4}$ of a turn and the spindle $\frac{1}{4}$ of $\frac{1}{40}$ or $\frac{1}{160}$ of a turn.

The following examples will serve to illustrate the principles of simple indexing.

Example 1

Index for 120 divisions.

SOLUTION. Since the spindle must make $\frac{1}{120}$ of a revolution, the worm must make $40 \times \frac{1}{120}$ or $\frac{1}{3}$ of a revolution. Multiplying the fraction $\frac{1}{3}$ by $\frac{5}{5}$, we get the equivalent fraction $\frac{5}{15}$. We therefore move the crank pin 5 holes in the 15-hole circle, giving a worm motion of $\frac{5}{15}$ or $\frac{1}{3}$ of a turn, and a spindle motion of $\frac{1}{3}$ of $\frac{1}{40}$, or $\frac{1}{120}$ of a turn, as required. The same result can be obtained by moving the pin 6 holes in the 18-hole circle, 7 holes in the 21-hole circle, etc.

Example 2

Index for 17 divisions.

SOLUTION. To turn the spindle $\frac{1}{17}$ of a turn, we must turn the worm $\frac{1}{17}$ of 40 or $\frac{40}{17}$ of a turn. This is equal to 2 whole turns and $\frac{6}{17}$ of a turn. We now adjust the crank pin to the 17-hole circle, make 2 complete turns and 6 holes more. The indexing is stated as 2 turns and 6 holes in the 17-circle.

Example 3

Index for 35 divisions.

SOLUTION. The spindle must be rotated $\frac{1}{35}$ of a turn. The worm must therefore be rotated $\frac{1}{35}$ of 40 or $\frac{40}{35}$, that is, 1 and $\frac{5}{35}$ turns or $1\frac{1}{7}$ turns. Using the 21-hole circle on the second plate, we make one complete turn and 3 holes beyond. The indexing is stated as 1 turn and 3 holes in the 21-circle.

Example 4

Index for 132 divisions.

SOLUTION. The crank must be turned $\frac{1}{132}$ of 40 turns, that is, $\frac{40}{132}$, which is equal to $\frac{10}{33}$ of a turn. Using the 33-hole circle, we move the crank 10 holes. The indexing therefore is 10 holes in the 33-circle.

Summary

We can summarize the rules for calculating simple indexing as follows:

1. Divide 40 by the number of divisions required.
2. Reduce the fraction to its lowest terms or to a mixed number consisting of a whole number and a fraction in its lowest terms.
3. The whole number is the number of whole turns; the fraction must be changed to an equivalent fraction whose denominator is the number of holes in one of the given hole circles; the numerator of the fraction is the number of holes required.
4. State the answer in turns and number of holes in a hole-circle; for example, 3 turns and 7 holes in the 51-circle.

311. Index Plates

A certain milling machine in common use has three index plates, with the following circles of holes:

First plate, 15, 16, 17, 18, 19, 20.
Second plate, 21, 23, 27, 29, 31, 33.
Third plate, 37, 39, 41, 43, 47, 49.

312. High Number Index Plates

The range of simple indexing can be increased still further by the use of special plates, called *high number index plates.*

These are larger than ordinary plates and contain circles with a large number of holes, making it possible to index for almost any number that is likely to occur in practical work.

PROBLEMS

1. How many turns of the crank are required to index for 20 divisions?

2. What is the simple indexing for cutting the grooves on a drill with 3 grooves?

3. How would you index for cutting the teeth on a milling cutter with 6 teeth? 7 teeth? 9 teeth? 11 teeth?

4. Figure the indexing necessary for cutting the teeth on a slitting saw with 12 teeth; 13 teeth; 14 teeth; 15 teeth.

5. What indexing is necessary for cutting a gear with 16 teeth? 17 teeth? 21 teeth? 22 teeth? 23 teeth?

6. What indexing is required for cutting a gear with 24 teeth? 25 teeth? 27 teeth? 31 teeth? 35 teeth?

7. What indexing is required for cutting a gear with 28 teeth? 32 teeth? 36 teeth? 37 teeth? 39 teeth?

8. What indexing is required for cutting a gear with 41 teeth? 58 teeth? 66 teeth? 72 teeth? 92 teeth?

9. What indexing is required for cutting a gear with 65 teeth? 84 teeth? 100 teeth? 90 teeth? 160 teeth?

10. Make a list of the numbers from 101 to 115 that can be indexed by simple indexing.

11. Make a list of the numbers from 225 to 240 that can be indexed by simple indexing.

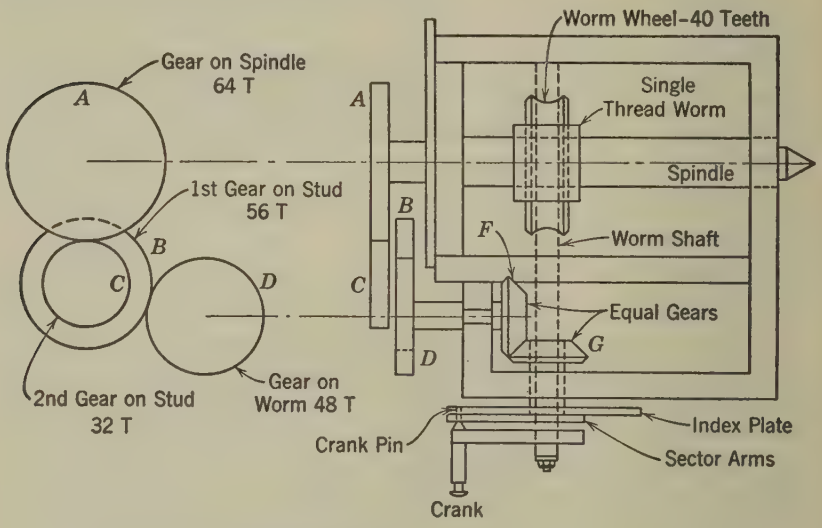
12. Index the following numbers by simple indexing: 170 divisions; 200 divisions; 260 divisions; 380 divisions; 470 divisions.

313. Differential Indexing

Differential indexing is used for numbers that cannot be indexed by simple indexing. Indexing by the differential method is done like simple indexing, the only difference being that, instead of the plate remaining stationary while the crank is being turned, the plate (which is free to rotate on the worm shaft) is made to revolve in the same direction as the index crank, or opposite to it. The rotation of the index plate is brought about by a series of change gears which con-

nect the plate to the main spindle of the index head in the following manner.

The index crank turns the worm shaft and worm, which in turn revolve the wormwheel and spindle. The spindle drives through gears *A*, *C*, and *B* to *D*. Gear *D* drives through the bevel gear *F* to *G* which is connected directly to the index plate. See Fig. 390.



Geared for 113

Fig. 390

In differential indexing, we index by means of the crank for a number which can be indexed by simple indexing and which is close to the actual indexing required. The difference between the movement of the spindle when indexing for the approximate number and the movement necessary for the required number is made up by the use of change gears, selected from the following set of gears furnished with the machine:

28, 32, 40, 44, 48, 56, 64, 72, 86, 100, 24, 24, 24

The following examples illustrate the effect of the gearing on the index plate and the indexing operation.

Example 1

Index by differential indexing for 113 divisions.

SOLUTION. We will index for 120 divisions as in simple indexing. This will require $\frac{1}{120}$ of $40 = \frac{40}{120} = \frac{1}{3}$ of a turn of the crank for each operation. But $113 \times \frac{1}{3} = 37\frac{2}{3}$ turns, whereas a complete revolution of the spindle requires 40 turns of the crank. The difference of $2\frac{1}{3}$ or $\frac{7}{3}$ turns is made up by causing the index plate to rotate $2\frac{1}{3}$ turns in the same direction as the crank. This is accomplished by means of the gears which connect the plate to the spindle. The ratio of the gears will have to be 7 to 3. Multiplying the ratio $\frac{7}{3}$ by $\frac{8}{8}$ gives $\frac{56}{24}$. The 56 gear is placed on the spindle shaft, and the 24 gear is placed on the shaft of gear F, which, through G, causes the plate to rotate.

If a 15-hole circle is used, moving the crank 5 holes in this circle will give $\frac{5}{15} = \frac{1}{3} = \frac{40}{120}$ of a revolution, the amount required for indexing 120 divisions. But we have seen that for 113 divisions we will be short $2\frac{1}{3}$ or $\frac{7}{3}$ turns of the crank; that is, the crank motion will be short $\frac{1}{113}$ of $\frac{7}{3} = \frac{7}{339}$ of a turn at each operation. Hence each time the crank moves 5 holes over the 15-hole circle, the plate moves $\frac{1}{113}$ of $\frac{7}{3}$ or $\frac{7}{339}$ of a turn in the same direction. The actual motion of the crank for each of the 113 operations is therefore equal to $\frac{5}{15} + \frac{7}{339} = \frac{120}{339}$ of a turn, while the resulting motion of the spindle is $\frac{1}{40}$ of $\frac{120}{339} = \frac{1}{113}$ of a turn. For the 113 operations necessary for the 113 divisions the crank will actually make $113 \times \frac{120}{339} = 40$ turns, while the spindle will make $113 \times \frac{1}{113} = 1$ turn. With the pair of single gears one idler will cause the plate to rotate in the same direction as the crank.

It is often more convenient to use compound gearing. The ratio $\frac{7}{3}$ would then be split up as follows: $\frac{7}{3} = \frac{2 \times 3\frac{1}{2}}{1 \times 3}$. Multiplying $\frac{2}{1}$ by $\frac{32}{32}$ gives $\frac{64}{32}$; multiplying $\frac{3\frac{1}{2}}{3}$ by $\frac{16}{16}$ gives $\frac{56}{48}$. That is, $\frac{7}{3} = \frac{64 \times 56}{32 \times 48}$. The gears in the numerator are drivers, and those in the denominator are followers. The 64 gear is placed on the spindle and the 56 gear as the first gear on the stud. The 32 gear will be the second gear on the stud and the 48 gear on the shaft on bevel gear F. No idlers are required. See Fig. 390.

Example 2

Index for 247 divisions.

SOLUTION. We will index with the crank for 240 divisions, that is, $\frac{40}{240}$ or $\frac{1}{6}$ of a turn, obtained by 3 holes in the 18-hole circle. We find that $247 \times \frac{1}{6} = \frac{247}{6} = 41\frac{1}{6}$ turns, which is too large by $1\frac{1}{6}$ turns. The gears must be arranged to revolve the index plate $1\frac{1}{6}$ turns in the opposite direction. The fraction $\frac{7}{6}$ represents the gearing ratio. Using simple gearing, we multiply the ratio $\frac{7}{6} \times \frac{8}{8}$, giving the equivalent ratio $\frac{56}{48}$. A 56 gear is placed on the spindle, and a 48 gear on the worm shaft. Two idlers are required to cause the plate to turn to the left while the crank is turned to the right 3 holes in the 18-hole circle. See Fig. 391.

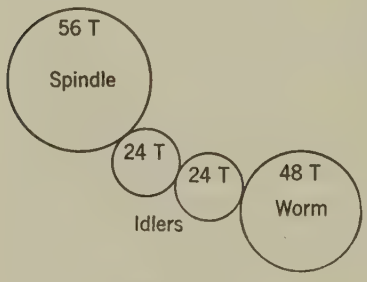


Fig. 391

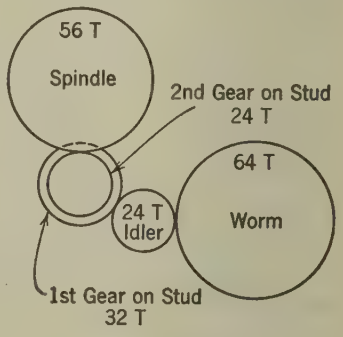


Fig. 392

If compound gearing is used, we split the ratio $\frac{56}{48}$ into $\frac{8 \times 7}{6 \times 8}$.

$$\frac{8}{6} \times \frac{4}{4} = \frac{32}{24}; \quad \frac{7}{8} \times \frac{8}{8} = \frac{56}{64}; \quad \frac{56}{48} = \frac{56 \times 32}{24 \times 64}$$

Gear on spindle, 56; first stud, 32; second stud, 24; gear on worm, 64. One idler will be required. See Fig. 392.

Example 3

Index for 173 divisions.

SOLUTION. Index for 180 divisions with the crank. $\frac{40}{180} = \frac{2}{9}$, that is, 4 holes in the 18-hole circle.

$173 \times \frac{2}{9} = \frac{346}{9} = 38\frac{4}{9}$ turns, which is short $1\frac{5}{9}$ turns. Since we have no single pair of gears which will give us a ratio of $\frac{14}{9}$, we use compound gearing.

$$\frac{14}{9} = \frac{2}{1} \times \frac{7}{9} = \left(\frac{2}{1} \times \frac{32}{32}\right) \times \left(\frac{7}{9} \times \frac{8}{8}\right) = \frac{64}{32} \times \frac{56}{72}$$

The 64 gear is placed on the spindle shaft, and the 56 gear as the first gear on the stud. The 32 gear will be the second gear on the stud and the 72 gear on the bevel gear shaft. No idlers are required. See Fig. 393.

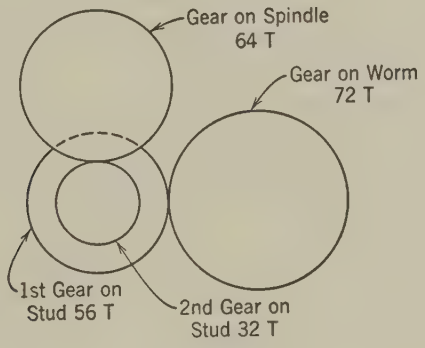


Fig. 393

NOTE. Differential indexing cannot be used when cutting spirals, as the dividing head must be geared to the lead screw of the machine.

PROBLEMS

1. How would you index, by differential indexing, for 101 divisions? Give the hole-circle, number of holes and gears required, with a sketch to show the arrangement of the gears.

2. Compute the differential indexing for cutting a gear with 111 teeth.

3. Compute the differential indexing for cutting a gear with 139 teeth.

4. Compute the differential indexing for cutting a gear with 217 teeth.

5. Compute the differential indexing for cutting a gear with 119 teeth.

6. Compute the differential indexing for cutting a gear with 311 teeth.

7. Compute the differential indexing for a gear with 201 teeth.

8. Compute the differential indexing for a gear with 238 teeth.

9. Compute the differential indexing for a gear with 179 teeth.

10. Compute the differential indexing for a gear with 149 teeth.

11. Index all the numbers from 60 to 70. Use simple indexing if possible.

12. Index all the numbers from 140 to 150. Use simple indexing if possible.

314. Angular Indexing

The angular measure of a complete circle being 360°, one complete turn of the crank turns the spindle $\frac{1}{40}$ of 360°, or 9°. Therefore any angle which is a multiple of 9° can be indexed by whole turns of the crank. An angle of 72°, for example, would require 8 turns of the crank.

An angle which is less than 9° requires a proportional part of a turn; for example, an angle of 1° requires $\frac{1}{9}$ of a turn; a 4° angle requires $\frac{4}{9}$ of a turn.

We can also index exactly for some angles less than 1°, as 20′, 30′, and 40′; that is, $\frac{1}{3}$°, $\frac{1}{2}$°, and $\frac{2}{3}$°. An angle of $\frac{1}{3}$° requires a rotation of $\frac{1}{3}$ of $\frac{1}{9}$, or $\frac{1}{27}$ of a turn, that is, 1 hole in the 27-hole circle.

Example 1

Index for 36°.

SOLUTION. $\frac{36}{9}$ = 4 turns of crank

EXPLANATION. Since each whole turn indexes 9°, we must turn the crank 4 times to index 36°.

Example 2

Index for 108°.

SOLUTION. $\frac{108}{9}$ = 12 turns of crank

Example 3

Index for 7°.

SOLUTION. The crank must be turned $\frac{7}{9}$ of a turn or 14 holes in the 18-circle.

Example 4

Index for 42°.

SOLUTION. $\frac{42}{9} = 4\frac{6}{9} = 4\frac{2}{3}$. This means that we must turn the crank 4 complete revolutions and $\frac{2}{3}$ of a revolution or 10 holes in the 15-circle.

Example 5

Index for 38° 40′.

SOLUTION. $38° 40' = 38\frac{2}{3}°$

$$38\tfrac{2}{3} \times \tfrac{1}{9} = \tfrac{116}{3} \times \tfrac{1}{9} = \tfrac{116}{27} = 4\tfrac{8}{27} \text{ turns}$$

This requires 4 turns and 8 holes in the 27-circle.

Example 6

Index for 3° 30′.

SOLUTION. $3° 30' = 3\frac{1}{2}°$

$$3\tfrac{1}{2} \times \tfrac{1}{9} = \tfrac{7}{2} \times \tfrac{1}{9} = \tfrac{7}{18} \text{ turns}$$

The indexing is 7 holes in the 18-circle.

Some angles cannot be indexed *exactly* by simple indexing with the index plates furnished. However, in most cases a very close approximation can be made by computation and the use of handbook tables. One of these tables consists of the decimal equivalents (to 4 decimal places) of the angular value of any number of holes in each of the hole circles. Thus, opposite to the 1-hole in the 15-circle appears the figure 0.0666, which is equal to $\frac{1}{15}$ changed to a decimal of four places; 9 holes in the 23-circle is 0.3913, which is the decimal equivalent of $\frac{9}{23}$. The following examples will illustrate the use of the table in angular indexing.

Example 7

Index for 0° 29′.

SOLUTION. Since one complete turn is equivalent to indexing for an angle of 9° or 540′, the indexing required for 29′ is $\frac{29}{540}$ of a turn. Converting this fraction into a decimal of four places, $\frac{29}{540} = 0.0537$. Consulting the table of decimal equivalents, we find that the figure nearest to 0.0537 is 0.0541, which is the decimal equivalent of $\frac{2}{37}$. The indexing required is 2 holes in the 37-circle.

The following computation will show the extent of the error. $\frac{2}{37}$ of $540' = 29\frac{7}{37}' = 29'\ 11''$, which is the angle for which we have indexed. The error is $(+)11''$.

Example 8

Index for 15° 37′.

SOLUTION. Since one complete turn gives a spindle rotation of 9°, we subtract 9° from 15° 37′ and make the computation for the part of a turn required.

$$15° 37′ - 9° = 6° 37′ = 6 \times 60 + 37 = 397′$$

$$\frac{397}{540} = 0.7352$$

The nearest figure to 0.7352 in the table is 0.7347, which is the decimal equivalent of $\frac{36}{49}$, that is, 36 holes in the 49-circle.

The indexing required is 1 turn and 36 holes in the 49-circle. $\frac{36}{49}$ of 540 minutes = $396\frac{36}{49}$ minutes = 396′ 44″ = 6° 36′ 44″. Adding the 9° for the whole turn, we find that we have indexed for 15° 36′ 44″. The error is (−)16 seconds.

PROBLEMS

Index for the following angles.

1. 45°	**2.** 99°	**3.** 71°	**4.** 27°	**5.** 114°
6. 1°	**7.** 7°	**8.** 66°	**9.** 43°	**10.** 214°
11. $7\frac{1}{4}$°	**12.** $21\frac{3}{4}$°	**13.** $16\frac{1}{2}$°	**14.** $40\frac{1}{3}$°	**15.** $65\frac{2}{3}$°
16. 2° 40′	**17.** 11° 20′	**18.** 56° 30′	**19.** 5° 20′	**20.** 0° 40′
21. 11° 31′	**22.** 75° 48′	**23.** 0° 53′	**24.** 2° 19′	**25.** 63° 09′

315. Multiple or Block Indexing

It is frequently found in cutting gears, especially cast iron gears of coarse pitch, that the material and the cutter tend to overheat, thereby lowering the speed and decreasing the efficiency of the machine. This can be obviated by skipping several teeth after each cut and revolving the gear several times before the entire gear is finished. That is, we index several teeth at one time. In a gear of 46 teeth, for example, we may index 5 teeth at a time. The teeth would be cut in the following rotation: 1, 6, 11, 16, 21, 26, 31, 36, 41, 46; 5, 10, 15, 20, etc., as indicated in Fig. 394. The gear would make 5 revolutions in the course of the operation.

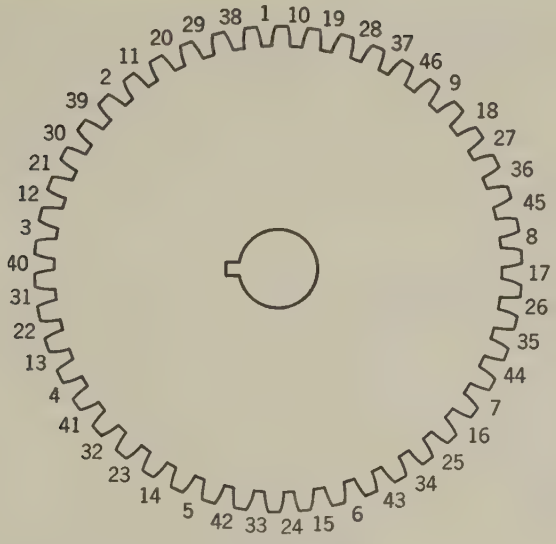


Fig. 394

During each revolution the cutter would be one tooth behind the previous revolution. *It is evident that the number of teeth indexed at one time must not be a factor of the number of teeth in the gear.* For cutting successive teeth the indexing would be $\frac{40}{46}$ or $\frac{20}{23}$, that is, 20 holes in the 23-circle. For indexing 5 teeth at a time the indexing is $5 \times \frac{20}{23}$ or $\frac{100}{23}$ or $4\frac{8}{23}$, that is, 4 turns and 8 holes in the 23-circle.

PROBLEMS

1. Index for milling a 100-tooth gear, cutting every third tooth.
2. Index for a 48-tooth gear, indexing 7 teeth at a time.
3. Index for a 75-tooth gear, indexing 4 teeth at a time.
4. Index for a 121-tooth gear, indexing 5 teeth at a time.
5. Index for a 64-tooth gear, indexing 3 teeth at a time.
6. Block index for a 95-tooth gear, selecting any convenient block.
7. Block index for a 38-tooth gear, selecting any convenient block.
8. Block index for a 175-tooth gear, selecting any convenient block.
9. Block index for a 69-tooth gear, selecting any convenient block.
10. Block index for a 127-tooth gear, selecting any convenient block.

NOTE. The entire subject of indexing has been very thoroughly discussed and the results tabulated in standard handbooks on machine shop

work. The makers of milling machines also furnish charts showing the indexing for most of the divisions the machine is capable of making.

Solve all the problems in this chapter by means of the indexing tables. The answers need not necessarily agree with the answers obtained by computation, since various combinations of plates and gears may be used to obtain the same result. Reduction to lowest terms of both results will serve as a check of the answers.

316. Milling Spirals

In cutting a spiral or helix on the milling machine, two adjustments are necessary: the table must be swiveled to correspond to the angle of spiral, and the machine must be geared to produce the required lead.

317. Spiral Angle

The table of the milling machine must be swung around until its axis coincides with the direction of the grooves in the spiral to be milled. To do this, the angle of the table must be the same as the spiral angle of the piece of work. The tangent of the spiral angle, as explained in the section on spiral gears, is found by *dividing the pitch circumference by the lead of the spiral.*

318. Lead

The lead of a spiral, when not stated, may be found by *dividing the pitch circumference by the tangent of the spiral angle.* The milling machine must be geared by means of the gears on the side of the index head so that the spindle will make one complete revolution while the table travels a distance equal to the lead of the spiral. This problem is similar to that of gearing for thread cutting on the lathe.

The pitch of the lead screw on standard milling machines is $\frac{1}{4}''$. If even gears (that is, ratio 1:1) are used between the lead screw and the worm spindle, the worm will make one revolution for each revolution of the lead screw. That is, the worm will make one revolution while the table travels $\frac{1}{4}''$. But 40 revolutions of the worm are required to produce one revolution of the spindle. The table, therefore, will

travel $40 \times \frac{1}{4}''$, or $10''$, while the spindle makes one complete turn. To vary the lead on the piece of work change gears are used.

Example 1

What change gears are required to mill a spiral with a $28''$ lead?

SOLUTION. $$\text{Required speed ratio} = \frac{\text{Lead of machine}}{\text{Required lead}}$$

$$= \frac{10}{28} = \frac{5}{7} \times \frac{2}{4} = \frac{5}{7} \times \frac{8}{8} \quad \text{and} \quad \frac{2}{4} \times \frac{12}{12} = \frac{40}{56} \quad \text{and} \quad \frac{24}{48}$$

EXPLANATION. First write the fraction $\frac{10}{28}$, where the numerator is the machine lead and the denominator is the required lead.

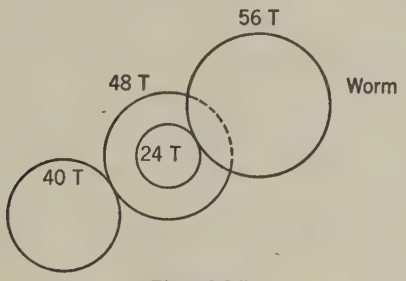


Fig. 395

Since there are four gears in the compound train, we split this ratio into the product of two ratios, $\frac{5}{7} \times \frac{2}{4}$, by factoring both terms. We then multiply both terms of each ratio by some convenient number which will give us gears that are in stock and will fit the machine. The ratio $\frac{5}{7}$ we multiply by $\frac{8}{8}$, giving $\frac{40}{56}$; the ratio $\frac{2}{4}$ we multiply by $\frac{12}{12}$, giving $\frac{24}{48}$. The numerators 40 and 24 represent driving gears and the denominators 56 and 48 driven gears or followers. We place one of the drivers, 40, on the lead screw and one of the followers, 48, on the stud. The second driver, 24, is placed on the same stud and the second follower, 56, on the worm spindle, as shown in Fig. 395.

Of the two gears on the intermediate stud the one which is in mesh with the worm is called the ***first stud gear;*** the one in mesh with the lead screw is called the ***second stud gear.***

Example 2

Find the change gears for milling a spiral with a lead of $4\frac{1}{2}''$.

SOLUTION. Required ratio $= \dfrac{\text{Machine lead}}{\text{Spiral lead}}$

$$= \frac{10}{4\frac{1}{2}} = \frac{5}{3} \times \frac{2}{1\frac{1}{2}} = \frac{5}{3} \times \frac{8}{8} \quad \text{and} \quad \frac{2}{1\frac{1}{2}} \times \frac{24}{24} = \frac{40}{24} \quad \text{and} \quad \frac{48}{36}$$

Lead screw gear = 40

Second stud gear = 36

First stud gear = 48

Worm gear = 24

See Fig. 396 for arrangement.

The method illustrated in the foregoing examples is very simple when applied to leads which form comparatively simple fractions with the machine lead. However, in the great majority of cases of spiral gearing, the lead is a decimal of two or more places. This is due to the fact that the lead is a computed figure based on the spiral angle and the pitch circumference. For instance, in the example in right angle spiral gears, Section 304, we see that the small gear has a lead of 23.73″, a number which cannot be factored into numbers that will give us available gears.

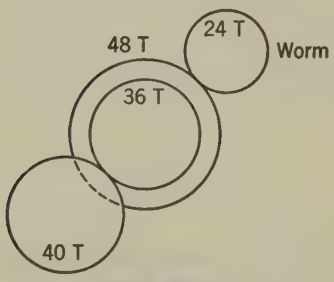


Fig. 396

In cases of this kind, resort may be had to the machinery handbooks or the charts furnished by the milling machine manufacturers. These handbooks contain tables showing the leads which can be obtained by every possible arrangement of the change gears furnished with the machine. We select the combination which gives us a lead nearest the required lead. Referring to such a table, we find the nearest

lead to 23.73″ is 23.571″, for which the gearing is as follows:

Screw gear	= 48
Second stud gear	= 44
First stud gear	= 28
Worm gear	= 72

This lead differs from the required lead by 0.159″, an error which may be disregarded in the majority of cases.

Where more accurate work is required, we can frequently make a closer approximation by the method of continued fractions. This consists of reducing the fraction $\frac{10}{23.73}$ to a considerably simpler fraction which is approximately equal to the original fraction. The method is as follows:

First multiply by $\frac{100}{100}$ to eliminate the decimal point:

$$\frac{10}{23.73} \times \frac{100}{100} = \frac{1000}{2373}$$

Dividing both terms by the numerator, we get

$$\frac{1}{2 + \frac{373}{1000}}$$

Repeating the operation on the fraction in the denominator, we get

$$\frac{1}{2 + \frac{1}{2 + \frac{254}{373}}}$$

Repeating the operation on the fraction in the denominator, we get

$$\frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{119}{254}}}}$$

Repeating the operation on the last fraction, we get

$$\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\frac{16}{119}}}}}$$

The remainder, $\frac{16}{119}$, being quite small, let us disregard it and proceed to evaluate the entire fraction, beginning at the bottom.

$$1+\frac{1}{2}=1\tfrac{1}{2};\quad \frac{1}{1\frac{1}{2}}=\frac{2}{3};\quad 2+\frac{2}{3}=2\tfrac{2}{3};$$

$$\frac{1}{2\frac{2}{3}}=\frac{3}{8};\quad 2+\frac{3}{8}=2\tfrac{3}{8};\quad \frac{1}{2\frac{3}{8}}=\frac{8}{19},$$

the required fraction. The following comparison will show the relative accuracy of the various methods:

Actual required lead = 23.73″

Nearest lead from table = 23.57″, an error of 0.16″

By method of continued fractions, lead

$$=\frac{19}{8}\times 10=\frac{190}{8}=23.75'',$$

an error of 0.02″.

Using $\frac{8}{19}$ as the required speed ratio, we get the following gears:

$$\frac{8}{19}=\frac{8}{9\frac{1}{2}}\times\frac{1}{2}=\frac{8}{9\frac{1}{2}}\times\frac{8}{8}=\frac{64}{76}\quad\text{and}\quad\frac{1}{2}\times\frac{24}{24}=\frac{24}{48}$$

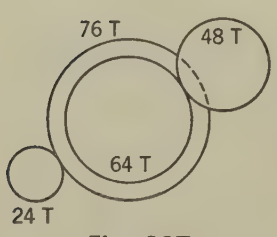


Fig. 397

The arrangement of the gearing may be as in Fig. 397.

Lead screw gear = 24

Second stud gear = 76

First stud gear = 64

Worm gear = 48

Sometimes such computations necessitate the making of special gears not part of the standard equipment of the machine; but where great accuracy is required, this may be done.

PROBLEMS

Compute sets of change gears for cutting spirals with the following leads. Check by tables in catalogs or handbooks, bearing in mind, however, that several combinations of gears may be used for obtaining the same ratio.

1. Lead 12″	**2.** Lead $8\frac{1}{2}$″	**3.** Lead 16″
4. Lead 32″	**5.** Lead 21″	**6.** Lead 14″
7. Lead $13\frac{1}{2}$″	**8.** Lead $3\frac{1}{2}$″	**9.** Lead 46″
10. Lead 27″	**11.** Lead 9.307″	**12.** Lead 18.41″
13. Lead 68.65″	**14.** Lead 3.764″	**15.** Lead 14.92″
16. Lead 31.9″	**17.** Lead 43.61″	**18.** Lead 29.31″
19. Lead 7.615″	**20.** Lead 23.27″	

24

Belting

319. Case I. Length of Open Belt Connecting Two Equal Pulleys

Figure 398 represents an open belt connecting two equal pulleys. It is evident that the arc of contact of belt and pulley is equal to a semi-circumference. Hence, to find the length of belt, we have the simple rule:

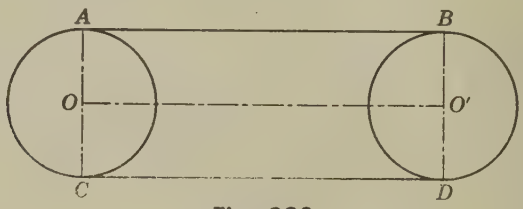


Fig. 398

Length = 2 × center distance + circumference of pulley

Example

Find the length of an open belt connecting two 14″ pulleys 7′ 0″ center to center.

SOLUTION. 7′ 0″ = 84″

Length of straight pieces = 2 × 84	= 168″
Circumference of 14″ pulley = $\frac{22}{7}$ × 14 =	44″
Total length of belt	= 212″
	= 17′ 8″

PROBLEMS

1. Compute length of open belt connecting two 18″ pulleys 6′ 6″ center to center.

2. Find length of open belt across two 24″ pulleys 8′ 0″ center to center.

3. How long must an open belt be to connect two 16″ pulleys 4′ 8″ center to center?

4. What is the length of an open belt connecting two 20″ pulleys 5′ 4″ center to center?

5. Find length of an open belt that connects two 30″ pulleys 13′ 0″ between centers.

320. Case II. Length of Open Belt Connecting Two Unequal Pulleys

Figure 399 shows an open belt connecting two pulleys of unequal diameters. Though the arc of contact on the *small* pulley is *less* than a semi-circumference and that on the

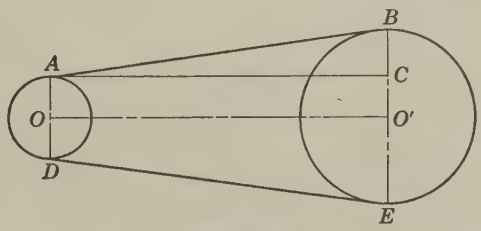


Fig. 399

large pulley *more* than a semi-circumference, it is correct for all practical purposes to assume that the arc of contact on each pulley is equal to a semi-circumference. The length of belt may then be regarded as consisting of the two straight pieces AB and DE plus half a circumference of each of the pulleys.

The straight part AB is the hypotenuse of the right triangle ABC, in which the base AC equals the center distance and the altitude BC is equal to the difference between the radii of the two pulleys.

Example

Find the length of an open belt connecting two pulleys 8′ 0″ between centers and having diameters of 10″ and 24″ respectively, Fig. 400.

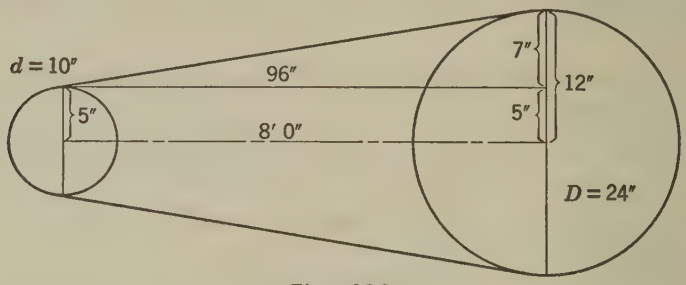


Fig. 400

SOLUTION.

$$2 \text{ straight parts} = 2\sqrt{96^2 + 7^2} = 2\sqrt{9216 + 49}$$

$$= 2\sqrt{9265} = 2 \times 96.25 = 192.50''$$

$\frac{1}{2}$ of a 10″ circumference $= \frac{1}{2} \times 3.14 \times 10 = 15.70''$

$\frac{1}{2}$ of a 24″ circumference $= \frac{1}{2} \times 3.14 \times 24 = 37.68''$

Total length of belt $= 245.88''$

$= 20' 6''$

PROBLEMS

1. Find the length of an open belt connecting two pulleys 7′ 6″ between centers and having diameters of 12″ and 20″ respectively.

2. How long must an open belt be to connect a 16″ pulley and a 22″ pulley if the distance center to center of pulleys is 9′ 5″?

3. Compute the length of an open belt across two pulleys 10′ 6″ between centers, with diameters of 14″ and 30″ respectively.

4. Compute the length of an open belt connecting an 18″ pulley and a 28″ pulley, the distance center to center of pulleys being 6′ 0″.

5. Two pulleys 8″ and 16″ in diameter respectively and 5′ 4″ between centers are connected by an open belt. How long is the belt?

321. Case III. Length of Crossed Belt Connecting Two Equal Pulleys

Figure 401 shows a crossed belt connecting two equal pulleys. Though the arcs of contact are greater than semi-circumferences, we assume that they are equal to semi-

circumferences. The belt is then figured as consisting of the two straight parts AB and DC plus one whole circumference. The straight part AB is the hypotenuse of the right

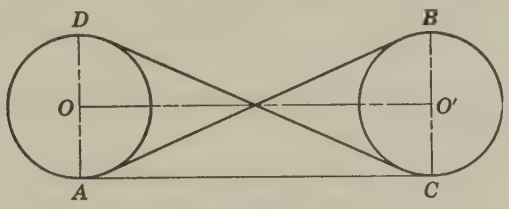


Fig. 401

triangle ABC, in which the base AC equals the distance between the centers of the pulleys and the altitude BC is the diameter of one of the pulleys.

Example

Find the length of a crossed belt connecting two 18″ pulleys 6′ 8″ center to center, Fig. 402.

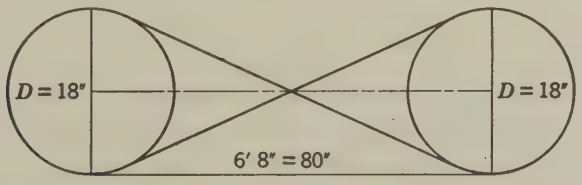


Fig. 402

Solution.

$$\text{2 straight parts} = 2\sqrt{80^2 + 18^2} = 2\sqrt{6400 + 324}$$

$$= 2\sqrt{6724} = 2 \times 82 = 164.00''$$

$$\text{Circumference of 18″ circle} = 3.14 \times 18 = 56.52''$$

$$\text{Total length of belt} = 220.52''$$

$$= 18' \, 4\tfrac{1}{2}''$$

PROBLEMS

1. Find length of crossed belt connecting two 18″ pulleys 6′ 0″ center to center.

2. Compute length of crossed belt connecting two 24″ pulleys 5′ 4″ center to center.

3. Compute length of crossed belt connecting two 20″ pulleys 9′ 0″ between centers.

4. How long is a crossed belt that connects two 16″ pulleys 8′ 0″ between centers?

5. Find the length of a crossed belt connecting two 14″ pulleys 10′ 0″ center to center.

322. Case IV. Length of Crossed Belt Connecting Two Unequal Pulleys

Figure 403 shows a crossed belt connecting two unequal pulleys. As in the two preceding cases, we assume that the arc of contact of belt on pulley is equal to a semi-circumference. The belt, according to this assumption, consists

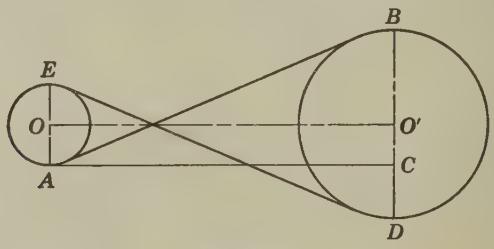


Fig. 403

of the two straight parts AB and ED plus half a circumference of each of the pulleys. The straight part AB is the hypotenuse of the right triangle ABC, in which the base AC equals the center-to-center distance and the altitude BC is equal to the sum of the radii of the two pulleys.

Example

Find the length of a crossed belt connecting two pulleys 8″ and 20″ in diameter and 8′ 4″ between centers, Fig. 404.

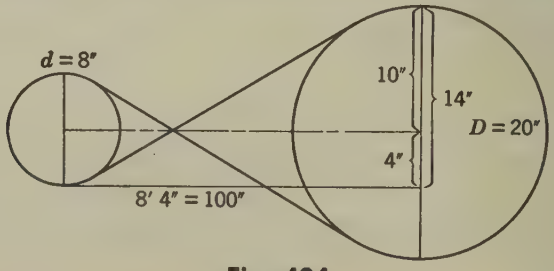


Fig. 404

SOLUTION.

$$2 \text{ straight parts} = 2\sqrt{100^2 + 14^2} = 2\sqrt{10000 + 196}$$

$$= 2\sqrt{10196} = 2 \times 100.97 = 201.94''$$

$\frac{1}{2}$ circumference of 8″ circle $= \frac{1}{2} \times 3.14 \times 8 = 12.56''$

$\frac{1}{2}$ circumference of 20″ circle $= \frac{1}{2} \times 3.14 \times 20 = 31.40''$

Total length of belt $= 245.90''$

$= 20' 6''$

PROBLEMS

1. Find length of crossed belt connecting two pulleys 12″ and 20″ in diameter and 7′ 6″ between centers.

2. Find length of crossed belt connecting two pulleys 16″ and 22″ in diameter and 9′ 0″ between centers.

3. Find length of crossed belt connecting two pulleys 14″ and 30″ in diameter and 10′ 6″ between centers.

4. Compute the length of a crossed belt connecting two pulleys 18″ and 28″ in diameter and 6′ 0″ between centers.

5. Two pulleys 8″ and 16″ in diameter respectively are connected by a crossed belt. Find the length of the belt if the distance between centers of pulleys is 5′ 4″.

NOTE. The methods outlined in Cases II, III, and IV, though only approximate, yield results accurate enough for all practical purposes. The exact length of the belt in these cases can be computed by trigonometric formulas found in standard handbooks.

323. Approximate Formulas for Length of Belt

An approximate formula often used for finding the length of an open belt connecting two unequal pulleys is:

$$\text{Length} = 2C + 3\tfrac{1}{4}(R + r)$$

in which C = Center distance
R = Radius of large pulley
r = Radius of small pulley

To find the part in contact with the pulleys we multiply the sum of the radii by $3\frac{1}{4}$ instead of 3.14 to compensate for the length of the straight parts which we assumed to be equal to twice the center distance, while in reality they are longer.

Example

Find the approximate length of an open belt connecting two pulleys 10″ and 24″ in diameter and 8′ 0″ between centers.

SOLUTION. Center distance = 8′ 0″ = 96″

$$R = 12''; r = 5''$$

$$\text{Length} = 2C + 3\tfrac{1}{4}(R + r)$$

$$= 2 \times 96 + 3\tfrac{1}{4}(12 + 5)$$

$$= 192 + 3\tfrac{1}{4} \times 17$$

$$= 192 + 55.25 = 247.25'' = 20'\ 7\tfrac{1}{4}''$$

The result obtained by Case II was 20′ 6″.

Another formula for finding the length of an open belt across two pulleys, and one giving more accurate results than the method just explained, is as follows:

$$\text{Length} = 2C + \pi(R + r) + \frac{(R - r)^2}{C}$$

in which C = Center distance
R = Radius of large pulley
r = Radius of small pulley

Example

Find the length of an open belt connecting two pullevs 10″ and 24″ in diameter and 8′ 0″ between centers.

SOLUTION. $\text{Length} = 2C + \pi(R + r) + \dfrac{(R - r)^2}{C}$

in which $C = 8'\ 0'' = 96''$
$R = \frac{1}{2}$ of $24'' = 12''$
$r = \frac{1}{2}$ of $10'' = 5''$

$$\text{Length} = 2 \times 96 + 3.14(12 + 5) + \frac{(12 - 5)^2}{96}$$

$$= 192 + 3.14 \times 17 + \frac{7^2}{96}$$

$$= 192 + 53.38 + 0.51 = 245.89''$$

$$= 20'\ 5.89''$$

This result is practically the same as that obtained in Case II, which was 20′ 6″.

For finding the approximate length of a crossed belt connecting two pulleys, the following formula is used.

$$\text{Length} = 2C + 3\tfrac{3}{8}(R + r)$$

in which C = Center distance
R = Radius of large pulley
r = Radius of small pulley

Example

Find the approximate length of a crossed belt connecting two pulleys 8″ and 20″ in diameter and 8′ 4″ between centers.

SOLUTION.

$$\text{Length} = 2C + 3\tfrac{3}{8}(R + r)$$

$$C = 8'\,4'' = 100''$$

$$R = \tfrac{1}{2} \text{ of } 20'' = 10''$$

$$r = \tfrac{1}{2} \text{ of } 8'' = 4''$$

$$\text{Length} = 2 \times 100 + 3\tfrac{3}{8}(10 + 4)$$

$$= 200 + 3\tfrac{3}{8} \times 14$$

$$= 200 + 47.25 = 247.25'' = 20'\,7\tfrac{1}{4}''$$

The result obtained by Case IV was 20′ 6″.

PROBLEMS IN USE OF FORMULAS

1. Solve the problems in Case II by both the formulas given and compare results.

2. Solve the problems in Case III and Case IV by the formula for approximate length of crossed belt and compare results.

324. Horsepower Transmitted by Belting

The unit of mechanical energy in common use is the ***horsepower.*** A horsepower represents 33,000 foot-pounds of work in 1 minute. To find the horsepower of a belt, we must multiply the speed of the belt in feet per minute by

the number of pounds pull exerted by the belt. The speed of the belt is the same as the rim speed of either of the two pulleys over which the belt is running. The pull of the belt depends upon its width and upon its thickness. It is considered good practice to allow an effective pull of 35 lb. per inch of width for single belts and about 60 lb. for double belts.

Example

Allowing an effective pull of 35 lb. per inch of width, what horsepower is transmitted by a single belt 5″ wide, if the driving pulley is 14″ in diameter and makes 240 r.p.m.?

SOLUTION. $$\text{Horsepower} = \frac{\text{Foot-pounds per minute}}{33{,}000}$$

$$\text{Speed in feet per minute} = \text{Circumference of pulley} \times \text{r.p.m.} \times \tfrac{1}{12}$$

$$= 14 \times \tfrac{22}{7} \times 240 \times \tfrac{1}{12}$$

$$= 880$$

$$\text{Effective pull} = 5 \times 35 = 175 \text{ lb.}$$

$$\text{Horsepower} = \frac{880 \times 175}{33{,}000} = 4.67$$

PROBLEMS

1. Find the horsepower transmitted by a single-ply 4″ belt running over a 16″ pulley at 180 r.p.m.

2. Compute the horsepower transmitted by a two-ply $6\frac{1}{2}$″ belt running over a 12″ pulley at 220 r.p.m. Allow an effective pull of 65 lb. per inch of width.

3. An 18″ pulley making 150 r.p.m. drives a two-ply 8″ belt. Compute the horsepower of the belt.

4. Allowing an effective pull of 36 lb. per inch of width, what is the horsepower of a 3″ single-ply belt running over a 24″ pulley making 160 r.p.m.?

5. Find the horsepower transmitted by a 5″ single-ply belt running over a 10″ pulley at 175 r.p.m.

325. To Find the Width of Belt Required to Transmit a Certain Horsepower

Example

A belt which runs over a 12″ pulley at 250 r.p.m. is required to transmit 5 H.P. What must be its width?

SOLUTION.

$$\text{Rim speed} = 12 \times \tfrac{22}{7} \times \tfrac{1}{12} \times 250$$

$$= 785.71 \text{ ft. per minute}$$

$$\text{Power of a 1″ belt} = 35 \times 785.71 = 27{,}500 \text{ ft-lb.}$$

$$5 \text{ H.P.} = 5 \times 33{,}000 = 165{,}000 \text{ ft-lb.}$$

$$\text{Width of belt required} = \frac{5 \text{ H.P.}}{\text{Power of 1″ belt}} = \frac{165{,}000}{27{,}500} = 6″$$

EXPLANATION. We first find the power in foot-pounds transmitted by a belt 1″ wide under the given conditions. We then find the total number of foot-pounds of work that the belt is required to transmit. Dividing the latter by the former gives the width of belt required.

PROBLEMS

1. A single belt running over an 18″ pulley and making 250 r.p.m. is required to transmit 8.5 H.P. Find the width of the belt.

2. What must be the width of a single belt running over a 14″ pulley which makes 210 r.p.m., in order to transmit 7 H.P.?

3. A single belt is driven by a 20″ pulley making 120 r.p.m. How wide must it be if it is required to transmit $3\frac{1}{2}$ H.P.?

4. With an effective pull of 65 lb. per inch of width, how wide must a two-ply belt be in order to transmit 10 H.P.? The belt runs over a 19-pulley which revolves at the rate of 160 r.p.m.

5. A double belt is to run over a 24″ pulley making 220 r.p.m. Allowing an effective pull of 70 lb. per inch of width, how wide must the belt be in order to transmit 12 H.P.?

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
1	1	1	1.0000	1.0000	3.142	0.7854	1
2	4	8	1.4142	1.2599	6.283	3.1416	2
3	9	27	1.7321	1.4422	9.425	7.0686	3
4	16	64	2.0000	1.5874	12.566	12.5664	4
5	25	125	2.2361	1.7100	15.708	19.6350	5
6	36	216	2.4495	1.8171	18.850	28.2743	6
7	49	343	2.6458	1.9129	21.991	38.4845	7
8	64	512	2.8284	2.0000	25.133	50.2655	8
9	81	729	3.0000	2.0801	28.274	63.6173	9
10	100	1000	3.1623	2.1544	31.416	78.5398	10
11	121	1331	3.3166	2.2240	34.558	95.0332	11
12	144	1728	3.4641	2.2894	37.699	113.097	12
13	169	2197	3.6056	2.3513	40.841	132.732	13
14	196	2744	3.7417	2.4101	43.982	153.938	14
15	225	3375	3.8730	2.4662	47.124	176.715	15
16	256	4096	4.0000	2.5198	50.265	201.062	16
17	289	4913	4.1231	2.5713	53.407	226.980	17
18	324	5832	4.2426	2.6207	56.549	254.469	18
19	361	6859	4.3589	2.6684	59.690	283.529	19
20	400	8000	4.4721	2.7144	62.832	314.159	20
21	441	9261	4.5826	2.7589	65.973	346.361	21
22	484	10648	4.6904	2.8020	69.115	380.133	22
23	529	12167	4.7958	2.8439	72.257	415.476	23
24	576	13824	4.8990	2.8845	75.398	452.389	24
25	625	15625	5.0000	2.9240	78.540	490.874	25
26	676	17576	5.0990	2.9625	81.681	530.929	26
27	729	19683	5.1962	3.0000	84.823	572.555	27
28	784	21952	5.2915	3.0366	87.965	615.752	28
29	841	24389	5.3852	3.0723	91.106	660.520	29
30	900	27000	5.4772	3.1072	94.248	706.858	30
31	961	29791	5.5678	3.1414	97.389	754.768	31
32	1024	32768	5.6569	3.1748	100.531	804.248	32
33	1089	35937	5.7446	3.2075	103.673	855.299	33
34	1156	39304	5.8310	3.2396	106.814	907.920	34
35	1225	42875	5.9161	3.2711	109.956	962.113	35
36	1296	46656	6.0000	3.3019	113.097	1017.88	36
37	1369	50653	6.0828	3.3322	116.239	1075.21	37
38	1444	54872	6.1644	3.3620	119.381	1134.11	38
39	1521	59319	6.2450	3.3912	122.522	1194.59	39

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
40	1600	64000	6.3246	3.4200	125.66	1256.64	40
41	1681	68921	6.4031	3.4482	128.81	1320.25	41
42	1764	74088	6.4807	3.4760	131.95	1385.44	42
43	1849	79507	6.5574	3.5034	135.09	1452.20	43
44	1936	85184	6.6332	3.5303	138.23	1520.53	44
45	2025	91125	6.7082	3.5569	141.37	1590.43	45
46	2116	97336	6.7823	3.5830	144.51	1661.90	46
47	2209	103823	6.8557	3.6088	147.65	1734.94	47
48	2304	110592	6.9282	3.6342	150.80	1809.56	48
49	2401	117649	7.0000	3.6593	153.94	1885.74	49
50	2500	125000	7.0711	3.6840	157.08	1963.50	50
51	2601	132651	7.1414	3.7084	160.22	2042.82	51
52	2704	140608	7.2111	3.7325	163.36	2123.72	52
53	2809	148877	7.2801	3.7563	166.50	2206.18	53
54	2916	157464	7.3485	3.7798	169.65	2290.22	54
55	3025	166375	7.4162	3.8030	172.79	2375.83	55
56	3136	175616	7.4833	3.8259	175.93	2463.01	56
57	3249	185193	7.5498	3.8485	179.07	2551.76	57
58	3364	195112	7.6158	3.8709	182.21	2642.08	58
59	3481	205379	7.6811	3.8930	185.35	2733.97	59
60	3600	216000	7.7460	3.9149	188.50	2827.43	60
61	3721	226981	7.8102	3.9365	191.64	2922.47	61
62	3844	238328	7.8740	3.9579	194.78	3019.07	62
63	3969	250047	7.9373	3.9791	197.92	3117.25	63
64	4096	262144	8.0000	4.0000	201.06	3216.99	64
65	4225	274625	8.0623	4.0207	204.20	3318.31	65
66	4356	287496	8.1240	4.0412	207.35	3421.19	66
67	4489	300763	8.1854	4.0615	210.49	3525.65	67
68	4624	314432	8.2462	4.0817	213.63	3631.68	68
69	4761	328509	8.3066	4.1016	216.77	3739.28	69
70	4900	343000	8.3666	4.1213	219.91	3848.45	70
71	5041	357911	8.4261	4.1408	223.05	3959.19	71
72	5184	373248	8.4853	4.1602	226.19	4071.50	72
73	5329	389017	8.5440	4.1793	229.34	4185.39	73
74	5476	405224	8.6023	4.1983	232.48	4300.84	74
75	5625	421875	8.6603	4.2172	235.62	4417.86	75
76	5776	438976	8.7178	4.2358	238.76	4536.46	76
77	5929	456533	8.7750	4.2543	241.90	4656.63	77
78	6084	474552	8.8318	4.2727	245.04	4778.36	78
79	6241	493039	8.8882	4.2908	248.19	4901.67	79

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
80	6400	512000	8.9443	4.3089	251.33	5026.55	80
81	6561	531441	9.0000	4.3267	254.47	5153.00	81
82	6724	551368	9.0554	4.3445	257.61	5281.02	82
83	6889	571787	9.1104	4.3621	260.75	5410.61	83
84	7056	592704	9.1652	4.3795	263.89	5541.77	84
85	7225	614125	9.2195	4.3968	267.04	5674.50	85
86	7396	636056	9.2736	4.4140	270.18	5808.80	86
87	7569	658503	9.3274	4.4310	273.32	5944.68	87
88	7744	681472	9.3808	4.4480	276.46	6082.12	88
89	7921	704969	9.4340	4.4647	279.60	6221.14	89
90	8100	729000	9.4868	4.4814	282.74	6361.73	90
91	8281	753571	9.5394	4.4979	285.88	6503.88	91
92	8464	778688	9.5917	4.5144	289.03	6647.61	92
93	8649	804357	9.6437	4.5307	292.17	6792.91	93
94	8836	830584	9.6954	4.5468	295.31	6939.78	94
95	9025	857375	9.7468	4.5629	298.45	7088.22	95
96	9216	884736	9.7980	4.5789	301.59	7238.23	96
97	9409	912673	9.8489	4.5947	304.73	7389.81	97
98	9604	941192	9.8995	4.6104	307.88	7542.96	98
99	9801	970299	9.9499	4.6261	311.02	7697.69	99
100	10000	1000000	10.0000	4.6416	314.16	7853.98	100
101	10201	1030301	10.0499	4.6570	317.30	8011.85	101
102	10404	1061208	10.0995	4.6723	320.44	8171.28	102
103	10609	1092727	10.1489	4.6875	323.58	8332.29	103
104	10816	1124864	10.1980	4.7027	326.73	8494.87	104
105	11025	1157625	10.2470	4.7177	329.87	8659.01	105
106	11236	1191016	10.2956	4.7326	333.01	8824.73	106
107	11449	1225043	10.3441	4.7475	336.15	8992.02	107
108	11664	1259712	10.3923	4.7622	339.29	9160.88	108
109	11881	1295029	10.4403	4.7769	342.43	9331.32	109
110	12100	1331000	10.4881	4.7914	345.58	9503.32	110
111	12321	1367631	10.5357	4.8059	348.72	9676.89	111
112	12544	1404928	10.5830	4.8203	351.86	9852.03	112
113	12769	1442897	10.6301	4.8346	355.00	10028.7	113
114	12996	1481544	10.6771	4.8488	358.14	10207.0	114
115	13225	1520875	10.7238	4.8629	361.28	10386.9	115
116	13456	1560896	10.7703	4.8770	364.42	10568.3	116
117	13689	1601613	10.8167	4.8910	367.57	10751.3	117
118	13924	1643032	10.8682	4.9049	370.71	10935.9	118
119	14161	1685159	10.9087	4.9187	373.85	11122.0	119

TABLE I

Powers, Roots, Circumferences, and Areas

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
120	14400	1728000	10.9545	4.9324	376.99	11309.7	120
121	14641	1771561	11.0000	4.9461	380.13	11499.0	121
122	14884	1815848	11.0454	4.9597	383.27	11689.9	122
123	15129	1860867	11.0905	4.9732	386.42	11882.3	123
124	15376	1906624	11.1355	4.9866	389.56	12076.3	124
125	15625	1953125	11.1803	5.0000	392.70	12271.8	125
126	15876	2000376	11.2250	5.0133	395.84	12469.0	126
127	16129	2048383	11.2694	5.0265	398.98	12667.7	127
128	16384	2097152	11.3137	5.0397	402.12	12868.0	128
129	16641	2146689	11.3578	5.0528	405.27	13069.8	129
130	16900	2197000	11.4018	5.0658	408.41	13273.2	130
131	17161	2248091	11.4455	5.0788	411.55	13478.2	131
132	17424	2299968	11.4891	5.0916	414.69	13684.8	132
133	17689	2352637	11.5326	5.1045	417.83	13892.9	133
134	17956	2406104	11.5758	5.1172	420.97	14102.6	134
135	18225	2460375	11.6190	5.1299	424.12	14313.9	135
136	18496	2515456	11.6619	5.1426	427.26	14526.7	136
137	18769	2571353	11.7047	5.1551	430.40	14741.1	137
138	19044	2628072	11.7473	5.1676	433.54	14957.1	138
139	19321	2685619	11.7898	5.1801	436.68	15174.7	139
140	19600	2744000	11.8322	5.1925	439.82	15393.8	140
141	19881	2803221	11.8743	5.2048	442.96	15614.5	141
142	20164	2863288	11.9164	5.2171	446.11	15836.8	142
143	20449	2924207	11.9583	5.2293	449.25	16060.6	143
144	20736	2985984	12.0000	2.2415	452.39	16286.0	144
145	21025	3048625	12.0416	5.2536	455.53	16513.0	145
146	21316	3112136	12.0830	5.2656	458.67	16741.5	146
147	21609	3176523	12.1244	5.2776	461.81	16971.7	147
148	21904	3241792	12.1655	5.2896	464.96	17203.4	148
149	22201	3307949	12.2066	5.3015	468.10	17436.6	149
150	22500	3375000	12.2474	5.3133	471.24	17671.5	150
151	22801	3442951	12.2882	5.3251	474.38	17907.9	151
152	23104	3511808	12.3288	5.3368	477.52	18145.8	152
153	23409	3581577	12.3693	5.3485	480.66	18385.4	153
154	23716	3652264	12.4097	5.3601	483.81	18626.5	154
155	24025	3723875	12.4499	5.3717	486.95	18869.2	155
156	24336	3796416	12.4900	5.3832	490.09	19113.4	156
157	24649	3869893	12.5300	5.3947	493.23	19359.3	157
158	24964	3944312	12.5698	5.4061	496.37	19606.7	158
159	25281	4019679	12.6095	5.4175	499.51	19855.7	159

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
160	25600	4096000	12.6491	5.4288	502.65	20106.2	160
161	25921	4173281	12.6886	5.4401	505.80	20358.3	161
162	26244	4251528	12.7279	5.4514	508.94	20612.0	162
163	26569	4330747	12.7671	5.4626	512.08	20867.2	163
164	26896	4410944	12.8062	5.4737	515.22	21124.1	164
165	27225	4492125	12.8452	5.4848	518.36	21382.5	165
166	27556	4574296	12.8841	5.4959	521.50	21642.4	166
167	27889	4657463	12.9228	5.5069	524.65	21904.0	167
168	28224	4741632	12.9615	5.5178	527.79	22167.1	168
169	28561	4826809	13.0000	5.5288	530.93	22431.8	169
170	28900	4913000	13.0384	5.5397	534.07	22698.0	170
171	29241	5000211	13.0767	5.5505	537.21	22965.8	171
172	29584	5088448	13.1149	5.5613	540.35	23235.2	172
173	29929	5177717	13.1529	5.5721	543.50	23506.2	173
174	30276	5268024	13.1909	5.5828	546.64	23778.7	174
175	30625	5359375	13.2288	5.5934	549.78	24052.8	175
176	30976	5451776	13.2665	5.6041	552.92	24328.5	176
177	31329	5545233	13.3041	5.6147	556.06	24605.7	177
178	31684	5639752	13.3417	5.6252	559.20	24884.6	178
179	32041	5735339	13.3791	5.6357	562.35	25164.9	179
180	32400	5832000	13.4164	5.6462	565.49	25446.9	180
181	32761	5929741	13.4536	5.6567	568.63	25730.4	181
182	33124	6028568	13.4907	5.6671	571.77	26015.5	182
183	33489	6128487	13.5277	5.6774	574.91	26302.2	183
184	33856	6229504	13.5647	5.6877	578.05	26590.4	184
185	34225	6331625	13.6015	5.6980	581.19	26880.3	185
186	34596	6434856	13.6382	5.7083	584.34	27171.6	186
187	34969	6539203	13.6748	5.7185	587.48	27464.6	187
188	35344	6644672	13.7113	5.7287	590.62	27759.1	188
189	35721	6751269	13.7477	5.7388	593.76	28055.2	189
190	36100	6859000	13.7840	5.7489	596.90	28352.9	190
191	36481	6967871	13.8203	5.7590	600.04	28652.1	191
192	36864	7077888	13.8564	5.7690	603.19	28952.9	192
193	37249	7189057	13.8924	5.7790	606.33	29255.3	193
194	37636	7301384	13.9284	5.7890	609.47	29559.2	194
195	38025	7414875	13.9642	5.7989	612.61	29864.8	195
196	38416	7529536	14.0000	5.8088	615.75	30171.9	196
197	38809	7645373	14.0357	5.8186	618.89	30480.5	197
198	39204	7762392	14.0712	5.8285	622.04	30790.7	198
199	39601	7880599	14.1067	5.8383	625.18	31102.6	199

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
200	40000	8000000	14.1421	5.8480	628.32	31415.9	200
201	40401	8120601	14.1774	5.8578	631.46	31730.9	201
202	40804	8242408	14.2127	5.8675	634.60	32047.4	202
203	41209	8365427	14.2478	5.8771	637.74	32365.5	203
204	41616	8489664	14.2829	5.8868	640.89	32685.1	204
205	42025	8615125	14.3178	5.8964	644.03	33006.4	205
206	42436	8741816	14.3527	5.9059	647.17	33329.2	206
207	42849	8869743	14.3875	5.9155	650.31	33653.5	207
208	43264	8998912	14.4222	5.9250	653.45	33979.5	208
209	43681	9129329	14.4568	5.9345	656.59	34307.0	209
210	44100	9261000	14.4914	5.9439	659.73	34636.1	210
211	44521	9393931	14.5258	5.9533	662.88	34966.7	211
212	44944	9528128	14.5602	5.9627	666.02	35298.9	212
213	45369	9663597	14.5945	5.9721	669.16	35632.7	213
214	45796	9800344	14.6287	5.9814	672.30	35968.1	214
215	46225	9938375	14.6629	5.9907	675.44	36305.0	215
216	46656	10077696	14.6969	6.0000	678.58	36643.5	216
217	47089	10218313	14.7309	6.0092	681.73	36983.6	217
218	47524	10360232	14.7648	6.0185	684.87	37325.3	218
219	47961	10503459	14.7986	6.0277	688.01	37668.5	219
220	48400	10648000	14.8324	6.0368	691.15	38013.3	220
221	48841	10793861	14.8661	6.0459	694.29	38359.6	221
222	49284	10941048	14.8997	6.0550	697.43	38707.6	222
223	49729	11089567	14.9332	6.0641	700.58	39057.1	223
224	50176	11239424	14.9666	6.0732	703.72	39408.1	224
225	50625	11390625	15.0000	6.0822	706.86	39760.8	225
226	51076	11543176	15.0333	6.0912	710.00	40115.0	226
227	51529	11697083	15.0665	6.1002	713.14	40470.8	227
228	51984	11852352	15.0997	6.1091	716.28	40828.1	228
229	52441	12008989	15.1327	6.1180	719.42	41187.1	229
230	52900	12167000	15.1658	6.1269	722.57	41547.6	230
231	53361	12326391	15.1987	6.1358	725.71	41909.6	231
232	53824	12487168	15.2315	6.1446	728.85	42273.3	232
233	54289	12649337	15.2643	6.1534	731.99	42638.5	233
234	54756	12812904	15.2971	6.1622	735.13	43005.3	234
235	55225	12977875	15.3297	6.1710	738.27	43373.6	235
236	55696	13144256	15.3623	6.1797	741.42	43743.5	236
237	56169	13312053	15.3948	6.1885	744.56	44115.0	237
238	56644	13481272	15.4272	6.1972	747.70	44488.1	238
239	57121	13651919	15.4596	6.2058	750.84	44862.7	239

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
240	57600	13824000	15.4919	6.2145	753.98	45238.9	240
241	58081	13997521	15.5242	6.2231	757.12	45616.7	241
242	58564	14172488	15.5563	6.2317	760.27	45996.1	242
243	59049	14348907	15.5885	6.2403	763.41	46377.0	243
244	59536	14526784	15.6205	6.2488	766.55	46759.5	244
245	60025	14706125	15.6525	6.2573	769.69	47143.5	245
246	60516	14886936	15.6844	6.2658	772.83	47529.2	246
247	61009	15069223	15.7162	6.2743	775.97	47916.4	247
248	61504	15252992	15.7480	6.2828	779.12	48305.1	248
249	62001	15438249	15.7797	6.2912	782.26	48695.5	249
250	62500	15625000	15.8114	6.2996	785.40	49087.4	250
251	63001	15813251	15.8430	6.3080	788.54	49480.9	251
252	63504	16003008	15.8745	6.3164	791.68	49875.9	252
253	64009	16194277	15.9060	6.3247	794.82	50272.6	253
254	64516	16387064	15.9374	6.3330	797.96	50670.7	254
255	65025	16581375	15.9687	6.3413	801.11	51070.5	255
256	65536	16777216	16.0000	6.3496	804.25	51471.9	256
257	66049	16974593	16.0312	6.3579	807.39	51874.8	257
258	66564	17173512	16.0624	6.3661	810.53	52279.2	258
259	67081	17373979	16.0935	6.3743	813.67	52685.3	259
260	67600	17576000	16.1245	6.3825	816.81	53092.9	260
261	68121	17779581	16.1555	6.3907	819.96	53502.1	261
262	68644	17984728	16.1864	6.3988	823.10	53912.9	262
263	69169	18191447	16.2173	6.4070	826.24	54325.2	263
264	69696	18399744	16.2481	6.4151	829.38	54739.1	264
265	70225	18609625	16.2788	6.4232	832.52	55154.6	265
266	70756	18821096	16.3095	6.4312	835.66	55571.6	266
267	71289	19034163	16.3401	6.4393	838.81	55990.3	267
268	71824	19248832	16.3707	6.4473	841.95	56410.4	268
269	72361	19465109	16.4012	6.4553	845.09	56832.2	269
270	72900	19683000	16.4317	6.4633	848.23	57255.5	270
271	73441	19902511	16.4621	6.4713	851.37	57680.4	271
272	73984	20123648	16.4924	6.4792	854.51	58106.9	272
273	74529	20346417	16.5227	6.4872	857.66	58534.9	273
274	75076	20570824	16.5529	6.4951	860.80	58964.6	274
275	75625	20796875	16.5831	6.5030	863.94	59395.7	275
276	76176	21024576	16.6132	6.5108	867.08	59828.5	276
277	76729	21253933	16.6433	6.5187	870.22	60262.8	277
278	77284	21484952	16.6733	6.5265	873.36	60698.7	278
279	77841	21717639	16.7033	6.5343	876.50	61136.2	279

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
280	78400	21952000	16.7332	6.5421	879.65	61575.2	280
281	78961	22188041	16.7631	6.5499	882.79	62015.8	281
282	79524	22425768	16.7929	6.5577	885.93	62458.0	282
283	80089	22665187	16.8226	6.5654	889.07	62901.8	283
284	80656	22906304	16.8523	6.5731	892.21	63347.1	284
285	81225	23149125	16.8819	6.5808	895.35	63794.0	285
286	81796	23393656	16.9115	6.5885	898.50	64242.4	286
287	82369	23639903	16.9411	6.5962	901.64	64692.5	287
288	82944	23887872	16.9706	6.6039	904.78	65144.1	288
289	83521	24137569	17.0000	6.6115	907.92	65597.2	289
290	84100	24389000	17.0294	6.6191	911.06	66052.0	290
291	84681	24642171	17.0587	6.6267	914.20	66508.3	291
292	85264	24897088	17.0880	6.6343	917.35	66966.2	292
293	85849	25153757	17.1172	6.6419	920.49	67425.6	293
294	86436	25412184	17.1464	6.6494	923.63	67886.7	294
295	87025	25672375	17.1756	6.6569	926.77	68349.3	295
296	87616	25934336	17.2047	6.6644	929.91	68813.5	296
297	88209	26198073	17.2337	6.6719	933.05	69279.2	297
298	88804	26463592	17.2627	6.6794	936.19	69746.5	298
299	89401	26730899	17.2916	6.6869	939.34	70215.4	299
300	90000	27000000	17.3205	6.6943	942.48	70685.8	300
301	90601	27270901	17.3494	6.7018	945.62	71157.9	301
302	91204	27543608	17.3781	6.7092	948.76	71631.5	302
303	91809	27818127	17.4069	6.7166	951.90	72106.6	303
304	92416	28094464	17.4356	6.7240	955.04	72583.4	304
305	93025	28372625	17.4642	6.7313	958.19	73061.7	305
306	93636	28652616	17.4929	6.7387	961.33	73541.5	306
307	94249	28934443	17.5214	6.7460	964.47	74023.0	307
308	94864	29218112	17.5499	6.7533	967.61	74506.0	308
309	95481	29503629	17.5784	6.7606	970.75	74990.6	309
310	96100	29791000	17.6068	6.7679	973.89	75476.8	310
311	96721	30080231	17.6352	6.7752	977.04	75964.5	311
312	97344	30371328	17.6635	6.7824	980.18	76453.8	312
313	97969	30664297	17.6918	6.7897	983.32	76944.7	313
314	98596	30959144	17.7200	6.7969	986.46	77437.1	314
315	99225	31255875	17.7482	6.8041	989.60	77931.1	315
316	99856	31554496	17.7764	6.8113	992.74	78426.7	316
317	100489	31855013	17.8045	6.8185	995.88	78923.9	317
318	101124	32157432	17.8326	6.8256	999.03	79422.6	318
319	101761	32461759	17.8606	6.8328	1002.2	79922.9	319

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
320	102400	32768000	17.8885	6.8399	1005.3	80424.8	320
321	103041	33076161	17.9165	6.8470	1008.5	80928.2	321
322	103684	33386248	17.9444	6.8541	1011.6	81433.2	322
323	104329	33698267	17.9722	6.8612	1014.7	81939.8	323
324	104976	34012224	18.0000	6.8683	1017.9	82448.0	324
325	105625	34328125	18.0278	6.8753	1021.0	82957.7	325
326	106276	34645976	18.0555	6.8824	1024.2	83469.0	326
327	106929	34965783	18.0831	6.8894	1027.3	83981.8	327
328	107584	35287552	18.1108	6.8964	1030.4	84496.3	328
329	108241	35611289	18.1384	6.9034	1033.6	85012.3	329
330	108900	35937000	18.1659	6.9104	1036.7	85529.9	330
331	109561	36264691	18.1934	6.9174	1039.9	86049.0	331
332	110224	36594368	18.2209	6.9244	1043.0	86569.7	332
333	110889	36926037	18.2483	6.9313	1046.2	87092.0	333
334	111556	37259704	18.2757	6.9382	1049.3	87615.9	334
335	112225	37595375	18.3030	6.9451	1052.4	88141.3	335
336	112896	37933056	18.3303	6.9521	1055.6	88668.3	336
337	113569	38272753	18.3576	6.9589	1058.7	89196.9	337
338	114244	38614472	18.3848	6.9658	1061.9	89727.0	338
339	114921	38958219	18.4120	6.9727	1065.0	90258.7	339
340	115600	39304000	18.4391	6.9795	1068.1	90792.0	340
341	116281	39651821	18.4662	6.9864	1071.3	91326.9	341
342	116964	40001688	18.4932	6.9932	1074.4	91863.3	342
343	117649	40353607	18.5203	7.0000	1077.6	92401.3	343
344	118336	40707584	18.5472	7.0068	1080.7	92940.9	344
345	119025	41063625	18.5742	7.0136	1083.8	93482.0	345
346	119716	41421736	18.6011	7.0203	1087.0	94024.7	346
347	120409	41781923	18.6279	7.0271	1090.1	94569.0	347
348	121104	42144192	18.6548	7.0338	1093.3	95114.9	348
349	121801	42508549	18.6815	7.0406	1096.4	95662.3	349
350	122500	42875000	18.7083	7.0473	1099.6	96211.3	350
351	123201	43243551	18.7350	7.0540	1102.7	96761.8	351
352	123904	43614208	18.7617	7.0607	1105.8	97314.0	352
353	124609	43986977	18.7883	7.0674	1109.0	97867.7	353
354	125316	44361864	18.8149	7.0740	1112.1	98423.0	354
355	126025	44738875	18.8414	7.0807	1115.3	98979.8	355
356	126736	45118016	18.8680	7.0873	1118.4	99538.2	356
357	127449	45499293	18.8944	7.0940	1121.5	100098	357
358	128164	45882712	18.9209	7.1006	1124.7	100660	358
359	128881	46268279	18.9473	7.1072	1127.8	101223	359

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
360	129600	46656000	18.9737	7.1138	1131.0	101788	360
361	130321	47045881	19.0000	7.1204	1134.1	102354	361
362	131044	47437928	19.0263	7.1269	1137.3	102922	362
363	131769	47832147	19.0526	7.1335	1140.4	103491	363
364	132496	48228544	19.0788	7.1400	1143.5	104062	364
365	133225	48627125	19.1050	7.1466	1146.7	104635	365
366	133956	49027896	19.1311	7.1531	1149.8	105209	366
367	134689	49430863	19.1572	7.1596	1153.0	105785	367
368	135424	49836032	19.1833	7.1661	1156.1	106362	368
369	136161	50243409	19.2094	7.1726	1159.2	106941	369
370	136900	50653000	19.2354	7.1791	1162.4	107521	370
371	137641	51064811	19.2614	7.1855	1165.5	108103	371
372	138384	51478848	19.2873	7.1920	1168.7	108687	372
373	139129	51895117	19.3132	7.1984	1171.8	109272	373
374	139876	52313624	19.3391	7.2048	1175.0	109858	374
375	140625	52734375	19.3649	7.2112	1178.1	110447	375
376	141376	53157376	19.3907	7.2177	1181.2	111036	376
377	142129	53582633	19.4165	7.2240	1184.4	111628	377
378	142884	54010152	19.4422	7.2304	1187.5	112221	378
379	143641	54439939	19.4679	7.2368	1190.7	112815	379
380	144400	54872000	19.4936	7.2432	1193.8	113411	380
381	145161	55306341	19.5192	7.2495	1196.9	114009	381
382	145924	55742968	19.5448	7.2558	1200.1	114608	382
383	146689	56181887	19.5704	7.2622	1203.2	115209	383
384	147456	56623104	19.5959	7.2685	1206.4	115812	384
385	148225	57066625	19.6214	7.2748	1209.5	116416	385
386	148996	57512456	19.6469	7.2811	1212.7	117021	386
387	149769	57960603	19.6723	7.2874	1215.8	117628	387
388	150544	58411072	19.6977	7.2936	1218.9	118237	388
389	151321	58863869	19.7231	7.2999	1221.1	118847	389
390	152100	59319000	19.7484	7.3061	1225.2	119459	390
391	152881	59776471	19.7737	7.3124	1228.4	120072	391
392	153664	60236288	19.7990	7.3186	1231.5	120687	392
393	154449	60698457	19.8242	7.3248	1234.6	121304	393
394	155236	61162984	19.8494	7.3310	1237.8	121922	394
395	156025	61629875	19.8746	7.3372	1240.9	122542	395
396	156816	62099136	19.8997	7.3434	1244.1	123163	396
397	157609	62570773	19.9249	7.3496	1247.2	123786	397
398	158404	63044792	19.9499	7.3558	1250.4	124410	398
399	159201	63521199	19.9750	7.3619	1253.5	125036	399

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
400	160000	64000000	20.0000	7.3681	1256.6	125664	400
401	160801	64481201	20.0250	7.3742	1259.8	126293	401
402	161604	64964808	20.0499	7.3803	1262.9	126923	402
403	162409	65450827	20.0749	7.3864	1266.1	127556	403
404	163216	65939264	20.0998	7.3925	1269.2	128190	404
405	164025	66430125	20.1246	7.3986	1272.3	128825	405
406	164836	66923416	20.1494	7.4047	1275.5	129462	406
407	165649	67419143	20.1742	7.4108	1278.6	130100	407
408	166464	67917312	20.1990	7.4169	1281.8	130741	408
409	167281	68417929	20.2237	7.4229	1284.9	131382	409
410	168100	68921000	20.2485	7.4290	1288.1	132025	410
411	168921	69426531	20.2731	7.4350	1291.2	132670	411
412	169744	69934528	20.2978	7.4410	1294.3	133317	412
413	170569	70444997	20.3224	7.4470	1297.5	133965	413
414	171396	70957944	20.3470	7.4530	1300.6	134614	414
415	172225	71473375	20.3715	7.4590	1303.8	135265	415
416	173056	71991296	20.3961	7.4650	1306.9	135918	416
417	173889	72511713	20.4206	7.4710	1310.0	136572	417
418	174724	73034632	20.4450	7.4770	1313.2	137228	418
419	175561	73560059	20.4695	7.4829	1316.3	137885	419
420	176400	74088000	20.4939	7.4889	1319.5	138544	420
421	177241	74618461	20.5183	7.4948	1322.6	139205	421
422	178084	75151448	20.5426	7.5007	1325.8	139867	422
423	178929	75686967	20.5670	7.5067	1328.9	140531	423
424	179776	76225024	20.5913	7.5126	1332.0	141196	424
425	180625	76765625	20.6155	7.5185	1335.2	141863	425
426	181476	77308776	20.6398	7.5244	1338.3	142531	426
427	182329	77854483	20.6640	7.5302	1341.5	143201	427
428	183184	78402752	20.6882	7.5361	1344.6	143872	428
429	184041	78953589	20.7123	7.5420	1347.7	144545	429
430	184900	79507000	20.7364	7.5478	1350.9	145220	430
431	185761	80062991	20.7605	7.5537	1354.0	145896	431
432	186624	80621568	20.7846	7.5595	1357.2	146574	432
433	187489	81182737	20.8087	7.5654	1360.3	147254	433
434	188356	81746504	20.8327	7.5712	1363.5	147934	434
435	189225	82312875	20.8567	7.5770	1366.6	148617	435
436	190096	82881856	20.8806	7.5828	1369.7	149301	436
437	190969	83453453	20.9045	7.5886	1372.9	149987	437
438	191844	84027672	20.9284	7.5944	1376.0	150674	438
439	192721	84604519	20.9523	7.6001	1379.2	151363	439

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
440	193600	85184000	20.9762	7.6059	1382.3	152053	440
441	194481	85766121	21.0000	7.6117	1385.4	152745	441
442	195364	86350888	21.0238	7.6174	1388.6	153439	442
443	196249	86938307	21.0476	7.6232	1391.7	154134	443
444	197136	87528384	21.0713	7.6289	1394.9	154830	444
445	198025	88121125	21.0950	7.6346	1398.0	155528	445
446	198916	88716536	21.1187	7.6403	1401.2	156228	446
447	199809	89314623	21.1424	7.6460	1404.3	156930	447
448	200704	89915392	21.1660	7.6517	1407.4	157633	448
449	201601	90518849	21.1896	7.6574	1410.6	158337	449
450	202500	91125000	21.2132	7.6631	1413.7	159043	450
451	203401	91733851	21.2368	7.6688	1416.9	159751	451
452	204304	92345408	21.2603	7.6744	1420.0	160460	452
453	205209	92959677	21.2838	7.6801	1423.1	161171	453
454	206116	93576664	21.3073	7.6857	1426.3	161883	454
455	207025	94196375	21.3307	7.6914	1429.4	162597	455
456	207936	94818816	21.3542	7.6970	1432.6	163313	456
457	208849	95443993	21.3776	7.7026	1435.7	164030	457
458	209764	96071912	21.4009	7.7082	1438.9	164748	458
459	210681	96702579	21.4243	7.7138	1442.0	165468	459
460	211600	97336000	21.4476	7.7194	1445.1	166190	460
461	212521	97972181	21.4709	7.7250	1448.3	166914	461
462	213444	98611128	21.4942	7.7306	1451.4	167639	462
463	214369	99252847	21.5174	7.7362	1454.6	168365	463
464	215296	99897344	21.5407	7.7418	1457.7	169093	464
465	216225	100544625	21.5639	7.7473	1460.8	169823	465
466	217156	101194696	21.5870	7.7529	1464.0	170554	466
467	218089	101847563	21.6102	7.7584	1467.1	171287	467
468	219024	102503232	21.6333	7.7639	1470.3	172021	468
469	219961	103161709	21.6564	7.7695	1473.4	172757	469
470	220900	103823000	21.6795	7.7750	1476.5	173494	470
471	221841	104487111	21.7025	7.7805	1479.7	174234	471
472	222784	105154048	21.7256	7.7860	1482.8	174974	472
473	223729	105823817	21.7486	7.7915	1486.0	175716	473
474	224676	106496424	21.7715	7.7970	1489.1	176460	474
475	225625	107171875	21.7945	7.8025	1492.3	177205	475
476	226576	107850176	21.8174	7.8079	1495.4	177952	476
477	227529	108531333	21.8403	7.8134	1498.5	178701	477
478	228484	109215352	21.8632	7.8188	1501.7	179451	478
479	229441	109902239	21.8861	7.8243	1504.8	180203	479

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
480	230400	110592000	21.9089	7.8297	1508.0	180956	480
481	231361	111284641	21.9317	7.8352	1511.1	181711	481
482	232324	111980168	21.9545	7.8406	1514.3	182467	482
483	233289	112678587	21.9773	7.8460	1517.4	183225	483
484	234256	113379904	22.0000	7.8514	1520.5	183984	484
485	235225	114084125	22.0227	7.8568	1523.7	184745	485
486	236196	114791256	22.0454	7.8622	1526.8	185508	486
487	237169	115501303	22.0681	7.8676	1530.0	186272	487
488	238144	116214272	22.0907	7.8730	1533.1	187038	488
489	239121	116930169	22.1133	7.8784	1536.2	187805	489
490	240100	117649000	22.1359	7.8837	1539.4	188574	490
491	241081	118370771	22.1585	7.8891	1542.5	189345	491
492	242064	119095488	22.1811	7.8944	1545.7	190117	492
493	243049	119823157	22.2036	7.8998	1548.8	190890	493
494	244036	120553784	22.2261	7.9051	1551.9	191665	494
495	245025	121287375	22.2486	7.9105	1555.1	192442	495
496	246016	122023936	22.2711	7.9158	1558.2	193221	496
497	247009	122763473	22.2935	7.9211	1561.4	194000	497
498	248004	123505992	22.3159	7.9264	1564.5	194782	498
499	249001	124251499	22.3383	7.9317	1567.7	195565	499
500	250000	125000000	22.3607	7.9370	1570.8	196350	500
501	251001	125751501	22.3830	7.9423	1573.9	197136	501
502	252004	126506008	22.4054	7.9476	1577.1	197923	502
503	253009	127263527	22.4277	7.9528	1580.2	198713	503
504	254016	128024064	22.4499	7.9581	1583.4	199504	504
505	255025	128787625	22.4722	7.9634	1586.5	200296	505
506	256036	129554216	22.4944	7.9686	1589.7	201090	506
507	257049	130323843	22.5167	7.9739	1592.8	201886	507
508	258064	131096512	22.5389	7.9791	1595.9	202683	508
509	259081	131872229	22.5610	7.9843	1599.1	202482	509
510	260100	132651000	22.5832	7.9896	1602.2	204282	510
511	261121	133432831	22.6053	7.9948	1605.4	205084	511
512	262144	134217728	22.6274	8.0000	1608.5	205887	512
513	263169	135005697	22.6495	8.0052	1611.6	206692	513
514	264196	135796744	22.6716	8.0104	1614.8	207499	514
515	265225	136590875	22.6936	8.0156	1617.9	208307	515
516	266256	137388096	22.7156	8.0208	1621.1	209117	516
517	267289	138188413	22.7376	8.0260	1624.2	209928	517
518	268324	138991832	22.7596	8.0311	1627.3	210741	518
519	269361	139798359	22.7816	8.0363	1630.5	211556	519

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POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
520	270400	140608000	22.8035	8.0415	1633.6	212372	520
521	271441	141420761	22.8254	8.0466	1636.8	213189	521
522	272484	142236648	22.8473	8.0517	1639.9	214008	522
523	273529	143055667	22.8692	8.0569	1643.1	214829	523
524	274576	143877824	22.8910	8.0620	1646.2	215651	524
525	275625	144703125	22.9129	8.0671	1649.3	216475	525
526	276676	145531576	22.9347	8.0723	1652.5	217301	526
527	277729	146363183	22.9565	8.0774	1655.6	218128	527
528	278784	147197952	22.9783	8.0825	1658.8	218956	528
529	279841	148035889	23.0000	8.0876	1661.9	219787	529
530	280900	148877000	23.0217	8.0927	1665.0	220618	530
531	281961	149721291	23.0434	8.0978	1668.2	221452	531
532	283024	150568768	23.0651	8.1028	1671.3	222287	532
533	284089	151419437	23.0868	8.1079	1674.5	223123	533
534	285156	152273304	23.1084	8.1130	1677.6	223961	534
535	286225	153130375	23.1301	8.1180	1680.8	224801	535
536	287296	153990656	23.1517	8.1231	1683.9	225642	536
537	288369	154854153	23.1733	8.1281	1687.0	226484	537
538	289444	155720872	23.1948	8.1332	1690.2	227329	538
539	290521	156590819	23.2164	8.1382	1693.3	228175	539
540	291600	157464000	23.2379	8.1433	1696.5	229022	540
541	292681	158340421	23.2594	8.1483	1699.6	229871	541
542	293764	159220088	23.2809	8.1533	1702.7	230722	542
543	294849	160103007	23.3024	8.1583	1705.9	231574	543
544	295936	160989184	23.3238	8.1633	1709.0	232428	544
545	297025	161878625	23.3452	8.1683	1712.2	233283	545
546	298116	162771336	23.3666	8.1733	1715.3	234140	546
547	299209	163667323	23.3880	8.1783	1718.5	234998	547
548	300304	164566592	23.4094	8.1833	1721.6	235858	548
549	301401	165469149	23.4307	8.1882	1724.7	236720	549
550	302500	166375000	23.4521	8.1932	1727.9	237583	550
551	303601	167284151	23.4734	8.1982	1731.0	238448	551
552	304704	168196608	23.4947	8.2031	1734.2	239314	552
553	305809	169112377	23.5160	8.2081	1737.3	240182	553
554	306916	170031464	23.5372	8.2130	1740.4	241051	554
555	308025	170953875	23.5584	8.2180	1743.6	241922	555
556	309136	171879616	23.5797	8.2229	1746.7	242795	556
557	310249	172808693	23.6008	8.2278	1749.9	243669	557
558	311364	173741112	23.6220	8.2327	1753.0	244545	558
559	312481	174676879	23.6432	8.2377	1756.2	245422	559

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
560	313600	175616000	23.6643	8.2426	1759.3	246301	560
561	314721	176558481	23.6854	8.2475	1762.4	247181	561
562	315844	177504328	23.7065	8.2524	1765.6	248063	562
563	316969	178453547	23.7276	8.2573	1768.7	248947	563
564	318096	179406144	23.7487	8.2621	1771.9	249832	564
565	319225	180362125	23.7697	8.2670	1775.0	250719	565
566	320356	181321496	23.7908	8.2719	1778.1	251607	566
567	321489	182284263	23.8118	8.2768	1781.3	252497	567
568	322624	183250432	23.8328	8.2816	1784.4	253388	568
569	323761	184220009	23.8537	8.2865	1787.6	254281	569
570	324900	185193000	23.8747	8.2913	1790.7	255176	570
571	326041	186169411	23.8956	8.2962	1793.9	256072	571
572	327184	187149248	23.9165	8.3010	1797.0	256970	572
573	328329	188132517	23.9374	8.3059	1800.1	257869	573
574	329476	189119224	23.9583	8.3107	1803.3	258770	574
575	330625	190109375	23.9792	8.3155	1806.4	259672	575
576	331776	191102976	24.0000	8.3203	1809.6	260576	576
577	332929	192100033	24.0208	8.3251	1812.7	261482	577
578	334084	193100552	24.0416	8.3300	1815.8	262389	578
579	335241	194104539	24.0624	8.3348	1819.0	263298	579
580	336400	195112000	24.0832	8.3396	1822.1	264208	580
581	337561	196122941	24.1039	8.3443	1825.3	265120	581
582	338724	197137368	24.1247	8.3491	1828.4	266033	582
583	339889	198155287	24.1454	8.3539	1831.6	266948	583
584	341056	199176704	24.1661	8.3587	1834.7	267865	584
585	342225	200201625	24.1868	8.3634	1837.8	268783	585
586	343396	201230056	24.2074	8.3682	1841.0	269701	586
587	344569	202262003	24.2281	8.3730	1844.1	270624	587
588	345744	203297472	24.2487	8.3777	1847.3	271547	588
589	346921	204336469	24.2693	8.3825	1850.4	272471	589
590	348100	205379000	24.2899	8.3872	1853.5	273397	590
591	349281	206425071	24.3105	8.3919	1856.7	274325	591
592	350464	207474688	24.3311	8.3967	1859.8	275254	592
593	351649	208527857	24.3516	8.4014	1863.0	276184	593
594	352836	209584584	24.3721	8.4061	1866.1	277117	594
595	354025	210644875	24.3926	8.4108	1869.3	278051	595
596	355216	211708736	24.4131	8.4155	1872.4	278986	596
597	356409	212776173	24.4336	8.4202	1875.5	279923	597
598	357604	213847192	24.4540	8.4249	1878.7	280862	598
599	358801	214921799	24.4745	8.4296	1881.8	281802	599

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
600	360000	216000000	24.4949	8.4343	1885.0	282743	600
601	361201	217081801	24.5153	8.4390	1888.1	283687	601
602	362404	218167208	24.5357	8.4437	1891.2	284631	602
603	363609	219256227	24.5561	8.4484	1894.4	285578	603
604	364816	220348864	24.5764	8.4530	1897.5	286526	604
605	366025	221445125	24.5967	8.4577	1900.7	287475	605
606	367236	222545016	24.6171	8.4623	1903.8	288426	606
607	368449	223648543	24.6374	8.4670	1907.0	289379	607
608	369664	224755712	24.6577	8.4716	1910.1	290333	608
609	370881	225866529	24.6779	8.4763	1913.2	291289	609
610	372100	226981000	24.6982	8.4809	1916.4	292247	610
611	373321	228099131	24.7184	8.4856	1919.5	293206	611
612	374544	229220928	24.7386	8.4902	1922.7	294166	612
613	375769	230346397	24.7588	8.4948	1925.8	295128	613
614	376996	231475544	24.7790	8.4994	1928.9	296092	614
615	378225	232608375	24.7992	8.5040	1932.1	297057	615
616	379456	233744896	24.8193	8.5086	1935.2	298024	616
617	380689	234885113	24.8395	8.5132	1938.4	298992	617
618	381924	236029032	24.8596	8.5178	1941.5	299962	618
619	383161	237176659	24.8797	8.5224	1944.7	300934	619
620	384400	238328000	24.8998	8.5270	1947.8	301907	620
621	385641	239483061	24.9199	8.5316	1950.9	302882	621
622	386884	240641848	24.9399	8.5462	1954.1	303858	622
623	388129	241804367	24.9600	8.5408	1957.2	304836	623
624	389376	242970624	24.9800	8.5453	1960.4	305815	624
625	390625	244140625	25.0000	8.5499	1963.5	306796	625
626	391876	245314376	25.0200	8.5544	1966.6	307779	626
627	393129	246491883	25.0400	8.5590	1969.8	308763	627
628	394384	247673152	25.0599	8.5635	1972.9	309748	628
629	395641	248858189	25.0799	8.5681	1976.1	310736	629
630	396900	250047000	25.0998	8.5726	1979.2	311725	630
631	398161	251239591	25.1197	8.5772	1982.4	312715	631
632	399424	252435968	25.1396	8.5817	1985.5	313707	632
633	400689	253636137	25.1595	8.5862	1988.6	314700	633
634	401956	254840104	25.1794	8.5907	1991.8	315696	634
635	403225	256047875	25.1992	8.5952	1994.9	316692	635
636	404496	257259456	25.2190	8.5997	1998.1	317690	636
637	405769	258474853	25.2389	8.6043	2001.2	318690	637
638	407044	259694072	25.2587	8.6088	2004.3	319692	638
639	408321	260917119	25.2784	8.6132	2007.5	320695	639

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
640	409600	262144000	25.2982	8.6177	2010.6	321699	640
641	410881	263374721	25.3180	8.6222	2013.8	322705	641
642	412164	264609288	25.3377	8.6267	2016.9	323713	642
643	413449	265847707	25.3574	8.6312	2020.0	324722	643
644	414736	267089984	25.3772	8.6357	2023.2	325733	644
645	416025	268336125	25.3969	8.6401	2026.3	326745	645
646	417316	269586136	25.4165	8.6446	2029.5	327759	646
647	418609	270840023	25.4362	8.6490	2032.6	328775	647
648	419904	272097792	25.4558	8.6535	2035.8	329792	648
649	421201	273359449	25.4755	8.6579	2038.9	330810	649
650	422500	274625000	25.4951	8.6624	2042.0	331831	650
651	423801	275894451	25.5147	8.6668	2045.2	332853	651
652	425104	277167808	25.5343	8.6713	2048.3	333876	652
653	426409	278445077	25.5539	8.6757	2051.5	334901	653
654	427716	279726264	25.5734	8.6801	2054.6	335927	654
655	429025	281011375	25.5930	8.6845	2057.7	336955	655
656	430336	282300416	25.6125	8.6890	2060.9	337985	656
657	431649	283593393	25.6320	8.6934	2064.0	339016	657
658	432964	284890312	25.6515	8.6978	2067.2	340049	658
659	434281	286191179	25.6710	8.7022	2070.3	341084	659
660	435600	287496000	25.6905	8.7066	2073.5	342119	660
661	436921	288804781	25.7099	8.7110	2076.6	343157	661
662	438244	290117528	25.7294	8.7154	2079.7	344196	662
663	439569	291434247	25.7488	8.7198	2082.9	345237	663
664	440896	292754944	25.7682	8.7241	2086.0	346279	664
665	442225	294079625	25.7876	8.7285	2089.2	347323	665
666	443556	295408296	25.8070	8.7329	2092.3	348368	666
667	444889	296740963	25.8263	8.7373	2095.4	349415	667
668	446224	298077632	25.8457	8.7416	2098.6	350464	668
669	447561	299418309	25.8650	8.7460	2101.7	351514	669
670	448900	300763000	25.8844	8.7503	2104.9	352565	670
671	450241	302111711	25.9037	8.7547	2108.0	353618	671
672	451584	303464448	25.9230	8.7590	2111.2	354673	672
673	452929	304821217	25.9422	8.7634	2114.3	355730	673
674	454276	306182024	25.9615	8.7677	2117.4	356788	674
675	455625	307546875	25.9808	8.7721	2120.6	357847	675
676	456976	308915776	26.0000	8.7764	2123.7	358908	676
677	458329	310288733	26.0192	8.7807	2126.9	359971	677
678	459684	311665752	26.0384	8.7850	2130.0	361035	678
679	461041	313046839	26.0576	8.7893	2133.1	362101	679

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
680	462400	314432000	26.0768	8.7937	2136.3	363168	680
681	463761	315821241	26.0960	8.7980	2139.4	364237	681
682	465124	317214568	26.1151	8.8023	2142.6	365308	682
683	466489	318611987	26.1343	8.8066	2145.7	366380	683
684	467856	320013504	26.1534	8.8109	2148.9	367453	684
685	469225	321419125	26.1725	8.8152	2152.0	368528	685
686	470596	322828856	26.1916	8.8194	2155.1	369605	686
687	471969	324242703	26.2107	8.8237	2158.3	370684	687
688	473344	325660672	26.2298	8.8280	2161.4	371764	688
689	474721	327082769	26.2488	8.8323	2164.6	372845	689
690	476100	328509000	26.2679	8.8366	2167.7	373928	690
691	477481	329939371	26.2869	8.8408	2170.8	375013	691
692	478864	331373888	26.3059	8.8451	2174.0	376099	692
693	480249	332812557	26.3249	8.8493	2177.1	377187	693
694	481636	334255384	26.3439	8.8536	2180.3	378276	694
695	483025	335702375	26.3629	8.8578	2183.4	379367	695
696	484416	337153536	26.3818	8.8621	2186.6	380459	696
697	485809	338608873	26.4008	8.8663	2189.7	381554	697
698	487204	340068392	26.4197	8.8706	2192.8	382649	698
699	488601	341532099	26.4386	8.8748	2196.0	383746	699
700	490000	343000000	26.4575	8.8790	2199.1	384845	700
701	491401	344472101	26.4764	8.8833	2202.3	385945	701
702	492804	345948408	26.4953	8.8875	2205.4	387047	702
703	494209	347428927	26.5141	8.8917	2208.5	388151	703
704	495616	348913664	26.5330	8.8959	2211.7	389256	704
705	497025	350402625	26.5518	8.9001	2214.8	390363	705
706	498436	351895816	26.5707	8.9043	2218.0	391471	706
707	499849	353393243	26.5895	8.9085	2221.1	392580	707
708	501264	354894912	26.6083	8.9127	2224.3	393692	708
709	502681	356400829	26.6271	8.9169	2227.4	394805	709
710	504100	357911000	26.6458	8.9211	2230.5	395919	710
711	505521	359425431	26.6646	8.9253	2233.7	397035	711
712	506944	360944128	26.6833	8.9295	2236.8	398153	712
713	508369	362467097	26.7021	8.9337	2240.0	399272	713
714	509796	363994344	26.7208	8.9378	2243.1	400393	714
715	511225	365525875	26.7395	8.9420	2246.2	401515	715
716	512656	367061696	26.7582	8.9462	2249.4	402639	716
717	514089	368601813	26.7769	8.9503	2252.5	403765	717
718	515524	370146232	26.7955	8.9545	2255.7	404892	718
719	516961	371694959	26.8142	8.9587	2258.8	406020	719

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
720	518400	373248000	26.8328	8.9628	2261.9	407150	720
721	519841	374805361	26.8514	8.9670	2265.1	408282	721
722	521284	376367048	26.8701	8.9711	2268.2	409416	722
723	522729	377933067	26.8887	8.9752	2271.4	410550	723
724	524176	379503424	26.9072	8.9794	2274.5	411687	724
725	525625	381078125	26.9258	8.9835	2277.7	412825	725
726	527076	382657176	26.9444	8.9876	2280.8	413965	726
727	528529	384240583	26.9629	8.9918	2283.9	415106	727
728	529984	385828352	26.9815	8.9959	2287.1	416248	728
729	531441	387420489	27.0000	9.0000	2290.2	417393	729
730	532900	389017000	27.0185	9.0041	2293.4	418539	730
731	534361	390617891	27.0370	9.0082	2296.5	419686	731
732	535824	392223168	27.0555	9.0123	2299.7	420835	732
733	537289	393832837	27.0740	9.0164	2302.8	421986	733
734	538756	395446904	27.0924	9.0205	2305.9	423138	734
735	540225	397065375	27.1109	9.0246	2309.1	424293	735
736	541696	398688256	27.1293	9.0287	2312.2	425448	736
737	543169	400315553	27.1477	9.0328	2315.4	426604	737
738	544644	401947272	27.1662	9.0369	2318.5	427762	738
739	546121	403583419	27.1846	9.0410	2321.6	428922	739
740	547600	405224000	27.2029	9.0450	2324.8	430084	740
741	549081	406869021	27.2213	9.0491	2327.9	431247	741
742	550564	408518488	27.2397	9.0532	2331.1	432412	742
743	552049	410172407	27.2580	9.0572	2334.2	433578	743
744	553536	411830784	27.2764	9.0613	2337.3	434746	744
745	555025	413493625	27.2947	9.0654	2340.5	435916	745
746	556516	415160936	27.3130	9.0694	2343.6	437087	746
747	558009	416832723	27.3313	9.0735	2346.8	438259	747
748	559504	418508992	27.3496	9.0775	2349.9	439433	748
749	561001	420189749	27.3679	9.0816	2353.1	440609	749
750	562500	421875000	27.3861	9.0856	2356.2	441786	750
751	564001	423564751	27.4044	9.0896	2359.3	442965	751
752	565504	425259008	27.4226	9.0937	2362.5	444146	752
753	567009	426957777	27.4408	9.0977	2365.6	445328	753
754	568516	428661064	27.4591	9.1017	2368.8	446511	754
755	570025	430368875	27.4773	9.1057	2371.9	447697	755
756	571536	432081216	27.4955	9.1098	2375.0	448883	756
757	573049	433798093	27.5136	9.1138	2378.2	450072	757
758	574564	435519512	27.5318	9.1178	2381.3	451262	758
759	576081	437245479	27.5500	9.1218	2384.5	452453	759

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
760	577600	438976000	27.5681	9.1258	2387.6	453646	760
761	579121	440711081	27.5862	9.1298	2390.8	454841	761
762	580644	442450728	27.6043	9.1338	2393.9	456037	762
763	582169	444194947	27.6225	9.1378	2397.0	457234	763
764	583696	445943744	27.6405	9.1418	2400.2	458434	764
765	585225	447697125	27.6586	9.1458	2403.3	459635	765
766	586756	449455096	27.6767	9.1498	2406.5	460837	766
767	588289	451217663	27.6948	9.1537	2409.6	462042	767
768	589824	452984832	27.7128	9.1577	2412.7	463247	768
769	591361	454756609	27.7308	9.1617	2415.9	464454	769
770	592900	456533000	27.7489	9.1657	2419.0	465663	770
771	594441	458314011	27.7669	9.1696	2422.2	466873	771
772	595984	460099648	27.7849	9.1736	2425.3	468085	772
773	597529	461889917	27.8029	9.1775	2428.5	469298	773
774	599076	463684824	27.8209	9.1815	2431.6	470513	774
775	600625	465484375	27.8388	9.1855	2434.7	471730	775
776	602176	467288576	27.8568	9.1894	2437.9	472948	776
777	603729	469097433	27.8747	9.1933	2441.0	474168	777
778	605284	470910952	27.8927	9.1973	2444.2	475389	778
779	606841	472729139	27.9106	9.2012	2447.3	476612	779
780	608400	474552000	27.9285	9.2052	2450.4	477836	780
781	609961	476379541	27.9464	9.2091	2453.6	479062	781
782	611524	478211768	27.9643	9.2130	2456.7	480290	782
783	613089	480048687	27.9821	9.2170	2459.9	481519	783
784	614656	481890304	28.0000	9.2209	2463.0	482750	784
785	616225	483736625	28.0179	9.2248	2466.2	483982	785
786	617796	485587656	28.0357	9.2287	2469.3	485216	786
787	619369	487443403	28.0535	9.2326	2472.4	486451	787
788	620944	489303872	28.0713	9.2365	2475.6	487688	788
789	622521	491169069	28.0891	9.2404	2478.7	488927	789
790	624100	493039000	28.1069	9.2443	2481.9	490167	790
791	625681	494913671	28.1247	9.2482	2485.0	491409	791
792	627264	496793088	28.1425	9.2521	2488.1	492652	792
793	628849	498677257	28.1603	9.2560	2491.3	493897	793
794	630436	500566184	28.1780	9.2599	2494.4	495143	794
795	632025	502459875	28.1957	9.2638	2497.6	496391	795
796	633616	504358336	28.2135	9.2677	2500.7	497641	796
797	635209	506261573	28.2312	9.2716	2503.8	498892	797
798	636804	508169592	28.2489	9.2754	2507.0	500145	798
799	638401	510082399	28.2666	9.2793	2510.1	501399	799

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
800	640000	512000000	28.2843	9.2832	2513.3	502655	800
801	641601	513922401	28.3019	9.2870	2516.4	503912	801
802	643204	515849608	28.3196	9.2909	2519.6	505171	802
803	644809	517781627	28.3373	9.2948	2522.7	506432	803
804	646416	519718464	28.3549	9.2986	2525.8	507694	804
805	648025	521660125	28.3725	9.3025	2529.0	508958	805
806	649636	523606616	28.3901	9.3063	2532.1	510223	806
807	651249	525557943	28.4077	9.3102	2535.3	511490	807
808	652864	527514112	28.4253	9.3140	2538.4	512758	808
809	654481	529475129	28.4429	9.3179	2541.5	514028	809
810	656100	531441000	28.4605	9.3217	2544.7	515300	810
811	657721	533411731	28.4781	9.3255	2547.8	516573	811
812	659344	535387328	28.4956	9.3294	2551.0	517848	812
813	660969	537367797	28.5132	9.3332	2554.1	519124	813
814	662596	539353144	28.5307	9.3370	2557.3	520402	814
815	664225	541343375	28.5482	9.3408	2560.4	521681	815
816	665856	543338496	28.5657	9.3447	2563.5	522962	816
817	667489	545338513	28.5832	9.3485	2566.7	524245	817
818	669124	547343432	28.6007	9.3523	2569.8	525529	818
819	670761	549353259	28.6182	9.3561	2573.0	526814	819
820	672400	551368000	28.6356	9.3599	2576.1	528102	820
821	674041	553387661	28.6531	9.3637	2579.2	529391	821
822	675684	555412248	28.6705	9.3675	2582.4	530681	822
823	677329	557441767	28.6880	9.3713	2585.5	531973	823
824	678976	559476224	28.7054	9.3751	2588.7	533267	824
825	680625	561515625	28.7228	9.3789	2591.8	534562	825
826	682276	563559976	28.7402	9.3827	2595.0	535858	826
827	683929	565609283	28.7576	9.3865	2598.1	537157	827
828	685584	567663552	28.7750	9.3902	2601.2	538456	828
829	687241	569722789	28.7924	9.3940	2604.4	539758	829
830	688900	571787000	28.8097	9.3978	2607.5	541061	830
831	690561	573856191	28.8271	9.4016	2610.7	542365	831
832	692224	575930368	28.8444	9.4053	2613.8	543671	832
833	693889	578009537	28.8617	9.4091	2616.9	544979	833
834	695556	580093704	28.8791	9.4129	2620.1	546288	834
835	697225	582182875	28.8964	9.4166	2623.2	547599	835
836	698896	584277056	28.9137	9.4204	2626.4	548912	836
837	700569	586376253	28.9310	9.4241	2629.5	550226	837
838	702244	588480472	28.9482	9.4279	2632.7	551541	838
839	703921	590589719	28.9655	9.4316	2635.8	552858	839

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
840	705600	592704000	28.9828	9.4354	2638.9	554177	840
841	707281	594823321	29.0000	9.4391	2642.1	555497	841
842	708964	596947688	29.0172	9.4429	2645.2	556819	842
843	710649	599077107	29.0345	9.4466	2648.4	558142	843
844	712336	601211584	29.0517	9.4503	2651.5	559467	844
845	714025	603351125	29.0689	9.4541	2654.6	560794	845
846	715716	605495736	29.0861	9.4578	2657.8	562122	846
847	717409	607645423	29.1033	9.4615	2660.9	563452	847
848	719104	609800192	29.1204	9.4652	2664.1	564783	848
849	720801	611960049	29.1376	9.4690	2667.2	566116	849
850	722500	614125000	29.1548	9.4727	2670.4	567450	850
851	724201	616295051	29.1719	9.4764	2673.5	568786	851
852	725904	618470208	29.1890	9.4801	2676.6	570124	852
853	727609	620650477	29.2062	9.4838	2679.8	571463	853
854	729316	622835864	29.2233	9.4875	2682.9	572803	854
855	731025	625026375	29.2404	9.4912	2686.1	574146	855
856	732736	627222016	29.2575	9.4949	2689.2	575490	856
857	734449	629422793	29.2746	9.4986	2692.3	576835	857
858	736164	631628712	29.2916	9.5023	2695.5	578182	858
859	737881	633839779	29.3087	9.5060	2698.6	579530	859
860	739600	636056000	29.3258	9.5097	2701.8	580880	860
861	741321	638277381	29.3428	9.5134	2704.9	582232	861
862	743044	640503928	29.3598	9.5171	2708.1	583585	862
863	744769	642735647	29.3769	9.5207	2711.2	584940	863
864	746496	644972544	29.3939	9.5244	2714.3	586297	864
865	748225	647214625	29.4109	9.5281	2717.5	587655	865
866	749956	649461896	29.4279	9.5317	2720.6	589014	866
867	751689	651714363	29.4449	9.5354	2723.8	590375	867
868	753424	653972032	29.4618	9.5391	2726.9	591738	868
869	755161	656234909	29.4788	9.5427	2730.0	593102	869
870	756900	658503000	29.4958	9.5464	2733.2	594468	870
871	758641	660776311	29.5127	9.5501	2736.3	595835	871
872	760384	663054848	29.5296	9.5537	2739.5	597204	872
873	762129	665338617	29.5466	9.5574	2742.6	598575	873
874	763876	667627624	29.5635	9.5610	2745.8	599947	874
875	765625	669921875	29.5804	9.5647	2748.9	601320	875
876	767376	672221376	29.5973	9.5683	2752.0	602696	876
877	769129	674526133	29.6142	9.5719	2755.2	604073	877
878	770884	676836152	29.6311	9.5756	2758.3	605451	878
879	772641	679151439	29.6479	9.5792	2761.5	606831	879

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
880	774400	681472000	29.6648	9.5828	2764.6	608212	880
881	776161	683797841	29.6816	9.5865	2767.7	609595	881
882	777924	686128968	29.6985	9.5901	2770.9	610980	882
883	779689	688465387	29.7153	9.5937	2774.0	612366	883
884	781456	690807104	29.7321	9.5973	2777.2	613754	884
885	783225	693154125	29.7489	9.6010	2780.3	615143	885
886	784996	695506456	29.7658	9.6046	2783.5	616534	886
887	786769	697864103	29.7825	9.6082	2786.6	617927	887
888	788544	700227072	29.7993	9.6118	2789.7	619321	888
889	790321	702595369	29.8161	9.6154	2792.9	620717	889
890	792100	704969000	29.8329	9.6190	2796.0	622114	890
891	793881	707347971	29.8496	9.6226	2799.2	623513	891
892	795664	709732288	29.8664	9.6262	2802.3	624913	892
893	797449	712121957	29.8831	9.6298	2805.4	626315	893
894	799236	714516984	29.8998	9.6334	2808.6	627718	894
895	801025	716917375	29.9166	9.6370	2811.7	629124	895
896	802816	719323136	29.9333	9.6406	2814.9	630530	896
897	804609	721734273	29.9500	9.6442	2818.0	631938	897
898	806404	724150792	29.9666	9.6477	2821.2	633348	898
899	808201	726572699	29.9833	9.6513	2824.3	634760	899
900	810000	729000000	30.0000	9.6549	2827.4	636173	900
901	811801	731432701	30.0167	9.6585	2830.6	637587	901
902	813604	733870808	30.0333	9.6620	2833.7	639003	902
903	815409	736314327	30.0500	9.6656	2836.9	640421	903
904	817216	738763264	30.0666	9.6692	2840.0	641840	904
905	819025	741217625	30.0832	9.6727	2843.1	643261	905
906	820836	743677416	30.0998	9.6763	2846.3	644683	906
907	822649	746142643	30.1164	9.6799	2849.4	646107	907
908	824464	748613312	30.1330	9.6834	2852.6	647533	908
909	826281	751089429	30.1496	9.6870	2855.7	648960	909
910	828100	753571000	30.1662	9.6905	2858.8	650388	910
911	829921	756058031	30.1828	9.6941	2862.0	651818	911
912	831744	758550528	30.1993	9.6976	2865.1	653250	912
913	833569	761048497	30.2159	9.7012	2868.3	654684	913
914	835396	763551944	30.2324	9.7047	2871.4	656118	914
915	837225	766060875	30.2490	9.7082	2874.6	657555	915
916	839056	768575296	30.2655	9.7118	2877.7	658993	916
917	840889	771095213	30.2820	9.7153	2880.8	660433	917
918	842724	773620632	30.2985	9.7188	2884.0	661874	918
919	844561	776151559	30.3150	9.7224	2887.1	663317	919

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
920	846400	778688000	30.3315	9.7259	2890.3	664761	920
921	848241	781229961	30.3480	9.7294	2893.4	666207	921
922	850084	783777448	30.3645	9.7329	2896.5	667654	922
923	851929	786330467	30.3809	9.7364	2899.7	669103	923
924	853776	788889024	30.3974	9.7400	2902.8	670554	924
925	855625	791453125	30.4138	9.7435	2906.0	672006	925
926	857476	794022776	30.4302	9.7470	2909.1	673460	926
927	859329	796597983	30.4467	9.7505	2912.3	674915	927
928	861184	799178752	30.4631	9.7540	2915.4	676372	928
929	863041	801765089	30.4795	9.7575	2918.5	677831	929
930	864900	804357000	30.4959	9.7610	2921.7	679291	930
931	866761	806954491	30.5123	9.7645	2924.8	680752	931
932	868624	809557568	30.5287	9.7680	2928.0	682216	932
933	870489	812166237	30.5450	9.7715	2931.1	683680	933
934	872356	814780504	30.5614	9.7750	2934.2	685147	934
935	874225	817400375	30.5778	9.7785	2937.4	686615	935
936	876096	820025856	30.5941	9.7819	2940.5	688084	936
937	877969	822656953	30.6105	9.7854	2943.7	689555	937
938	879844	825293672	30.6268	9.7889	2946.8	691028	938
939	881721	827936019	30.6431	9.7924	2950.0	692502	939
940	883600	830584000	30.6594	9.7959	2953.1	693978	940
941	885481	833237621	30.6757	9.7993	2956.2	695455	941
942	887364	835896888	30.6920	9.8028	2959.4	696934	942
943	889249	838561807	30.7083	9.8063	2962.5	698415	943
944	891136	841232384	30.7246	9.8097	2965.7	699897	944
945	893025	843908625	30.7409	9.8132	2968.8	701380	945
946	894916	846590536	30.7571	9.8167	2971.9	702865	946
947	896809	849278123	30.7734	9.8201	2975.1	704352	947
948	898704	851971392	30.7896	9.8236	2978.2	705840	948
949	900601	854670349	30.8058	9.8270	2981.4	707330	949
950	902500	857375000	30.8221	9.8305	2984.5	708822	950
951	904401	860085351	30.8383	9.8339	2987.7	710315	951
952	906304	862801408	30.8545	9.8374	2990.8	711809	952
953	908209	865523177	30.8707	9.8408	2993.9	713306	953
954	910116	868250664	30.8869	9.8443	2997.1	714803	954
955	912025	870983875	30.9031	9.8477	3000.2	716303	955
956	913936	873722816	30.9192	9.8511	3003.4	717804	956
957	915849	876467493	30.9354	9.8546	3006.5	719306	957
958	917764	879217912	30.9516	9.8580	3009.6	720810	958
959	919681	881974079	30.9677	9.8614	3012.8	722316	959

TABLE I

POWERS, ROOTS, CIRCUMFERENCES, AND AREAS

No.	Square	Cube	Square Root	Cube Root	No. = Diam.		No.
					Circum.	Area	
960	921600	884736000	30.9839	9.8648	3015.9	723823	960
961	923521	887503681	31.0000	9.8683	3019.1	725332	961
962	925444	890277128	31.0161	9.8717	3022.2	726842	962
963	927369	893056347	31.0322	9.8751	3025.4	728354	963
964	929296	895841344	31.0483	9.8785	3028.5	729867	964
965	931225	898632125	31.0644	9.8819	3031.6	731382	965
966	933156	901428696	31.0805	9.8854	3034.8	732899	966
967	935089	904231063	31.0966	9.8888	3037.9	734417	967
968	937024	907039232	31.1127	9.8922	3041.1	735937	968
969	938961	909853209	31.1288	9.8956	3044.2	737458	969
970	940900	912673000	31.1448	9.8990	3047.3	738981	970
971	942841	915498611	31.1609	9.9024	3050.5	740506	971
972	944784	918330048	31.1769	9.9058	3053.6	742032	972
973	946729	921167317	31.1929	9.9092	3056.8	743559	973
974	948676	924010424	31.2090	9.9126	3059.9	745088	974
975	950625	926859375	31.2250	9.9160	3063.1	746619	975
976	952576	929714176	31.2410	9.9194	3066.2	748151	976
977	954529	932574833	31.2570	9.9227	3069.3	749685	977
978	956484	935441352	31.2730	9.9261	3072.5	751221	978
979	958441	938313739	31.2890	9.9295	3075.6	752758	979
980	960400	941192000	31.3050	9.9329	3078.8	754296	980
981	962361	944076141	31.3209	9.9363	3081.9	755837	981
982	964324	946966168	31.3369	9.9396	3085.0	757378	982
983	966289	949862087	31.3528	9.9430	3088.2	758922	983
984	968256	952763904	31.3688	9.9464	3091.3	760466	984
985	970225	955671625	31.3847	9.9497	3094.5	762013	985
986	972196	958585256	31.4006	9.9531	3097.6	763561	986
987	974169	961504803	31.4166	9.9565	3100.8	765111	987
988	976144	964430272	31.4325	9.9598	3103.9	766662	988
989	978121	967361669	31.4484	9.9632	3107.0	768214	989
990	980100	970299000	31.4643	9.9666	3110.2	769769	990
991	982081	973242271	31.4802	9.9699	3113.3	771325	991
992	984064	976191488	31.4960	9.9733	3116.5	772882	992
993	986049	979146657	31.5119	9.9766	3119.6	774441	993
994	988036	982107784	31.5278	9.9800	3122.7	776002	994
995	990025	985074875	31.5436	9.9833	3125.9	777564	995
996	992016	988047936	31.5595	9.9866	3129.0	779128	996
997	994009	991026973	31.5753	9.9900	3132.2	780693	997
998	996004	994011992	31.5911	9.9933	3135.3	782260	998
999	998001	997002999	31.6070	9.9967	3138.5	783828	999

TABLE II

Sines and Cosines, Natural

′	0°		1°		2°		3°		4°		′
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	89°		88°		87°		86°		85°		

TABLE II

Sines and Cosines, Natural

′	5°		6°		7°		8°		9°		′
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	84°		83°		82°		81°		80°		

TABLE II

Sines and Cosines, Natural

′	10°		11°		12°		13°		14°		′
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	79°		78°		77°		76°		75°		

TABLE II

Sines and Cosines, Natural

′	15°		16°		17°		18°		19°		′
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	74°		73°		72°		71°		70°		

TABLE II

Sines and Cosines, Natural

′	20°		21°		22°		23°		24°		′
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	69°		68°		67°		66°		65°		

TABLE II

Sines and Cosines, Natural

′	25°		26°		27°		28°		29°		′
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.42262	.90631	.43837	.89879	.45339	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	64°		63°		62°		61°		60°		

TABLE II

Sines and Cosines, Natural

	30°		31°		32°		33°		34°		
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	59°		58°		57°		56°		55°		

TABLE II

Sines and Cosines, Natural

′	35°		36°		37°		38°		39°		′
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	54°		53°		52°		51°		50°		

TABLE II

Sines and Cosines, Natural

′	40°		41°		42°		43°		44°		′
	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	Cos	Sin	′
	49°		48°		47°		46°		45°		

TABLE III

Tangents and Cotangents, Natural

′	0°		1°		2°		3°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	89°		88°		87°		86°		

TABLE III

Tangents and Cotangents, Natural

′	4°		5°		6°		7°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	85°		84°		83°		82°		

TABLE III

Tangents and Cotangents, Natural

′	8°		9°		10°		11°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	81°		80°		79°		78°		

TABLE III

Tangents and Cotangents, Natural

′	12°		13°		14°		15°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	77°		76°		75°		74°		

TABLE III

Tangents and Cotangents, Natural

′	16°		17°		18°		19°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	73°		72°		71°		70°		

TABLE III

Tangents and Cotangents, Natural

′	20°		21°		22°		23°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	69°		68°		67°		66°		

TABLE III

Tangents and Cotangents, Natural

′	24°		25°		26°		27°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	65°		64°		63°		62°		

TABLE III

Tangents and Cotangents, Natural

′	28°		29°		30°		31°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65554	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56501	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58905	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76629	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	61°		60°		59°		58°		

TABLE III

Tangents and Cotangents, Natural

′	32°		33°		34°		35°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65024	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72167	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	57°		56°		55°		54°		

TABLE III

Tangents and Cotangents, Natural

′	36°		37°		38°		39°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	53°		52°		51°		50°		

TABLE III

Tangents and Cotangents, Natural

′	40°		41°		42°		43°		′
	Tan	Cot	Tan	Cot	Tan	Cot	Tan	Cot	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cot	Tan	Cot	Tan	Cot	Tan	Cot	Tan	′
	49°		48°		47°		46°		

TABLE III

Tangents and Cotangents, Natural

′	44°		′	′	44°		′	′	44°		′
	Tan	Cot			Tan	Cot			Tan	Cot	
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
′	Cot	Tan	′	′	Cot	Tan	′	′	Cot	Tan	′
	45°				45°				45°		

TABLE IV

Secants and Cosecants, Natural

′	0°		1°		2°		3°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1	Infinite.	1.0001	57.299	1.0006	28.654	1.0014	19.107	60
1	1	3437.70	1.0001	56.359	1.0006	28.417	1.0014	19.002	59
2	1	1718.90	1.0002	55.450	1.0006	28.184	1.0014	18.897	58
3	1	1145.90	1.0002	54.570	1.0006	27.955	1.0014	18.794	57
4	1	859.44	1.0002	53.718	1.0006	27.730	1.0014	18.692	56
5	1	687.55	1.0002	52.891	1.0007	27.508	1.0014	18.591	55
6	1	572.96	1.0002	52.090	1.0007	27.290	1.0015	18.491	54
7	1	491.11	1.0002	51.313	1.0007	27.075	1.0015	18.393	53
8	1	429.72	1.0002	50.558	1.0007	26.864	1.0015	18.295	52
9	1	381.97	1.0002	49.826	1.0007	26.655	1.0015	18.198	51
10	1	343.77	1.0002	49.114	1.0007	26.450	1.0015	18.103	50
11	1	312.52	1.0002	48.422	1.0007	26.249	1.0015	18.008	49
12	1	286.48	1.0002	47.750	1.0007	26.050	1.0016	17.914	48
13	1	264.44	1.0002	47.096	1.0007	25.854	1.0016	17.821	47
14	1	245.55	1.0002	46.460	1.0008	25.661	1.0016	17.730	46
15	1	229.18	1.0002	45.840	1.0008	25.471	1.0016	17.639	45
16	1	214.86	1.0002	45.237	1.0008	25.284	1.0016	17.549	44
17	1	202.22	1.0002	44.650	1.0008	25.100	1.0016	17.460	43
18	1	190.99	1.0002	44.077	1.0008	24.918	1.0017	17.372	42
19	1	180.73	1.0003	43.520	1.0008	24.739	1.0017	17.285	41
20	1	171.89	1.0003	42.976	1.0008	24.562	1.0017	17.198	40
21	1	163.70	1.0003	42.445	1.0008	24.358	1.0017	17.113	39
22	1	156.26	1.0003	41.928	1.0008	24.216	1.0017	17.028	38
23	1	149.47	1.0003	41.423	1.0009	24.047	1.0017	16.944	37
24	1	143.24	1.0003	40.930	1.0009	23.880	1.0018	16.861	36
25	1	137.51	1.0003	40.448	1.0009	23.716	1.0018	16.779	35
26	1	132.22	1.0003	39.978	1.0009	23.553	1.0018	16.698	34
27	1	127.32	1.0003	39.518	1.0009	23.393	1.0018	16.617	33
28	1	122.78	1.0003	39.069	1.0009	23.235	1.0018	16.538	32
29	1	118.54	1.0003	38.631	1.0009	23.079	1.0018	16.459	31
30	1	114.59	1.0003	38.201	1.0009	22.925	1.0019	16.380	30
31	1	110.90	1.0003	37.782	1.0010	22.774	1.0019	16.303	29
32	1	107.43	1.0003	37.371	1.0010	22.624	1.0019	16.226	28
33	1	104.17	1.0004	36.969	1.0010	22.476	1.0019	16.150	27
34	1	101.11	1.0004	36.576	1.0010	22.330	1.0019	16.075	26
35	1	95.223	1.0004	36.191	1.0010	22.186	1.0019	16.000	25
36	1	95.405	1.0004	35.814	1.0010	22.044	1.0020	15.926	24
37	1	92.914	1.0004	35.445	1.0010	21.904	1.0020	15.853	23
38	1.0001	92.469	1.0004	35.084	1.0010	21.765	1.0020	15.780	22
39	1.0001	88.149	1.0004	34.729	1.0011	21.629	1.0020	15.708	21
40	1.0001	85.946	1.0004	34.382	1.0011	21.494	1.0020	15.637	20
41	1.0001	83.849	1.0004	34.042	1.0011	21.360	1.0021	15.566	19
42	1.0001	81.853	1.0004	33.708	1.0011	21.228	1.0021	15.496	18
43	1.0001	79.950	1.0004	33.381	1.0011	21.098	1.0021	15.427	17
44	1.0001	78.133	1.0004	33.060	1.0011	20.970	1.0021	15.358	16
45	1.0001	76.396	1.0005	32.745	1.0011	20.843	1.0021	15.290	15
46	1.0001	74.736	1.0005	32.437	1.0012	20.717	1.0022	15.222	14
47	1.0001	73.146	1.0005	32.134	1.0012	20.593	1.0022	15.155	13
48	1.0001	71.622	1.0005	31.836	1.0012	20.471	1.0022	15.089	12
49	1.0001	71.160	1.0005	31.544	1.0012	20.350	1.0022	15.023	11
50	1.0001	68.757	1.0005	31.257	1.0012	20.230	1.0022	14.958	10
51	1.0001	67.409	1.0005	30.976	1.0012	20.112	1.0023	14.893	9
52	1.0001	66.113	1.0005	30.699	1.0012	19.995	1.0023	14.829	8
53	1.0001	64.866	1.0005	30.428	1.0013	19.880	1.0023	14.765	7
54	1.0001	63.664	1.0005	30.161	1.0013	19.766	1.0023	14.702	6
55	1.0001	62.507	1.0005	29.899	1.0013	19.653	1.0023	14.640	5
56	1.0001	61.391	1.0006	29.641	1.0013	19.541	1.0024	14.578	4
57	1.0001	61.314	1.0006	29.388	1.0013	19.431	1.0024	14.517	3
58	1.0001	59.274	1.0006	29.139	1.0013	19.322	1.0024	14.456	2
59	1.0001	58.270	1.0006	28.894	1.0013	19.214	1.0024	14.395	1
60	1.0001	57.299	1.0006	28.654	1.0014	19.107	1.0024	14.335	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	89°		88°		87°		86°		

TABLE IV

Secants and Cosecants, Natural

′	4°		5°		6°		7°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.0024	14.335	1.0038	11.474	1.0055	9.5668	1.0075	8.2055	60
1	1.0025	14.276	1.0038	11.436	1.0055	9.5404	1.0075	8.1861	59
2	1.0025	14.217	1.0039	11.398	1.0056	9.5141	1.0076	8.1668	58
3	1.0025	14.159	1.0039	11.360	1.0056	9.4880	1.0076	8.1476	57
4	1.0025	14.101	1.0039	11.323	1.0056	9.4620	1.0076	8.1285	56
5	1.0025	14.043	1.0039	11.286	1.0057	9.4362	1.0077	8.1094	55
6	1.0026	13.986	1.0040	11.249	1.0057	9.4105	1.0077	8.0905	54
7	1.0026	13.930	1.0040	11.213	1.0057	9.3850	1.0078	8.0717	53
8	1.0026	13.874	1.0040	11.176	1.0057	9.3596	1.0078	8.0529	52
9	1.0026	13.818	1.0040	11.140	1.0058	9.3343	1.0078	8.0342	51
10	1.0026	13.763	1.0041	11.104	1.0058	9.3092	1.0079	8.0156	50
11	1.0027	13.708	1.0041	11.069	1.0058	9.2842	1.0079	7.9971	49
12	1.0027	13.654	1.0041	11.033	1.0059	9.2593	1.0079	7.9787	48
13	1.0027	13.600	1.0041	10.988	1.0059	9.2346	1.0080	7.9604	47
14	1.0027	13.547	1.0042	10.963	1.0059	9.2100	1.0080	7.9421	46
15	1.0027	13.494	1.0042	10.929	1.0060	9.1855	1.0080	7.9240	45
16	1.0028	13.441	1.0042	10.894	1.0060	9.1612	1.0081	7.9059	44
17	1.0028	13.389	1.0043	10.860	1.0060	9.1370	1.0081	7.8879	43
18	1.0028	13.337	1.0043	10.826	1.0061	9.1129	1.0082	7.8700	42
19	1.0028	13.286	1.0043	10.792	1.0061	9.0890	1.0082	7.8522	41
20	1.0029	13.235	1.0043	10.758	1.0061	9.0651	1.0082	7.8344	40
21	1.0029	13.184	1.0044	10.725	1.0062	9.0414	1.0083	7.8168	39
22	1.0029	13.134	1.0044	10.692	1.0062	9.0179	1.0083	7.7992	38
23	1.0029	13.084	1.0044	10.659	1.0062	8.9944	1.0084	7.7817	37
24	1.0029	13.034	1.0044	10.626	1.0063	8.9711	1.0084	7.7642	36
25	1.0030	12.985	1.0045	10.593	1.0063	8.9479	1.0084	7.7469	35
26	1.0030	12.937	1.0045	10.561	1.0063	8.9248	1.0085	7.7296	34
27	1.0030	12.888	1.0045	10.529	1.0064	8.9018	1.0085	7.7124	33
28	1.0030	12.840	1.0046	10.497	1.0064	8.8790	1.0085	7.6953	32
29	1.0031	12.793	1.0046	10.465	1.0064	8.8563	1.0086	7.6783	31
30	1.0031	12.745	1.0046	10.433	1.0065	8.8337	1.0086	7.6613	30
31	1.0031	12.698	1.0046	10.402	1.0065	8.8112	1.0087	7.6444	29
32	1.0031	12.652	1.0047	10.371	1.0065	8.7888	1.0087	7.6276	28
33	1.0032	12.606	1.0047	10.340	1.0066	8.7665	1.0087	7.6108	27
34	1.0032	12.560	1.0047	10.309	1.0066	8.7444	1.0088	7.5942	26
35	1.0032	12.514	1.0048	10.278	1.0066	8.7223	1.0088	7.5776	25
36	1.0032	12.469	1.0048	10.248	1.0067	8.7004	1.0089	7.5611	24
37	1.0032	12.424	1.0048	10.217	1.0067	8.6786	1.0089	7.5446	23
38	1.0033	12.379	1.0048	10.187	1.0067	8.6569	1.0089	7.5282	22
39	1.0033	12.335	1.0049	10.157	1.0068	8.6353	1.0090	7.5119	21
40	1.0033	12.291	1.0049	10.127	1.0068	8.6138	1.0090	7.4957	20
41	1.0033	12.248	1.0049	10.098	1.0068	8.5924	1.0090	7.4795	19
42	1.0034	12.204	1.0050	10.068	1.0069	8.5711	1.0091	7.4634	18
43	1.0034	12.161	1.0050	10.039	1.0069	8.5499	1.0091	7.4474	17
44	1.0034	12.118	1.0050	10.010	1.0069	8.5289	1.0092	7.4315	16
45	1.0034	12.076	1.0050	9.9812	1.0070	8.5079	1.0092	7.4156	15
46	1.0035	12.034	1.0051	9.9525	1.0070	8.4871	1.0092	7.3998	14
47	1.0035	11.992	1.0051	9.9239	1.0070	8.4663	1.0093	7.3840	13
48	1.0035	11.950	1.0051	9.8955	1.0071	8.4457	1.0093	7.3683	12
49	1.0035	11.909	1.0052	9.8672	1.0071	8.4251	1.0094	7.3527	11
50	1.0036	11.868	1.0052	9.8391	1.0071	8.4046	1.0094	7.3372	10
51	1.0036	11.828	1.0052	9.8112	1.0072	8.3843	1.0094	7.3217	9
52	1.0036	11.787	1.0053	9.7834	1.0072	8.3640	1.0095	7.3063	8
53	1.0036	11.747	1.0053	9.7558	1.0073	8.3439	1.0095	7.2909	7
54	1.0037	11.707	1.0053	9.7283	1.0073	8.3238	1.0096	7.2757	6
55	1.0037	11.668	1.0053	9.7010	1.0073	8.3039	1.0096	7.2604	5
56	1.0037	11.628	1.0054	9.6739	1.0074	8.2840	1.0097	7.2453	4
57	1.0037	11.589	1.0054	9.6469	1.0074	8.2642	1.0097	7.2302	3
58	1.0038	11.550	1.0054	9.6200	1.0074	8.2446	1.0097	7.2152	2
59	1.0038	11.512	1.0055	9.5933	1.0075	8.2250	1.0098	7.2002	1
60	1.0038	11.474	1.0055	9.5668	1.0075	8.2055	1.0098	7.1853	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	85°		84°		83°		82°		

TABLE IV

Secants and Cosecants, Natural

′	8°		9°		10°		11°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.0098	7.1853	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	60
1	1.0099	7.1704	1.0125	6.3807	1.0155	5.7493	1.0188	5.2330	59
2	1.0099	7.1557	1.0125	6.3690	1.0155	5.7398	1.0188	5.2252	58
3	1.0099	7.1409	1.0126	6.3574	1.0156	5.7304	1.0189	5.2174	57
4	1.0100	7.1263	1.0126	6.3458	1.0156	5.7210	1.0189	5.2097	56
5	1.0100	7.1117	1.0127	6.3343	1.0157	5.7117	1.0190	5.2019	55
6	1.0101	7.0972	1.0127	6.3228	1.0157	5.7023	1.0191	5.1942	54
7	1.0101	7.0827	1.0128	6.3113	1.0158	5.6930	1.0191	5.1865	53
8	1.0102	7.0683	1.0128	6.2999	1.0158	5.6838	1.0192	5.1788	52
9	1.0102	7.0539	1.0129	6.2885	1.0159	5.6745	1.0192	5.1712	51
10	1.0102	7.0396	1.0129	6.2772	1.0159	5.6653	1.0193	5.1636	50
11	1.0103	7.0254	1.0130	6.2659	1.0160	5.6561	1.0193	5.1560	49
12	1.0103	7.0112	1.0130	6.2546	1.0160	5.6470	1.0194	5.1484	48
13	1.0104	6.9971	1.0131	6.2434	1.0161	5.6379	1.0195	5.1409	47
14	1.0104	6.9830	1.0131	6.2322	1.0162	5.6288	1.0195	5.1333	46
15	1.0104	6.9690	1.0132	6.2211	1.0162	5.6197	1.0196	5.1258	45
16	1.0105	6.9550	1.0132	6.2100	1.0163	5.6107	1.0196	5.1183	44
17	1.0105	6.9411	1.0133	6.1990	1.0163	5.6017	1.0197	5.1109	43
18	1.0106	6.9273	1.0133	6.1880	1.0164	5.5928	1.0198	5.1034	42
19	1.0106	6.9135	1.0134	6.1770	1.0164	5.5838	1.0198	5.0960	41
20	1.0107	6.8998	1.0134	6.1661	1.0165	5.5749	1.0199	5.0886	40
21	1.0107	6.8861	1.0135	6.1552	1.0165	5.5660	1.0199	5.0812	39
22	1.0107	6.8725	1.0135	6.1443	1.0166	5.5572	1.0200	5.0739	38
23	1.0108	6.8589	1.0136	6.1335	1.0166	5.5484	1.0201	5.0666	37
24	1.0108	6.8454	1.0136	6.1227	1.0167	5.5396	1.0201	5.0593	36
25	1.0109	6.8320	1.0136	6.1120	1.0167	5.5308	1.0202	5.0520	35
26	1.0109	6.8185	1.0137	6.1013	1.0168	5.5221	1.0202	5.0447	34
27	1.0110	6.8052	1.0137	6.0906	1.0169	5.5134	1.0203	5.0375	33
28	1.0110	6.7919	1.0138	6.0800	1.0169	5.5047	1.0204	5.0302	32
29	1.0111	6.7787	1.0138	6.0694	1.0170	5.4960	1.0204	5.0230	31
30	1.0111	6.7655	1.0139	6.0588	1.0170	5.4874	1.0205	5.0158	30
31	1.0111	6.7523	1.0139	6.0483	1.0171	5.4788	1.0205	5.0087	29
32	1.0112	6.7392	1.0140	6.0379	1.0171	5.4702	1.0206	5.0015	28
33	1.0112	6.7262	1.0140	6.0274	1.0172	5.4617	1.0207	4.9944	27
34	1.0113	6.7132	1.0141	6.0170	1.0172	5.4532	1.0207	4.9873	26
35	1.0113	6.7003	1.0141	6.0066	1.0173	5.4447	1.0208	4.9802	25
36	1.0114	6.6874	1.0142	5.9963	1.0174	5.4362	1.0208	4.9732	24
37	1.0114	6.6745	1.0142	5.9860	1.0174	5.4278	1.0209	4.9661	23
38	1.0115	6.6617	1.0143	5.9758	1.0175	5.4194	1.0210	4.9591	22
39	1.0115	6.6490	1.0143	5.9655	1.0175	5.4110	1.0210	4.9521	21
40	1.0115	6.6363	1.0144	5.9554	1.0176	5.4026	1.0211	4.9452	20
41	1.0116	6.6237	1.0144	5.9452	1.0176	5.3943	1.0211	4.9382	19
42	1.0116	6.6111	1.0145	5.9351	1.0177	5.3860	1.0212	4.9313	18
43	1.0117	6.5985	1.0145	5.9250	1.0177	5.3777	1.0213	4.9243	17
44	1.0117	6.5860	1.0146	5.9150	1.0178	5.3695	1.0213	4.9175	16
45	1.0118	6.5736	1.0146	5.9049	1.0179	5.3612	1.0214	4.9106	15
46	1.0118	6.5612	1.0147	5.8950	1.0179	5.3530	1.0215	4.9037	14
47	1.0119	6.5488	1.0147	5.8850	1.0180	5.3449	1.0215	4.8969	13
48	1.0119	6.5365	1.0148	5.8751	1.0180	5.3367	1.0216	4.8901	12
49	1.0119	6.5243	1.0148	5.8652	1.0181	5.3286	1.0216	4.8833	11
50	1.0120	6.5121	1.0149	5.8554	1.0181	5.3205	1.0217	4.8765	10
51	1.0120	6.4999	1.0150	5.8456	1.0182	5.3124	1.0218	4.8697	9
52	1.0121	6.4878	1.0150	5.8358	1.0182	5.3044	1.0218	4.8630	8
53	1.0121	6.4757	1.0151	5.8261	1.0183	5.2963	1.0219	4.8563	7
54	1.0122	6.4637	1.0151	5.8163	1.0184	5.2883	1.0220	4.8496	6
55	1.0122	6.4517	1.0152	5.8067	1.0184	5.2803	1.0220	4.8429	5
56	1.0123	6.4398	1.0152	5.7970	1.0185	5.2724	1.0221	4.8362	4
57	1.0123	6.4279	1.0153	5.7874	1.0185	5.2645	1.0221	4.8296	3
58	1.0124	6.4160	1.0153	5.7778	1.0186	5.2566	1.0222	4.8229	2
59	1.0124	6.4042	1.0154	5.7683	1.0186	5.2487	1.0223	4.8163	1
60	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	1.0223	4.8097	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	81°		80°		79°		78°		

TABLE IV

Secants and Cosecants, Natural

′	12°		13°		14°		15°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.0223	4.8097	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637	60
1	1.0224	4.8032	1.0264	4.4398	1.0307	4.1287	1.0353	3.8595	59
2	1.0225	4.7966	1.0264	4.4342	1.0308	4.1239	1.0354	3.8553	58
3	1.0225	4.7901	1.0265	4.4287	1.0308	4.1191	1.0355	3.8512	57
4	1.0226	4.7835	1.0266	4.4231	1.0309	4.1144	1.0356	3.8470	56
5	1.0226	4.7770	1.0266	4.4176	1.0310	4.1096	1.0357	3.8428	55
6	1.0227	4.7706	1.0267	4.4121	1.0311	4.1048	1.0358	3.8387	54
7	1.0228	4.7641	1.0268	4.4065	1.0311	4.1001	1.0358	3.8346	53
8	1.0228	4.7576	1.0268	4.4011	1.0312	4.0953	1.0359	3.8304	52
9	1.0229	4.7512	1.0269	4.3956	1.0313	4.0906	1.0360	3.8263	51
10	1.0230	4.7448	1.0270	4.3910	1.0314	4.0859	1.0361	3.8222	50
11	1.0230	4.7384	1.0271	4.3847	1.0314	4.0812	1.0362	3.8181	49
12	1.0231	4.7320	1.0271	4.3792	1.0315	4.0765	1.0362	3.8140	48
13	1.0232	4.7257	1.0272	4.3738	1.0316	4.0718	1.0363	3.8100	47
14	1.0232	4.7193	1.0273	4.3684	1.0317	4.0672	1.0364	3.8059	46
15	1.0233	4.7130	1.0273	4.3630	1.0317	4.0625	1.0365	3.8018	45
16	1.0234	4.7067	1.0274	4.3576	1.0318	4.0579	1.0366	3.7978	44
17	1.0234	4.7004	1.0275	4.3522	1.0319	4.0532	1.0367	3.7937	43
18	1.0235	4.6942	1.0276	4.3469	1.0320	4.0486	1.0367	3.7897	42
19	1.0235	4.6879	1.0276	4.3415	1.0320	4.0440	1.0368	3.7857	41
20	1.0236	4.6817	1.0277	4.3362	1.0321	4.0394	1.0369	3.7816	40
21	1.0237	4.6754	1.0278	4.3309	1.0322	4.0348	1.0370	3.7776	39
22	1.0237	4.6692	1.0278	4.3256	1.0323	4.0302	1.0371	3.7736	38
23	1.0238	4.6631	1.0279	4.3203	1.0323	4.0256	1.0371	3.7697	37
24	1.0239	4.6569	1.0280	4.3150	1.0324	4.0211	1.0372	3.7657	36
25	1.0239	4.6507	1.0280	4.3098	1.0325	4.0165	1.0373	3.7617	35
26	1.0240	4.6446	1.0281	4.3045	1.0326	4.0120	1.0374	3.7577	34
27	1.0241	4.6385	1.0282	4.2993	1.0327	4.0074	1.0375	3.7538	33
28	1.0241	4.6324	1.0283	4.2941	1.0327	4.0029	1.0376	3.7498	32
29	1.0242	4.6263	1.0283	4.2888	1.0328	3.9984	1.0376	3.7459	31
30	1.0243	4.6202	1.0284	4.2836	1.0329	3.9939	1.0377	3.7420	30
31	1.0243	4.6142	1.0285	4.2785	1.0330	3.9894	1.0378	3.7380	29
32	1.0244	4.6081	1.0285	4.2733	1.0330	3.9850	1.0379	3.7341	28
33	1.0245	4.6021	1.0286	4.2681	1.0331	3.9805	1.0380	3.7302	27
34	1.0245	4.5961	1.0287	4.2630	1.0332	3.9760	1.0381	3.7263	26
35	1.0246	4.5901	1.0288	4.2579	1.0333	3.9716	1.0382	3.7224	25
36	1.0247	4.5841	1.0288	4.2527	1.0334	3.9672	1.0382	3.7186	24
37	1.0247	4.5782	1.0289	4.2476	1.0334	3.9627	1.0383	3.7147	23
38	1.0248	4.5722	1.0290	4.2425	1.0335	3.9583	1.0384	3.7108	22
39	1.0249	4.5663	1.0291	4.2375	1.0336	3.9539	1.0385	3.7070	21
40	1.0249	4.5604	1.0291	4.2324	1.0337	3.9495	1.0386	3.7031	20
41	1.0250	4.5545	1.0292	4.2273	1.0338	3.9451	1.0387	3.6993	19
42	1.0251	4.5486	1.0293	4.2223	1.0338	3.9408	1.0387	3.6955	18
43	1.0251	4.5428	1.0293	4.2173	1.0339	3.9364	1.0388	3.6917	17
44	1.0252	4.5369	1.0294	4.2122	1.0340	3.9320	1.0389	3.6878	16
45	1.0253	4.5311	1.0295	4.2072	1.0341	3.9277	1.0390	3.6840	15
46	1.0253	4.5253	1.0296	4.2022	1.0341	3.9234	1.0391	3.6802	14
47	1.0254	4.5195	1.0296	4.1972	1.0342	3.9199	1.0392	3.6765	13
48	1.0255	4.5137	1.0297	4.1923	1.0343	3.9147	1.0393	3.6727	12
49	1.0255	4.5079	1.0298	4.1873	1.0344	3.9104	1.0393	3.6689	11
50	1.0256	4.5021	1.0299	4.1824	1.0345	3.9061	1.0394	3.6651	10
51	1.0257	4.4964	1.0299	4.1774	1.0345	3.9018	1.0395	3.6614	9
52	1.0257	4.4907	1.0300	4.1725	1.0346	3.8976	1.0396	3.6576	8
53	1.0258	4.4850	1.0301	4.1676	1.0347	3.8933	1.0397	3.6539	7
54	1.0259	4.4793	1.0302	4.1627	1.0348	3.8990	1.0398	3.6502	6
55	1.0260	4.4736	1.0302	4.1578	1.0349	3.8848	1.0399	3.6464	5
56	1.0260	4.4679	1.0303	4.1529	1.0349	3.8805	1.0399	3.6427	4
57	1.0261	4.4623	1.0304	4.1481	1.0350	3.8763	1.0400	3.6390	3
58	1.0262	4.4566	1.0305	4.1432	1.0351	3.8721	1.0401	3.6353	2
59	1.0262	4.4510	1.0305	4.1384	1.0352	3.8679	1.0402	3.6316	1
60	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637	1.0403	3.6279	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	77°		76°		75°		74°		

TABLE IV

Secants and Cosecants, Natural

′	16°		17°		18°		19°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.0403	3.6279	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	60
1	1.0404	3.6243	1.0458	3.4170	1.0516	3.2332	1.0577	3.0690	59
2	1.0405	3.6206	1.0459	3.4138	1.0517	3.2303	1.0578	3.0664	58
3	1.0406	3.6169	1.0460	3.4106	1.0518	3.2274	1.0579	3.0638	57
4	1.0406	3.6133	1.0461	3.4073	1.0519	3.2245	1.0580	3.0612	56
5	1.0407	3.6096	1.0461	3.4041	1.0520	3.2216	1.0581	3.0586	55
6	1.0408	3.6060	1.0462	3.4009	1.0521	3.2188	1.0582	3.0561	54
7	1.0409	3.6024	1.0463	3.3977	1.0522	3.2159	1.0584	3.0535	53
8	1.0410	3.5987	1.0464	3.3945	1.0523	3.2131	1.0585	3.0509	52
9	1.0411	3.5951	1.0465	3.3913	1.0524	3.2102	1.0586	3.0484	51
10	1.0412	3.5915	1.0466	3.3881	1.0525	3.2074	1.0587	3.0458	50
11	1.0413	3.5879	1.0467	3.3849	1.0526	3.2045	1.0588	3.0433	49
12	1.0413	3.5843	1.0468	3.3817	1.0527	3.2017	1.0589	3.0407	48
13	1.0414	3.5807	1.0469	3.3785	1.0528	3.1989	1.0590	3.0382	47
14	1.0415	3.5772	1.0470	3.3754	1.0529	3.1960	1.0591	3.0357	46
15	1.0416	3.5736	1.0471	3.3722	1.0530	3.1932	1.0592	3.0331	45
16	1.0417	3.5700	1.0472	3.3690	1.0531	3.1904	1.0593	3.0306	44
17	1.0418	3.5665	1.0473	3.3659	1.0532	3.1876	1.0594	3.0281	43
18	1.0419	3.5629	1.0474	3.3627	1.0533	3.1848	1.0595	3.0256	42
19	1.0420	3.5594	1.0475	3.3596	1.0534	3.1820	1.0596	3.0231	41
20	1.0420	3.5559	1.0476	3.3565	1.0535	3.1792	1.0598	3.0206	40
21	1.0421	3.5523	1.0477	3.3534	1.0536	3.1764	1.0599	3.0181	39
22	1.0422	3.5488	1.0478	3.3502	1.0537	3.1736	1.0600	3.0156	38
23	1.0423	3.5453	1.0478	3.3471	1.0538	3.1708	1.0601	3.0131	37
24	1.0424	3.5418	1.0479	3.3440	1.0539	3.1681	1.0602	3.0106	36
25	1.0425	3.5383	1.0480	3.3409	1.0540	3.1653	1.0603	3.0081	35
26	1.0426	3.5348	1.0481	3.3378	1.0541	3.1625	1.0604	3.0056	34
27	1.0427	3.5313	1.0482	3.3347	1.0542	3.1598	1.0605	3.0031	33
28	1.0428	3.5279	1.0483	3.3316	1.0543	3.1570	1.0606	3.0007	32
29	1.0428	3.5244	1.0484	3.3286	1.0544	3.1543	1.0607	2.9982	31
30	1.0429	3.5209	1.0485	3.3255	1.0545	3.1515	1.0608	2.9957	30
31	1.0430	3.5175	1.0486	3.3224	1.0546	3.1488	1.0609	2.9933	29
32	1.0431	3.5140	1.0487	3.3194	1.0547	3.1461	1.0611	2.9908	28
33	1.0432	3.5106	1.0488	3.3163	1.0548	3.1433	1.0612	2.9884	27
34	1.0433	3.5072	1.0489	3.3133	1.0549	3.1406	1.0613	2.9859	26
35	1.0434	3.5037	1.0490	3.3102	1.0550	3.1379	1.0614	2.9835	25
36	1.0435	3.5003	1.0491	3.3072	1.0551	3.1352	1.0615	2.9810	24
37	1.0436	3.4969	1.0492	3.3042	1.0552	3.1325	1.0616	2.9786	23
38	1.0437	3.4935	1.0493	3.3011	1.0553	3.1298	1.0617	2.9762	22
39	1.0438	3.4901	1.0494	3.2981	1.0554	3.1271	1.0618	2.9738	21
40	1.0438	3.4867	1.0495	3.2951	1.0555	3.1244	1.0619	2.9713	20
41	1.0439	3.4833	1.0496	3.2921	1.0556	3.1217	1.0620	2.9689	19
42	1.0440	3.4799	1.0497	3.2891	1.0557	3.1190	1.0622	2.9665	18
43	1.0441	3.4766	1.0498	3.2861	1.0558	3.1163	1.0623	2.9641	17
44	1.0442	3.4732	1.0499	3.2831	1.0559	3.1137	1.0624	2.9617	16
45	1.0443	3.4698	1.0500	3.2801	1.0560	3.1110	1.0625	2.9593	15
46	1.0444	3.4665	1.0501	3.2772	1.0561	3.1083	1.0626	2.9569	14
47	1.0445	3.4632	1.0502	3.2742	1.0562	3.1057	1.0627	2.9545	13
48	1.0446	3.4598	1.0503	3.2712	1.0563	3.1030	1.0628	2.9521	12
49	1.0447	3.4565	1.0504	3.2683	1.0565	3.1004	1.0629	2.9497	11
50	1.0448	3.4532	1.0505	3.2653	1.0566	3.0977	1.0630	2.9474	10
51	1.0448	3.4498	1.0506	3.2624	1.0567	3.0951	1.0632	2.9450	9
52	1.0449	3.4465	1.0507	3.2594	1.0568	3.0925	1.0633	2.9426	8
53	1.0450	3.4432	1.0508	3.2565	1.0569	3.0898	1.0634	2.9402	7
54	1.0451	3.4399	1.0509	3.2535	1.0570	3.0872	1.0635	2.9379	6
55	1.0452	3.4366	1.0510	3.2506	1.0571	3.0846	1.0636	2.9355	5
56	1.0453	3.4334	1.0511	3.2477	1.0572	3.0820	1.0637	2.9332	4
57	1.0454	3.4301	1.0512	3.2448	1.0573	3.0793	1.0638	2.9308	3
58	1.0455	3.4268	1.0513	3.2419	1.0574	3.0767	1.0639	2.9285	2
59	1.0456	3.4236	1.0514	3.2390	1.0575	3.0741	1.0641	2.9261	1
60	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	1.0642	2.9238	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	73°		72°		71°		70°		

TABLE IV

Secants and Cosecants, Natural

′	20°		21°		22°		23°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.0642	2.9238	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593	60
1	1.0643	2.9215	1.0713	2.7883	1.0787	2.6675	1.0865	2.5575	59
2	1.0644	2.9191	1.0714	2.7862	1.0788	2.6656	1.0866	2.5558	58
3	1.0645	2.9168	1.0715	2.7841	1.0789	2.6637	1.0868	2.5540	57
4	1.0646	2.9145	1.0716	2.7820	1.0790	2.6618	1.0869	2.5523	56
5	1.0647	2.9122	1.0717	2.7799	1.0792	2.6599	1.0870	2.5506	55
6	1.0648	2.9098	1.0719	2.7778	1.0793	2.6580	1.0872	2.5488	54
7	1.0650	2.9075	1.0720	2.7757	1.0794	2.6561	1.0873	2.5471	53
8	1.0651	2.9052	1.0721	2.7736	1.0795	2.6542	1.0874	2.5453	52
9	1.0652	2.9029	1.0722	2.7715	1.0797	2.6523	1.0876	2.5436	51
10	1.0653	2.9006	1.0723	2.7694	1.0798	2.6504	1.0877	2.5419	50
11	1.0654	2.8983	1.0725	2.7674	1.0799	2.6485	1.0878	2.5402	49
12	1.0655	2.8960	1.0726	2.7653	1.0801	2.6466	1.0880	2.5384	48
13	1.0656	2.8937	1.0727	2.7632	1.0802	2.6447	1.0881	2.5367	47
14	1.0658	2.8915	1.0728	2.7611	1.0803	2.6428	1.0882	2.5350	46
15	1.0659	2.8892	1.0729	2.7591	1.0804	2.6410	1.0884	2.5333	45
16	1.0660	2.8869	1.0731	2.7570	1.0806	2.6391	1.0885	2.5316	44
17	1.0661	2.8846	1.0732	2.7550	1.0807	2.6372	1.0886	2.5299	43
18	1.0662	2.8824	1.0733	2.7529	1.0808	2.6353	1.0888	2.5281	42
19	1.0663	2.8801	1.0734	2.7509	1.0810	2.6335	1.0889	2.5264	41
20	1.0664	2.8778	1.0736	2.7488	1.0811	2.6316	1.0891	2.5247	40
21	1.0666	2.8756	1.0737	2.7468	1.0812	2.6297	1.0892	2.5230	39
22	1.0667	2.8733	1.0738	2.7447	1.0813	2.6279	1.0893	2.5213	38
23	1.0668	2.8711	1.0739	2.7427	1.0815	2.6260	1.0895	2.5196	37
24	1.0669	2.8688	1.0740	2.7406	1.0816	2.6242	1.0896	2.5179	36
25	1.0670	2.8666	1.0742	2.7386	1.0817	2.6223	1.0897	2.5163	35
26	1.0671	2.8644	1.0743	2.7366	1.0819	2.6205	1.0899	2.5146	34
27	1.0673	2.8621	1.0744	2.7346	1.0820	2.6186	1.0900	2.5129	33
28	1.0674	2.8599	1.0745	2.7325	1.0821	2.6168	1.0902	2.5112	32
29	1.0675	2.8577	1.0747	2.7305	1.0823	2.6150	1.0903	2.5095	31
30	1.0676	2.8554	1.0748	2.7285	1.0824	2.6131	1.0904	2.5078	30
31	1.0677	2.8532	1.0749	2.7265	1.0825	2.6113	1.0906	2.5062	29
32	1.0678	2.8510	1.0750	2.7245	1.0826	2.6095	1.0907	2.5045	28
33	1.0679	2.8488	1.0751	2.7225	1.0828	2.6076	1.0908	2.5028	27
34	1.0681	2.8466	1.0753	2.7205	1.0829	2.6058	1.0910	2.5011	26
35	1.0682	2.8444	1.0754	2.7185	1.0830	2.6040	1.0911	2.4995	25
36	1.0683	2.8422	1.0755	2.7165	1.0832	2.6022	1.0913	2.4978	24
37	1.0684	2.8400	1.0756	2.7145	1.0833	2.6003	1.0914	2.4961	23
38	1.0685	2.8378	1.0758	2.7125	1.0834	2.5985	1.0915	2.4945	22
39	1.0686	2.8356	1.0759	2.7105	1.0836	2.5967	1.0917	2.4928	21
40	1.0688	2.8334	1.0760	2.7085	1.0837	2.5949	1.0918	2.4912	20
41	1.0689	2.8312	1.0761	2.7065	1.0838	2.5931	1.0920	2.4895	19
42	1.0690	2.8290	1.0763	2.7045	1.0840	2.5913	1.0921	2.4879	18
43	1.0691	2.8269	1.0764	2.7026	1.0841	2.5895	1.0922	2.4862	17
44	1.0692	2.8247	1.0765	2.7006	1.0842	2.5877	1.0924	2.4846	16
45	1.0694	2.8225	1.0766	2.6986	1.0844	2.5859	1.0925	2.4829	15
46	1.0695	2.8204	1.0768	2.6967	1.0845	2.5841	1.0927	2.4813	14
47	1.0696	2.8182	1.0769	2.6947	1.0846	2.5823	1.0928	2.4797	13
48	1.0697	2.8160	1.0770	2.6927	1.0847	2.5805	1.0929	2.4780	12
49	1.0698	2.8139	1.0771	2.6908	1.0849	2.5787	1.0931	2.4764	11
50	1.0699	2.8117	1.0773	2.6888	1.0850	2.5770	1.0932	2.4748	10
51	1.0701	2.8096	1.0774	2.6869	1.0851	2.5752	1.0934	2.4731	9
52	1.0702	2.8074	1.0775	2.6849	1.0853	2.5734	1.0935	2.4715	8
53	1.0703	2.8053	1.0776	2.6830	1.0854	2.5716	1.0936	2.4699	7
54	1.0704	2.8032	1.0778	2.6810	1.0855	2.5699	1.0938	2.4683	6
55	1.0705	2.8010	1.0779	2.6791	1.0857	2.5681	1.0939	2.4666	5
56	1.0707	2.7989	1.0780	2.6772	1.0858	2.5663	1.0941	2.4650	4
57	1.0708	2.7968	1.0781	2.6752	1.0859	2.5646	1.0942	2.4634	3
58	1.0709	2.7947	1.0783	2.6733	1.0861	2.5628	1.0943	2.4618	2
59	1.0710	2.7925	1.0784	2.6714	1.0862	2.5610	1.0945	2.4602	1
60	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593	1.0946	2.4586	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	69°		68°		67°		66°		

TABLE IV

Secants and Cosecants, Natural

′	24°		25°		26°		27°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.0946	2.4586	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	60
1	1.0948	2.4570	1.1035	2.3647	1.1127	2.2798	1.1225	2.2014	59
2	1.0949	2.4554	1.1037	2.3632	1.1129	2.2784	1.1226	2.2002	58
3	1.0951	2.4538	1.1038	2.3618	1.1131	2.2771	1.1228	2.1989	57
4	1.0952	2.4522	1.1040	2.3603	1.1132	2.2757	1.1230	2.1977	56
5	1.0953	2.4506	1.1041	2.3588	1.1134	2.2744	1.1231	2.1964	55
6	1.0955	2.4490	1.1043	2.3574	1.1135	2.2730	1.1233	2.1952	54
7	1.0956	2.4474	1.1044	2.3559	1.1137	2.2717	1.1235	2.1939	53
8	1.0958	2.4458	1.1046	2.3544	1.1139	2.2703	1.1237	2.1927	52
9	1.0959	2.4442	1.1047	2.3530	1.1140	2.2690	1.1238	2.1914	51
10	1.0961	2.4426	1.1049	2.3515	1.1142	2.2676	1.1240	2.1902	50
11	1.0962	2.4411	1.1050	2.3501	1.1143	2.2663	1.1242	2.1889	49
12	1.0963	2.4395	1.1052	2.3486	1.1145	2.2650	1.1243	2.1877	48
13	1.0965	2.4379	1.1053	2.3472	1.1147	2.2636	1.1245	2.1865	47
14	1.0966	2.4363	1.1055	2.3457	1.1148	2.2623	1.1247	2.1852	46
15	1.0968	2.4347	1.1056	2.3443	1.1150	2.2610	1.1248	2.1840	45
16	1.0969	2.4332	1.1058	2.3428	1.1151	2.2596	1.1250	2.1828	44
17	1.0971	2.4316	1.1059	2.3414	1.1153	2.2583	1.1252	2.1815	43
18	1.0972	2.4300	1.1061	2.3399	1.1155	2.2570	1.1253	2.1803	42
19	1.0973	2.4285	1.1062	2.3385	1.1156	2.2556	1.1255	2.1791	41
20	1.0975	2.4269	1.1064	2.3371	1.1158	2.2543	1.1257	2.1778	40
21	1.0976	2.4254	1.1065	2.3356	1.1159	2.2530	1.1258	2.1766	39
22	1.0978	2.4238	1.1067	2.3342	1.1161	2.2517	1.1260	2.1754	38
23	1.0979	2.4222	1.1068	2.3328	1.1163	2.2503	1.1262	2.1742	37
24	1.0981	2.4207	1.1070	2.3313	1.1164	2.2490	1.1264	2.1730	36
25	1.0982	2.4191	1.1072	2.3299	1.1166	2.2477	1.1265	2.1717	35
26	1.0984	2.4176	1.1073	2.3285	1.1167	2.2464	1.1267	2.1705	34
27	1.0985	2.4160	1.1075	2.3271	1.1169	2.2451	1.1269	2.1693	33
28	1.0986	2.4145	1.1076	2.3256	1.1171	2.2438	1.1270	2.1681	32
29	1.0988	2.4130	1.1078	2.3242	1.1172	2.2425	1.1272	2.1669	31
30	1.0989	2.4114	1.1079	2.3228	1.1174	2.2411	1.1274	2.1657	30
31	1.0991	2.4099	1.1081	2.3214	1.1176	2.2398	1.1275	2.1645	29
32	1.0992	2.4083	1.1082	2.3200	1.1177	2.2385	1.1277	2.1633	28
33	1.0994	2.4068	1.1084	2.3186	1.1179	2.2372	1.1279	2.1620	27
34	1.0995	2.4053	1.1085	2.3172	1.1180	2.2359	1.1281	2.1608	26
35	1.0997	2.4037	1.1087	2.3158	1.1182	2.2346	1.1282	2.1596	25
36	1.0998	2.4022	1.1088	2.3143	1.1184	2.2333	1.1284	2.1584	24
37	1.1000	2.4007	1.1090	2.3129	1.1185	2.2320	1.1286	2.1572	23
38	1.1001	2.3992	1.1092	2.3115	1.1187	2.2307	1.1287	2.1560	22
39	1.1003	2.3976	1.1093	2.3101	1.1189	2.2294	1.1289	2.1548	21
40	1.1004	2.3961	1.1095	2.3087	1.1190	2.2282	1.1291	2.1536	20
41	1.1005	2.3946	1.1096	2.3073	1.1192	2.2269	1.1293	2.1525	19
42	1.1007	2.3931	1.1098	2.3059	1.1193	2.2256	1.1294	2.1513	18
43	1.1008	2.3916	1.1099	2.3046	1.1195	2.2243	1.1296	2.1501	17
44	1.1010	2.3901	1.1101	2.3032	1.1197	2.2230	1.1298	2.1489	16
45	1.1011	2.3886	1.1102	2.3018	1.1198	2.2217	1.1299	2.1477	15
46	1.1013	2.3871	1.1104	2.3004	1.1200	2.2204	1.1301	2.1465	14
47	1.1014	2.3856	1.1106	2.2990	1.1202	2.2192	1.1303	2.1453	13
48	1.1016	2.3841	1.1107	2.2976	1.1203	2.2179	1.1305	2.1441	12
49	1.1017	2.3826	1.1109	2.2962	1.1205	2.2166	1.1306	2.1430	11
50	1.1019	2.3811	1.1110	2.2949	1.1207	2.2153	1.1308	2.1418	10
51	1.1020	2.3796	1.1112	2.2935	1.1208	2.2141	1.1310	2.1406	9
52	1.1022	2.3781	1.1113	2.2921	1.1210	2.2128	1.1312	2.1394	8
53	1.1023	2.3766	1.1115	2.2907	1.1212	2.2115	1.1313	2.1382	7
54	1.1025	2.3751	1.1116	2.2894	1.1213	2.2103	1.1315	2.1371	6
55	1.1026	2.3736	1.1118	2.2880	1.1215	2.2090	1.1317	2.1359	5
56	1.1028	2.3721	1.1120	2.2866	1.1217	2.2077	1.1319	2.1347	4
57	1.1029	2.3706	1.1121	2.2853	1.1218	2.2065	1.1320	2.1335	3
58	1.1031	2.3691	1.1123	2.2839	1.1220	2.2052	1.1322	2.1324	2
59	1.1032	2.3677	1.1124	2.2825	1.1222	2.2039	1.1324	2.1312	1
60	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	1.1326	2.1300	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	65°		64°		63°		62°		

TABLE IV

Secants and Cosecants, Natural

′	28°		29°		30°		31°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.1326	2.1300	1.1433	2.0627	1.1547	2.0000	1.1666	1.9416	60
1	1.1327	2.1289	1.1435	2.0616	1.1549	1.9990	1.1668	1.9407	59
2	1.1329	2.1277	1.1437	2.0605	1.1551	1.9980	1.1670	1.9397	58
3	1.1331	2.1266	1.1439	2.0594	1.1553	1.9970	1.1672	1.9388	57
4	1.1333	2.1254	1.1441	2.0583	1.1555	1.9960	1.1674	1.9378	56
5	1.1334	2.1242	1.1443	2.0573	1.1557	1.9950	1.1676	1.9369	55
6	1.1336	2.1231	1.1445	2.0562	1.1559	1.9940	1.1678	1.9360	54
7	1.1338	2.1219	1.1446	2.0551	1.1561	1.9930	1.1681	1.9350	53
8	1.1340	2.1208	1.1448	2.0540	1.1562	1.9920	1.1683	1.9341	52
9	1.1341	2.1196	1.1450	2.0530	1.1564	1.9910	1.1685	1.9332	51
10	1.1343	2.1185	1.1452	2.0519	1.1566	1.9900	1.1687	1.9322	50
11	1.1345	2.1173	1.1454	2.0508	1.1568	1.9890	1.1689	1.9313	49
12	1.1347	2.1162	1.1456	2.0498	1.1570	1.9880	1.1691	1.9304	48
13	1.1349	2.1150	1.1458	2.0487	1.1572	1.9870	1.1693	1.9295	47
14	1.1350	2.1139	1.1459	2.0476	1.1574	1.9860	1.1695	1.9285	46
15	1.1352	2.1127	1.1461	2.0466	1.1576	1.9850	1.1697	1.9276	45
16	1.1354	2.1116	1.1463	2.0455	1.1578	1.9840	1.1699	1.9267	44
17	1.1356	2.1104	1.1465	2.0444	1.1580	1.9830	1.1701	1.9258	43
18	1.1357	2.1093	1.1467	2.0434	1.1582	1.9820	1.1703	1.9248	42
19	1.1359	2.1082	1.1469	2.0423	1.1584	1.9811	1.1705	1.9239	41
20	1.1361	2.1070	1.1471	2.0413	1.1586	1.9801	1.1707	1.9230	40
21	1.1363	2.1059	1.1473	2.0402	1.1588	1.9791	1.1709	1.9221	39
22	1.1365	2.1048	1.1474	2.0392	1.1590	1.9781	1.1712	1.9212	38
23	1.1366	2.1036	1.1476	2.0381	1.1592	1.9771	1.1714	1.9203	37
24	1.1368	2.1025	1.1478	2.0370	1.1594	1.9761	1.1716	1.9193	36
25	1.1370	2.1014	1.1480	2.0360	1.1596	1.9752	1.1718	1.9184	35
26	1.1372	2.1002	1.1482	2.0349	1.1598	1.9742	1.1720	1.9175	34
27	1.1373	2.0991	1.1484	2.0339	1.1600	1.9732	1.1722	1.9166	33
28	1.1375	2.0980	1.1486	2.0329	1.1602	1.9722	1.1724	1.9157	32
29	1.1377	2.0969	1.1488	2.0318	1.1604	1.9713	1.1726	1.9148	31
30	1.1379	2.0957	1.1489	2.0308	1.1606	1.9703	1.1728	1.9139	30
31	1.1381	2.0946	1.1491	2.0297	1.1608	1.9693	1.1730	1.9130	29
32	1.1382	2.0935	1.1493	2.0287	1.1610	1.9683	1.1732	1.9121	28
33	1.1384	2.0924	1.1495	2.0276	1.1612	1.9774	1.1734	1.9112	27
34	1.1386	2.0912	1.1497	2.0266	1.1614	1.9664	1.1737	1.9102	26
35	1.1388	2.0901	1.1499	2.0256	1.1616	1.9654	1.1739	1.9093	25
36	1.1390	2.0890	1.1501	2.0245	1.1618	1.9645	1.1741	1.9084	24
37	1.1391	2.0879	1.1503	2.0235	1.1620	1.9635	1.1743	1.9075	23
38	1.1393	2.0868	1.1505	2.0224	1.1622	1.9625	1.1745	1.9066	22
39	1.1395	2.0857	1.1507	2.0214	1.1624	1.9616	1.1747	1.9057	21
40	1.1397	2.0846	1.1508	2.0204	1.1626	1.9606	1.1749	1.9048	20
41	1.1399	2.0835	1.1510	2.0194	1.1628	1.9596	1.1751	1.9039	19
42	1.1401	2.0824	1.1512	2.0183	1.1630	1.9587	1.1753	1.9030	18
43	1.1402	2.0812	1.1514	2.0173	1.1632	1.9577	1.1756	1.9021	17
44	1.1404	2.0801	1.1516	2.0163	1.1634	1.9568	1.1758	1.9013	16
45	1.1406	2.0790	1.1518	2.0152	1.1636	1.9558	1.1760	1.9004	15
46	1.1408	2.0779	1.1520	2.0142	1.1638	1.9549	1.1762	1.8995	14
47	1.1410	2.0768	1.1522	2.0132	1.1640	1.9539	1.1764	1.8986	13
48	1.1411	2.0757	1.1524	2.0122	1.1642	1.9530	1.1766	1.8977	12
49	1.1413	2.0746	1.1526	2.0111	1.1644	1.9520	1.1768	1.8968	11
50	1.1415	2.0735	1.1528	2.0101	1.1646	1.9510	1.1770	1.8959	10
51	1.1417	2.0725	1.1530	2.0091	1.1648	1.9501	1.1772	1.8950	9
52	1.1419	2.0714	1.1531	2.0081	1.1650	1.9491	1.1775	1.8941	8
53	1.1421	2.0703	1.1533	2.0071	1.1652	1.9482	1.1777	1.8932	7
54	1.1422	2.0692	1.1535	2.0061	1.1654	1.9473	1.1779	1.8924	6
55	1.1424	2.0681	1.1537	2.0050	1.1656	1.9463	1.1781	1.8915	5
56	1.1426	2.0670	1.1539	2.0040	1.1658	1.9454	1.1783	1.8906	4
57	1.1428	2.0659	1.1541	2.0030	1.1660	1.9444	1.1785	1.8897	3
58	1.1430	2.0648	1.1543	2.0020	1.1662	1.9435	1.1787	1.8888	2
59	1.1432	2.0637	1.1545	2.0010	1.1664	1.9425	1.1790	1.8879	1
60	1.1433	2.0627	1.1547	2.0000	1.1666	1.9416	1.1792	1.8871	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	61°		60°		59°		58°		

TABLE IV

Secants and Cosecants, Natural

′	32°		33°		34°		35°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.1792	1.8871	1.1924	1.8361	1.2062	1.7883	1.2208	1.7434	60
1	1.1794	1.8862	1.1926	1.8352	1.2064	1.7875	1.2210	1.7427	59
2	1.1796	1.8853	1.1928	1.8344	1.2067	1.7867	1.2213	1.7420	58
3	1.1798	1.8844	1.1930	1.8336	1.2069	1.7860	1.2215	1.7413	57
4	1.1800	1.8836	1.1933	1.8328	1.2072	1.7852	1.2218	1.7405	56
5	1.1802	1.8827	1.1935	1.8320	1.2074	1.7844	1.2220	1.7398	55
6	1.1805	1.8818	1.1937	1.8311	1.2076	1.7837	1.2223	1.7391	54
7	1.1807	1.8809	1.1939	1.8303	1.2079	1.7829	1.2225	1.7384	53
8	1.1809	1.8801	1.1942	1.8295	1.2081	1.7821	1.2228	1.7377	52
9	1.1811	1.8792	1.1944	1.8287	1.2083	1.7814	1.2230	1.7369	51
10	1.1813	1.8783	1.1946	1.8279	1.2086	1.7806	1.2233	1.7362	50
11	1.1815	1.8785	1.1948	1.8271	1.2088	1.7798	1.2235	1.7355	49
12	1.1818	1.8766	1.1951	1.8263	1.2091	1.7791	1.2238	1.7348	48
13	1.1820	1.8757	1.1953	1.8255	1.2093	1.7783	1.2240	1.7341	47
14	1.1822	1.8749	1.1955	1.8246	1.2095	1.7776	1.2243	1.7334	46
15	1.1824	1.8740	1.1958	1.8238	1.2098	1.7768	1.2245	1.7327	45
16	1.1826	1.8731	1.1960	1.8230	1.2100	1.7760	1.2248	1.7319	44
17	1.1828	1.8723	1.1962	1.8222	1.2103	1.7753	1.2250	1.7312	43
18	1.1831	1.8714	1.1964	1.8214	1.2105	1.7745	1.2253	1.7305	42
19	1.1833	1.8706	1.1967	1.8206	1.2107	1.7738	1.2255	1.7298	41
20	1.1835	1.8697	1.1969	1.8198	1.2110	1.7730	1.2258	1.7291	40
21	1.1837	1.8688	1.1971	1.8190	1.2112	1.7723	1.2260	1.7284	39
22	1.1839	1.8680	1.1974	1.8182	1.2115	1.7715	1.2263	1.7277	38
23	1.1841	1.8671	1.1976	1.8174	1.2117	1.7708	1.2265	1.7270	37
24	1.1844	1.8663	1.1978	1.8166	1.2119	1.7700	1.2268	1.7263	36
25	1.1846	1.8654	1.1980	1.8158	1.2122	1.7693	1.2270	1.7256	35
26	1.1848	1.8646	1.1983	1.8150	1.2124	1.7685	1.2273	1.7249	34
27	1.1850	1.8637	1.1985	1.8142	1.2127	1.7678	1.2276	1.7242	33
28	1.1852	1.8629	1.1987	1.8134	1.2129	1.7670	1.2278	1.7234	32
29	1.1855	1.8620	1.1990	1.8126	1.2132	1.7663	1.2281	1.7227	31
30	1.1857	1.8611	1.1992	1.8118	1.2134	1.7655	1.2283	1.7220	30
31	1.1859	1.8603	1.1994	1.8110	1.2136	1.7648	1.2286	1.7213	29
32	1.1861	1.8595	1.1997	1.8102	1.2139	1.7640	1.2288	1.7206	28
33	1.1863	1.8586	1.1999	1.8094	1.2141	1.7633	1.2291	1.7199	27
34	1.1866	1.8578	1.2001	1.8086	1.2144	1.7625	1.2293	1.7192	26
35	1.1868	1.8569	1.2004	1.8078	1.2146	1.7618	1.2296	1.7185	25
36	1.1870	1.8561	1.2006	1.8070	1.2149	1.7610	1.2298	1.7178	24
37	1.1872	1.8552	1.2008	1.8062	1.2151	1.7603	1.2301	1.7171	23
38	1.1874	1.8544	1.2010	1.8054	1.2153	1.7596	1.2304	1.7164	22
39	1.1877	1.8535	1.2013	1.8047	1.2156	1.7588	1.2306	1.7157	21
40	1.1879	1.8527	1.2015	1.8039	1.2158	1.7581	1.2309	1.7151	20
41	1.1881	1.8519	1.2017	1.8031	1.2161	1.7573	1.2311	1.7144	19
42	1.1883	1.8510	1.2020	1.8023	1.2163	1.7566	1.2314	1.7137	18
43	1.1886	1.8502	1.2022	1.8015	1.2166	1.7559	1.2316	1.7130	17
44	1.1888	1.8493	1.2024	1.8007	1.2168	1.7551	1.2319	1.7123	16
45	1.1890	1.8485	1.2027	1.7999	1.2171	1.7544	1.2322	1.7116	15
46	1.1892	1.8477	1.2029	1.7992	1.2173	1.7537	1.2324	1.7109	14
47	1.1894	1.8468	1.2031	1.7984	1.2175	1.7529	1.2327	1.7102	13
48	1.1897	1.8460	1.2034	1.7976	1.2178	1.7522	1.2329	1.7095	12
49	1.1899	1.8452	1.2036	1.7968	1.2180	1.7514	1.2332	1.7088	11
50	1.1901	1.8443	1.2039	1.7960	1.2183	1.7507	1.2335	1.7081	10
51	1.1903	1.8435	1.2041	1.7953	1.2185	1.7500	1.2337	1.7075	9
52	1.1906	1.8427	1.2043	1.7945	1.2188	1.7493	1.2340	1.7068	8
53	1.1908	1.8418	1.2046	1.7937	1.2190	1.7485	1.2342	1.7061	7
54	1.1910	1.8410	1.2048	1.7929	1.2193	1.7478	1.2345	1.7054	6
55	1.1912	1.8402	1.2050	1.7921	1.2195	1.7471	1.2348	1.7047	5
56	1.1915	1.8394	1.2053	1.7914	1.2198	1.7463	1.2350	1.7040	4
57	1.1917	1.8385	1.2055	1.7906	1.2200	1.7456	1.2353	1.7033	3
58	1.1919	1.8377	1.2057	1.7898	1.2203	1.7449	1.2355	1.7027	2
59	1.1921	1.8369	1.2060	1.7891	1.2205	1.7442	1.2358	1.7020	1
60	1.1922	1.8361	1.2062	1.7883	1.2208	1.7434	1.2361	1.7013	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	
	57°		56°		55°		54°		

TABLE IV

Secants and Cosecants, Natural

′	36°		37°		38°		39°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.2361	1.7013	1.2521	1.6616	1.2690	1.6243	1.2867	1.5890	60
1	1.2363	1.7006	1.2524	1.6610	1.2693	1.6227	1.2871	1.5884	59
2	1.2366	1.6999	1.2527	1.6603	1.2696	1.6231	1.2874	1.5879	58
3	1.2368	1.6993	1.2530	1.6597	1.2699	1.6224	1.2877	1.5873	57
4	1.2371	1.6986	1.2532	1.6591	1.2702	1.6218	1.2880	1.5867	56
5	1.2374	1.6979	1.2535	1.6584	1.2705	1.6212	1.2883	1.5862	55
6	1.2376	1.6972	1.2538	1.6578	1.2707	1.6206	1.2886	1.5856	54
7	1.2379	1.6965	1.2541	1.6572	1.2710	1.6200	1.2889	1.5850	53
8	1.2382	1.6959	1.2543	1.6565	1.2713	1.6194	1.2892	1.5845	52
9	1.2384	1.6952	1.2546	1.6559	1.2716	1.6188	1.2895	1.5839	51
10	1.2387	1.6945	1.2549	1.6552	1.2719	1.6182	1.2898	1.5833	50
11	1.2389	1.6938	1.2552	1.6546	1.2722	1.6176	1.2901	1.5828	49
12	1.2392	1.6932	1.2554	1.6540	1.2725	1.6170	1.2904	1.5822	48
13	1.2395	1.6925	1.2557	1.6533	1.2728	1.6164	1.2907	1.5816	47
14	1.2397	1.6918	1.2560	1.6527	1.2731	1.6159	1.2910	1.5811	46
15	1.2400	1.6912	1.2563	1.6521	1.2734	1.6153	1.2913	1.5805	45
16	1.2403	1.6905	1.2565	1.6514	1.2737	1.6147	1.2916	1.5799	44
17	1.2405	1.6898	1.2568	1.6508	1.2739	1.6141	1.2919	1.5794	43
18	1.2408	1.6891	1.2571	1.6502	1.2742	1.6135	1.2922	1.5788	42
19	1.2411	1.6885	1.2574	1.6496	1.2745	1.6129	1.2926	1.5783	41
20	1.2413	1.6878	1.2577	1.6489	1.2748	1.6123	1.2929	1.5777	40
21	1.2416	1.6871	1.2579	1.6483	1.2751	1.6117	1.2932	1.5771	39
22	1.2419	1.6865	1.2582	1.6477	1.2754	1.6111	1.2935	1.5766	38
23	1.2421	1.6858	1.2585	1.6470	1.2757	1.6105	1.2938	1.5760	37
24	1.2424	1.6851	1.2588	1.6464	1.2760	1.6099	1.2941	1.5755	36
25	1.2427	1.6845	1.2591	1.6458	1.2763	1.6093	1.2944	1.5749	35
26	1.2429	1.6838	1.2593	1.6452	1.2766	1.6087	1.2947	1.5743	34
27	1.2432	1.6831	1.2596	1.6445	1.2769	1.6081	1.2950	1.5738	33
28	1.2435	1.6825	1.2599	1.6439	1.2772	1.6077	1.2953	1.5732	32
29	1.2437	1.6818	1.2602	1.6433	1.2775	1.6070	1.2956	1.5727	31
30	1.2440	1.6812	1.2605	1.6427	1.2778	1.6064	1.2960	1.5721	30
31	1.2443	1.6805	1.2607	1.6420	1.2781	1.6058	1.2963	1.5716	29
32	1.2445	1.6798	1.2610	1.6414	1.2784	1.6052	1.2966	1.5710	28
33	1.2448	1.6792	1.2613	1.6408	1.2787	1.6046	1.2969	1.5705	27
34	1.2451	1.6785	1.2616	1.6402	1.2790	1.6040	1.2972	1.5699	26
35	1.2453	1.6779	1.2619	1.6396	1.2793	1.6034	1.2975	1.5694	25
36	1.2456	1.6772	1.2622	1.6389	1.2795	1.6029	1.2978	1.5688	24
37	1.2459	1.6766	1.2624	1.6383	1.2798	1.6023	1.2981	1.5683	23
38	1.2461	1.6759	1.2627	1.6377	1.2801	1.6017	1.2985	1.5677	22
39	1.2464	1.6752	1.2630	1.6371	1.2804	1.6011	1.2988	1.5672	21
40	1.2467	1.6746	1.2633	1.6365	1.2807	1.6005	1.2991	1.5666	20
41	1.2470	1.6739	1.2636	1.6359	1.2810	1.6000	1.2994	1.5661	19
42	1.2472	1.6733	1.2639	1.6352	1.2813	1.5994	1.2997	1.5655	18
43	1.2475	1.6726	1.2641	1.6346	1.2816	1.5988	1.3000	1.5650	17
44	1.2478	1.6720	1.2644	1.6340	1.2819	1.5982	1.3003	1.5644	16
45	1.2480	1.6713	1.2647	1.6334	1.2822	1.5976	1.3006	1.5639	15
46	1.2483	1.6707	1.2650	1.6328	1.2825	1.5971	1.3010	1.5633	14
47	1.2486	1.6700	1.2653	1.6322	1.2828	1.5965	1.3013	1.5628	13
48	1.2488	1.6694	1.2656	1.6316	1.2831	1.5959	1.3016	1.5622	12
49	1.2490	1.6687	1.2659	1.6309	1.2834	1.5953	1.3019	1.5617	11
50	1.2494	1.6681	1.2661	1.6303	1.2837	1.5947	1.3022	1.5611	10
51	1.2497	1.6674	1.2664	1.6297	1.2840	1.5942	1.3025	1.5606	9
52	1.2499	1.6668	1.2667	1.6291	1.2843	1.5936	1.3029	1.5600	8
53	1.2502	1.6661	1.2670	1.6285	1.2846	1.5930	1.3032	1.5595	7
54	1.2505	1.6655	1.2673	1.6279	1.2849	1.5924	1.3035	1.5590	6
55	1.2508	1.6648	1.2676	1.6273	1.2852	1.5919	1.3038	1.5584	5
56	1.2510	1.6642	1.2679	1.6267	1.2855	1.5913	1.3041	1.5579	4
57	1.2513	1.6636	1.2681	1.6261	1.2858	1.5907	1.3044	1.5573	3
58	1.2516	1.6629	1.2684	1.6255	1.2861	1.5901	1.3048	1.5568	2
59	1.2519	1.6623	1.2687	1.6249	1.2864	1.5896	1.3051	1.5563	1
60	1.2521	1.6616	1.2690	1.6243	1.2867	1.5890	1.3054	1.5557	0
′	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	53°		52°		51°		50°		

TABLE IV

Secants and Cosecants, Natural

′	40°		41°		42°		43°		′
	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	
0	1.3054	1.5557	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	60
1	1.3057	1.5552	1.3253	1.5237	1.3460	1.4940	1.3677	1.4658	59
2	1.3060	1.5546	1.3257	1.5232	1.3463	1.4935	1.3681	1.4654	58
3	1.3064	1.5541	1.3260	1.5227	1.3467	1.4930	1.3684	1.4649	57
4	1.3067	1.5536	1.3263	1.5222	1.3470	1.4925	1.3688	1.4644	56
5	1.3070	1.5530	1.3267	1.5217	1.3474	1.4921	1.3692	1.4640	55
6	1.3073	1.5525	1.3270	1.5212	1.3477	1.4916	1.3695	1.4635	54
7	1.3076	1.5520	1.3274	1.5207	1.3481	1.4911	1.3699	1.4631	53
8	1.3080	1.5514	1.3277	1.5202	1.3485	1.4906	1.3703	1.4626	52
9	1.3083	1.5509	1.3280	1.5197	1.3488	1.4901	1.3707	1.4622	51
10	1.3086	1.5503	1.3284	1.5192	1.3492	1.4897	1.3710	1.4617	50
11	1.3089	1.5498	1.3287	1.5187	1.3495	1.4892	1.3714	1.4613	49
12	1.3092	1.5493	1.3290	1.5182	1.3499	1.4887	1.3718	1.4608	48
13	1.3096	1.5487	1.3294	1.5177	1.3502	1.4882	1.3722	1.4604	47
14	1.3099	1.5482	1.3297	1.5171	1.3506	1.4877	1.3725	1.4599	46
15	1.3102	1.5477	1.3301	1.5166	1.3509	1.4873	1.3729	1.4595	45
16	1.3105	1.5471	1.3304	1.5161	1.3513	1.4868	1.3733	1.4590	44
17	1.3109	1.5466	1.3307	1.5156	1.3517	1.4863	1.3737	1.4586	43
18	1.3112	1.5461	1.3311	1.5151	1.3520	1.4858	1.3740	1.4581	42
19	1.3115	1.5456	1.3314	1.5146	1.3524	1.4854	1.3744	1.4577	41
20	1.3118	1.5450	1.3318	1.5141	1.3527	1.4849	1.3748	1.4572	40
21	1.3121	1.5445	1.3321	1.5136	1.3531	1.4844	1.3752	1.4568	39
22	1.3125	1.5440	1.3324	1.5131	1.3534	1.4839	1.3756	1.4563	38
23	1.3128	1.5434	1.3328	1.5126	1.3538	1.4835	1.3759	1.4559	37
24	1.3131	1.5429	1.3331	1.5121	1.3542	1.4830	1.3763	1.4554	36
25	1.3134	1.5424	1.3335	1.5116	1.3545	1.4825	1.3767	1.4550	35
26	1.3138	1.5419	1.3338	1.5111	1.3549	1.4821	1.3771	1.4545	34
27	1.3141	1.5413	1.3342	1.5106	1.3552	1.4816	1.3774	1.4541	33
28	1.3144	1.5408	1.3345	1.5101	1.3556	1.4811	1.3778	1.4536	32
29	1.3148	1.5403	1.3348	1.5096	1.3560	1.4806	1.3782	1.4532	31
30	1.3151	1.5398	1.3352	1.5092	1.3563	1.4802	1.3786	1.4527	30
31	1.3154	1.5392	1.3355	1.5087	1.3567	1.4797	1.3790	1.4523	29
32	1.3157	1.5387	1.3359	1.5082	1.3571	1.4792	1.3794	1.4518	28
33	1.3161	1.5382	1.3362	1.5077	1.3574	1.4788	1.3797	1.4514	27
34	1.3164	1.5377	1.3366	1.5072	1.3578	1.4783	1.3801	1.4510	26
35	1.3167	1.5371	1.3369	1.5067	1.3581	1.4778	1.3805	1.4505	25
36	1.3170	1.5366	1.3372	1.5062	1.3585	1.4774	1.3809	1.4501	24
37	1.3174	1.5361	1.3376	1.5057	1.3589	1.4769	1.3813	1.4496	23
38	1.3177	1.5356	1.3379	1.5052	1.3592	1.4764	1.3816	1.4492	22
39	1.3180	1.5351	1.3383	1.5047	1.3596	1.4760	1.3820	1.4487	21
40	1.3184	1.5345	1.3386	1.5042	1.3600	1.4755	1.3824	1.4483	20
41	1.3187	1.5340	1.3390	1.5037	1.3603	1.4750	1.3828	1.4479	19
42	1.3190	1.5335	1.3393	1.5032	1.3607	1.4746	1.3832	1.4474	18
43	1.3193	1.5330	1.3397	1.5027	1.3611	1.4741	1.3836	1.4470	17
44	1.3197	1.5325	1.3400	1.5022	1.3614	1.4736	1.3839	1.4465	16
45	1.3200	1.5319	1.3404	1.5018	1.3618	1.4732	1.3843	1.4461	15
46	1.3203	1.5314	1.3407	1.5013	1.3622	1.4727	1.3847	1.4457	14
47	1.3207	1.5309	1.3411	1.5008	1.3625	1.4723	1.3851	1.4452	13
48	1.3210	1.5304	1.3414	1.5003	1.3629	1.4718	1.3855	1.4448	12
49	1.3213	1.5299	1.3418	1.4998	1.3633	1.4713	1.3859	1.4443	11
50	1.3217	1.5294	1.3421	1.4993	1.3636	1.4709	1.3863	1.4439	10
51	1.3220	1.5289	1.3425	1.4988	1.3640	1.4704	1.3867	1.4435	9
52	1.3223	1.5283	1.3428	1.4983	1.3644	1.4699	1.3870	1.4430	8
53	1.3227	1.5278	1.3432	1.4979	1.3647	1.4695	1.3874	1.4426	7
54	1.3230	1.5273	1.3435	1.4974	1.3651	1.4690	1.3878	1.4422	6
55	1.3233	1.5268	1.3439	1.4969	1.3655	1.4686	1.3882	1.4417	5
56	1.3237	1.5263	1.3442	1.4964	1.3658	1.4681	1.3886	1.4413	4
57	1.3240	1.5258	1.3446	1.4959	1.3662	1.4676	1.3890	1.4408	3
58	1.3243	1.5253	1.3449	1.4954	1.3666	1.4672	1.3894	1.4404	2
59	1.3247	1.5248	1.3453	1.4949	1.3669	1.4667	1.3898	1.4400	1
60	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	1.3902	1.4395	0
	Cosec	Sec	Cosec	Sec	Cosec	Sec	Cosec	Sec	′
	49°		48°		47°		46°		

TABLE IV

Secants and Cosecants, Natural

′	44°		′	′	44°		′	′	44°		′
	Sec	Cosec			Sec	Cosec			Sec	Cosec	
0	1.3902	1.4395	60	21	1.3984	1.4305	39	41	1.4065	1.4221	19
1	1.3905	1.4391	59	22	1.3988	1.4301	38	42	1.4069	1.4217	18
2	1.3909	1.4387	58	23	1.3992	1.4297	37	43	1.4073	1.4212	17
3	1.3913	1.4382	57	24	1.3996	1.4292	36	44	1.4077	1.4208	16
4	1.3917	1.4378	56	25	1.4000	1.4288	35	45	1.4081	1.4204	15
5	1.3921	1.4374	55	26	1.4004	1.4284	34	46	1.4085	1.4200	14
6	1.3925	1.4370	54	27	1.4008	1.4280	33	47	1.4089	1.4196	13
7	1.3929	1.4365	53	28	1.4012	1.4276	32	48	1.4093	1.4192	12
8	1.3933	1.4361	52	29	1.4016	1.4271	31	49	1.4097	1.4188	11
9	1.3937	1.4357	51	30	1.4020	1.4267	30	50	1.4101	1.4183	10
10	1.3941	1.4352	50								
				31	1.4024	1.4263	29	51	1.4105	1.4179	9
11	1.3945	1.4348	49	32	1.4028	1.4259	28	52	1.4109	1.4175	8
12	1.3949	1.4344	48	33	1.4032	1.4254	27	53	1.4113	1.4171	7
13	1.3953	1.4339	47	34	1.4036	1.4250	26	54	1.4117	1.4167	6
14	1.3957	1.4335	46	35	1.4040	1.4246	25	55	1.4122	1.4163	5
15	1.3960	1.4331	45	36	1.4044	1.4242	24	56	1.4126	1.4159	4
16	1.3964	1.4327	44	37	1.4048	1.4238	23	57	1.4130	1.4154	3
17	1.3968	1.4322	43	38	1.4052	1.4233	22	58	1.4134	1.4150	2
18	1.3972	1.4318	42	39	1.4056	1.4229	21	59	1.4138	1.4146	1
19	1.3976	1.4314	41	40	1.4060	1.4225	20	60	1.4142	1.4142	0
20	1.3980	1.4310	40								
′	Cosec	Sec	′	′	Cosec	Sec	′	′	Cosec	Sec	′
	45°				45°				45°		

TABLE V

Weights of Materials

Material	Average Weight in Pounds	
	Per Cubic Foot	Per Cubic Inch
Aluminum	160	.0924
Brass	512	.2960
Brick, pressed	150	
Brick, common, hard	125	
Cement, American, Rosendale	56	
Cement, Portland	90	
Clay, loose	63	
Coal, broken, loose, Anthracite	54	
Coal, broken, loose, Bituminous	49	
Concrete	154	
Copper	550	.3184
Earth, common loam	76	
Earth, packed	95	
Gravel, dry, loose	90 to 106	
Gravel, well shaken	99 to 117	
Gold	1206	.6975
Ice	58.7	
Iron, cast	450	.2600
Iron, wrought	480	.2778
Lead	710	.4109
Lime	53	
Masonry, well dressed	165	
Masonry, dry rubble	138	
Mortar, hardened	103	
Nickel	549	.3177
Quartz	165	
Sand, dry, loose	90 to 106	
Sand, well shaken	99 to 117	
Silver	657	.3802
Snow, freshly fallen	5 to 12	
Snow, wet and compacted	15 to 50	
Steel	490	.2835
Stone, gneiss	168	
Stone, granite	170	
Stone, limestone	168	
Stone, marble	168	
Stone, sandstone	151	
Stone, shale	162	
Stone, slate	175	
Tar	62	
Tin	455	.2632
Water	62.5	
Wood, dry, ash	38	
Wood, dry, cherry	42	
Wood, dry, chestnut	41	
Wood, dry, elm	35	
Wood, dry, hemlock	25	
Wood, dry, hickory	53	
Wood, dry, lignum vitae	83	
Wood, dry, mahogany	53	
Wood, dry, maple	49	
Wood, dry, oak, white	50	
Wood, dry, oak, other kinds	32 to 45	
Wood, dry, pine, white	25	
Wood, dry, pine, yellow	34 to 45	
Wood, dry, spruce	25	
Wood, dry, sycamore	37	
Wood, dry, walnut, black	38	
Zinc	438	.2528

TABLE VI

AMERICAN STANDARD COARSE-THREAD SERIES

Size	Threads per Inch	Basic Diameters		Cross-section at Root of Thread (Square Inches)	Tap Drill Size	
		Major Diam. (Inches)	Minor Diam. (Inches)		Commercial Size $\frac{3}{4}$ Depth of Thread	$\frac{5}{6}$ Depth of Thread
1	64	0.073	0.0527	0.0022	53	0.0561
2	56	.086	.0628	.0031	50	.0667
3	48	.099	.0719	.0041	47	.0764
4	40	.112	.0795	.0050	43	.0849
5	40	.125	.0925	.0067	38	.0979
6	32	.138	.0974	.0075	36	.1042
8	32	.164	.1234	.0120	29	.1302
10	24	.190	.1359	.0145	25	.1449
12	24	.216	.1619	.0206	16	.1709
$\frac{1}{4}$	20	.2500	.1850	.0269	7	.1959
$\frac{5}{16}$	18	.3125	.2403	.0454	F	.2524
$\frac{3}{8}$	16	.3750	.2938	.0678	$\frac{5}{16}$	.3073
$\frac{7}{16}$	14	.4375	.3447	.0933	U	.3602
$\frac{1}{2}$	13	.5000	.4001	.1257	$\frac{27}{64}$	.4167
$\frac{9}{16}$	12	.5625	.4542	.1620	$\frac{31}{64}$	.4723
$\frac{5}{8}$	11	.6250	.5069	.2018	$\frac{17}{32}$	.5266
$\frac{3}{4}$	10	.7500	.6201	.3020	$\frac{21}{32}$	.6417
$\frac{7}{8}$	9	.8750	.7307	.4193	$\frac{49}{64}$	.7547
1	8	1.0000	.8376	.5510	$\frac{7}{8}$	.8647
$1\frac{1}{8}$	7	1.1250	.9394	.6931	$\frac{63}{64}$	.9704
$1\frac{1}{4}$	7	1.2500	1.0644	.8898	$1\frac{7}{64}$	1.0954
$1\frac{3}{8}$	6	1.3750	1.1585	1.0541	$1\frac{7}{32}$	1.1946
$1\frac{1}{2}$	6	1.5000	1.2835	1.2938	$1\frac{11}{32}$	1.3196
$1\frac{3}{4}$	5	1.7500	1.4902	1.7441	$1\frac{9}{16}$	1.5335
2	$4\frac{1}{2}$	2.0000	1.7113	2.3001	$1\frac{25}{32}$	1.7594
$2\frac{1}{4}$	$4\frac{1}{2}$	2.2500	1.9613	3.0212	$2\frac{1}{32}$	2.0094
$2\frac{1}{2}$	4	2.5000	2.1752	3.7161	$2\frac{1}{4}$	2.2294
$2\frac{3}{4}$	4	2.7500	2.4252	4.6194	$2\frac{1}{2}$	2.4794
3	4	3.0000	2.6752	5.6209	$2\frac{23}{32}$	2.7294
$3\frac{1}{4}$	4	3.2500	2.9252	6.7205	$2\frac{31}{32}$	2.9794
$3\frac{1}{2}$	4	3.5000	3.1752	7.9183	$3\frac{7}{32}$	3.2294
$3\frac{3}{4}$	4	3.7500	3.4252	9.2143	$3\frac{15}{32}$	3.4794
4	4	4.0000	3.6752	10.6084	$3\frac{23}{32}$	3.7294

TABLE VII

AMERICAN STANDARD FINE-THREAD SERIES

Size	Threads per Inch	Basic Diameters		Cross-section at Root of Thread (Square Inches)	Tap Drill Size	
		Major (Inches)	Minor (Inches)		Commercial Size $\frac{3}{4}$ Depth of Thread	$\frac{5}{6}$ Depth of Thread
0	80	0.060	0.0438	0.0015	$\frac{3}{64}$	0.0465
1	72	.073	.0550	.0024	53	.0580
2	64	.086	.0657	.0034	50	.0691
3	56	.099	.0758	.0045	45	.0797
4	48	.112	.0849	.0057	42	.0894
5	44	.125	.0955	.0072	37	.1004
6	40	.138	.1055	.0087	33	.1109
8	36	.164	.1279	.0128	29	.1339
10	32	.190	.1494	.0175	21	.1562
12	28	.216	.1696	.0226	14	.1773
$\frac{1}{4}$	28	.2500	.2036	.0326	3	.2113
$\frac{5}{16}$	24	.3125	.2584	.0524	1	.2674
$\frac{3}{8}$	24	.3750	.3209	.0809	Q	.3299
$\frac{7}{16}$	20	.4375	.3725	.1090	$\frac{25}{64}$	.3834
$\frac{1}{2}$	20	.5000	.4350	.1486	$\frac{29}{64}$	.4459
$\frac{9}{16}$	18	.5625	.4903	.1888	$\frac{33}{64}$	.5024
$\frac{5}{8}$	18	.6250	.5528	.2400	$\frac{37}{64}$	.5649
$\frac{3}{4}$	16	.7500	.6688	.3513	$\frac{11}{16}$	.6823
$\frac{7}{8}$	14	.8750	.7822	.4805	$\frac{13}{16}$	.7977
1	14	1.0000	.9072	.6464	$\frac{15}{16}$	.9227
$1\frac{1}{8}$	12	1.1250	1.0167	.8118	$1\frac{3}{64}$	1.0348
$1\frac{1}{4}$	12	1.2500	1.1417	1.0238	$1\frac{11}{64}$	1.1598
$1\frac{3}{8}$	12	1.3750	1.2667	1.2602	$1\frac{19}{64}$	1.2848
$1\frac{1}{2}$	12	1.5000	1.3917	1.5212	$1\frac{27}{64}$	1.4098

TABLE VIII

Metric Conversion Tables

Metric Measures of Length

10 millimeters (symbol mm.)	= 1 centimeter	(symbol cm.)
10 centimeters	= 1 decimeter	(symbol dm.)
10 decimeters	= 1 meter	(symbol m.)
10 meters	= 1 dekameter	(symbol Dm.)
10 dekameters	= 1 hektometer	(symbol Hm.)
10 hektometers	= 1 kilometer	(symbol Km.)

Conversion Tables

1 meter = 39.37 inches = 3.28083 feet = 1.0936 yards
1 centimeter = .3937 inch
1 millimeter = .03937 inch = $\frac{1}{25}$ inch, approximately
1 kilometer = .62137 mile
1 foot = .3048 meter
1 inch = 2.54 centimeters = 25.4 millimeters

Table for Converting Millimeters to Inches and Decimals

mm.	inches	mm.	inches	mm.	inches	mm.	inches
1	.03937	26	1.02362	51	2.00787	76	2.99212
2	.07874	27	1.06299	52	2.04724	77	3.03149
3	.11811	28	1.10236	53	2.08661	78	3.07086
4	.15748	29	1.14173	54	2.12598	79	3.11023
5	.19685	30	1.18110	55	2.16535	80	3.14960
6	.23622	31	1.22047	56	2.20472	81	3.18897
7	.27559	32	1.25984	57	2.24409	82	3.22834
8	.31496	33	1.29921	58	2.28346	83	3.26771
9	.35433	34	1.33858	59	2.32283	84	3.30708
10	.39370	35	1.37795	60	2.36220	85	3.34645
11	.43307	36	1.41732	61	2.40157	86	3.38582
12	.47244	37	1.45669	62	2.44094	87	3.42519
13	.51181	38	1.49606	63	2.48031	88	3.46456
14	.55118	39	1.53543	64	2.51968	89	3.50393
15	.59055	40	1.57480	65	2.55905	90	3.54330
16	.62992	41	1.61417	66	2.59842	91	3.58267
17	.66929	42	1.65354	67	2.63779	92	3.62204
18	.70866	43	1.69291	68	2.67716	93	3.66141
19	.74803	44	1.73228	69	2.71653	94	3.70078
20	.78740	45	1.77165	70	2.75590	95	3.74015
21	.82677	46	1.81102	71	2.79527	96	3.77952
22	.86614	47	1.85039	72	2.83464	97	3.81889
23	.90551	48	1.88976	73	2.87401	98	3.85826
24	.94488	49	1.92913	74	2.91338	99	3.89763
25	.98425	50	1.96850	75	2.95275	100	3.93700

TABLE IX

DECIMAL AND METRIC EQUIVALENTS OF FRACTIONS OF AN INCH

Fraction	32ds	64ths	Decimal	Millimeters	Fraction	32ds	64ths	Decimal	Millimeters
		1	.015625	0.3968			33	.515625	13.0966
	1	2	.03125	0.7937		17	34	.53125	13.4934
		3	.046875	1.1906			35	.546875	13.8903
1/16	2	4	.0625	1.5875	9/16	18	36	.5625	14.2872
		5	.078125	1.9843			37	.578125	14.6841
	3	6	.09375	2.3812		19	38	.59375	15.0809
		7	.109375	2.7780			39	.609375	15.4778
1/8	4	8	.125	3.1749	5/8	20	40	.625	15.8747
		9	.140625	3.5718			41	.640625	16.2715
	5	10	.15625	3.9686		21	42	.65625	16.6684
		11	.171875	4.3655			43	.671875	17.0653
3/16	6	12	.1875	4.7624	11/16	22	44	.6875	17.4621
		13	.203125	5.1592			45	.703125	17.8590
	7	14	.21875	5.5561		23	46	.71875	18.2559
		15	.234375	5.9530			47	.734375	18.6527
1/4	8	16	.25	6.3498	3/4	24	48	.75	19.0496
		17	.265625	6.7467			49	.765625	19.4465
	9	18	.28125	7.1436		25	50	.78125	19.8433
		19	.296875	7.5404			51	.796875	20.2402
5/16	10	20	.3125	7.9373	13/16	26	52	.8125	20.6371
		21	.328125	8.3342			53	.828125	21.0339
	11	22	.34375	8.7310		27	54	.84375	21.4308
		23	.359375	9.1279			55	.859375	21.8277
3/8	12	24	.375	9.5248	7/8	28	56	.875	22.2245
		25	.390625	9.9216			57	.890625	22.6214
	13	26	.40625	10.3185		29	58	.90625	23.0183
		27	.421875	10.7154			59	.921875	23.4151
7/16	14	28	.4375	11.1122	15/16	30	60	.9375	23.8120
		29	.453125	11.5091			61	.953125	24.2089
	15	30	.46875	11.9060		31	62	.96875	24.6057
		31	.484375	12.3029			63	.984375	25.0026
1/2	16	32	.5	12.6997	1	32	64	1.	25.3995

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